5

CONCEPTION AND DESIGN

A different paradigm: As every mechanical part, a composite part has to withstand loadings. In addition, the conception process has to extend over a range much larger than for a component made of "pre-established" material. In fact,

- For isotropic materials, the classical process of conception consists of selection of an existing material and then design of the piece.
- For a component made of composites, the designer "creates" the material based on the functional requirements. The designer chooses the reinforcement, the matrix, and the process for curing.

Following that the designer must define the component architecture, i.e., the arrangement and dimensions of plies, the representation of these on the designs, etc. These subjects are covered in this chapter.

5.1 DESIGN OF A COMPOSITE PIECE

The following characteristic properties always have to be kept in mind by the designer:

- Fiber orientation enables the optimization of the mechanical behavior along a specific direction.
- The material is elastic up to rupture. It **cannot yield** by local plastic deformation as can classical metallic materials.
- Fatigue resistance is excellent.

A Very Good Fatigue Resistance

The specific fatigue resistance is expressed by the ratio (σ/ρ), with ρ being the specific mass. For composite materials, this specific resistance is three times higher than for aluminum alloys and two times higher than that of high strength steel and titanium alloys because the fatigue resistance is equal to 90% of the static fracture strength for a composite, instead of 35% for aluminum alloys and 50% for steels and titanium alloys (see Figure 5.1).[1]

[1] See Section 5.4.4.

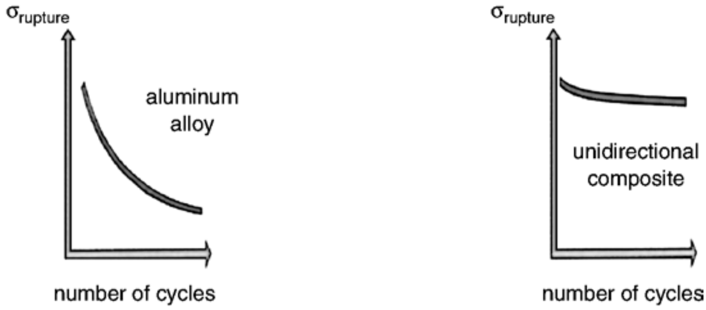


Figure 5.1 Comparison of Fatigue Behavior Between Composite and Aluminum

- The percent elongation is not the same as that for metals (attention should be paid to the metal/composite joints).
- Complex forms can be easily molded.
- It is possible to reduce the number of parts and to limit the amount of processing work.
- One must adapt the classical techniques of attachments and take into account their induced problems: fragility, delustering, fatigue, thermal stresses.

5.1.1 Guidelines for Values for Predesign

Figure 5.2 shows a comparison between different materials, which can help in the choice of composite in the predesign phase.

Figure 5.3 allows the comparison of principal specific properties of fibers which make up the plies. The specific modulus and specific strength are presented in the spirit of lightweight structural materials.

The safety factors are defined to take care of uncertainties on

- The magnitude of mechanical characteristics of reinforcement and matrix
- The stress concentrations
- The imperfection of the hypotheses for calculation
- The fabrication process
- The aging of materials

The orders of magnitude of safety factors are as follows:

High volume composites:		
Static loading	short duration:	2
	long duration:	4
Intermittent loading over long term:		4
Cyclic loading:		5
Impact loading:		10
High performance composites:		1.3 to 1.8

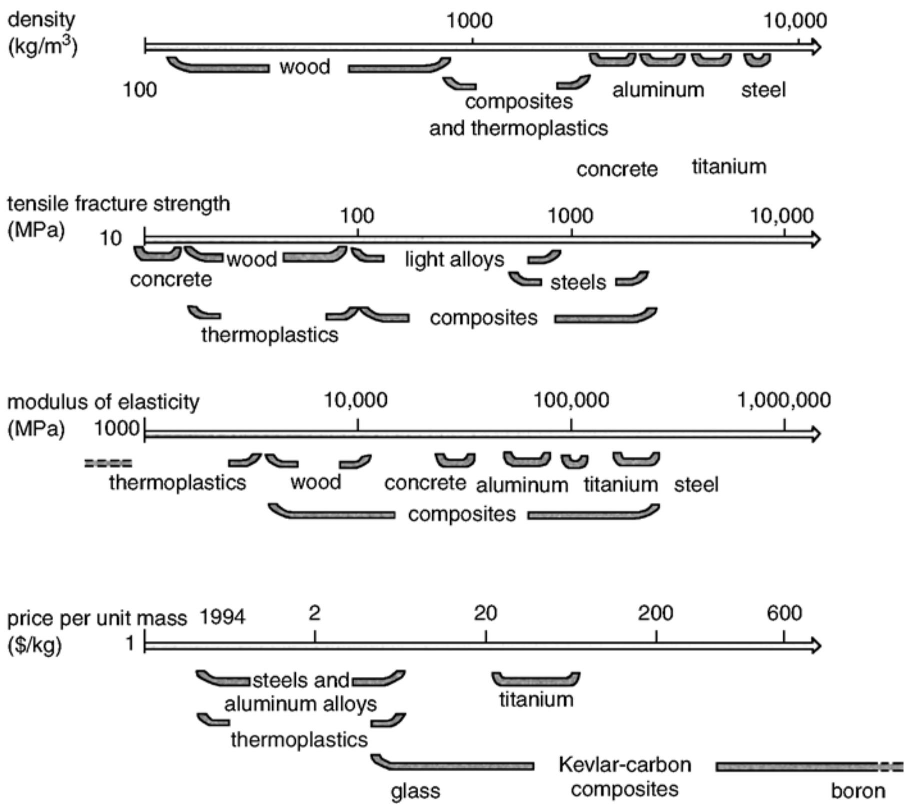


Figure 5.2 Comparison of Characteristics of Different Materials

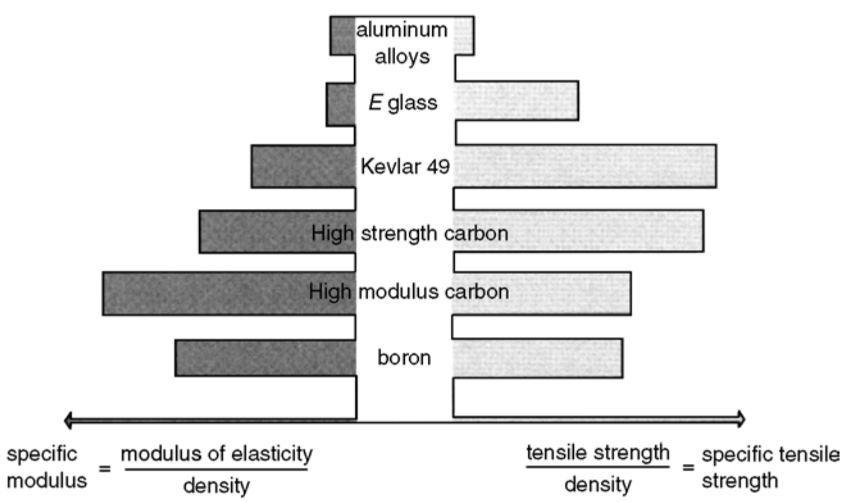


Figure 5.3 Specific Characteristics of Different Fibers

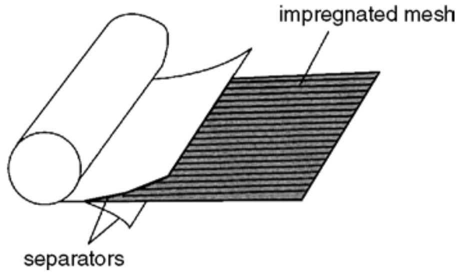


Figure 5.4 Unidirectional Layer

5.2 THE LAMINATE

Recall that laminates result in the superposition of many layers, or plies, or sheets, made of unidirectional layers, fabrics or mats, with proper orientations in each ply. This is the operation of **hand-lay-up**.

5.2.1 Unidirectional Layers and Fabrics

Unidirectional layers are as shown in Figure 5.4. The advantages of unidirectional layers are:

- They have high rigidity (maximum number of fibers in one direction).
- The ply can be used to wrap over long distance. Then the load transmission of the fibers is continuous over large distance.
- They have less waste.

The disadvantages of unidirectional layers are

- The time for wrapping is long.
- One cannot cover complex shapes using wrapping.

Example: Carbon/epoxy unidirectionals: Width 300 or 1000 mm, preimpregnated with resin; usable over a few years when stored at cold temperature (–18°C).

Fabrics can be found in rolls in dry form or impregnated with resin (Figure 5.5). The advantages of fabrics are

- Reduced wrapping time
- Possibility to shape complex form using the deformation of the fabric
- Possibility to combine different types of fibers in the same fabric

The disadvantages of fabrics are

- Lower modulus and strength than the case of unidirectionals
- Larger amount of waste material after cutting
- Requirement of joints when wrapping large parts

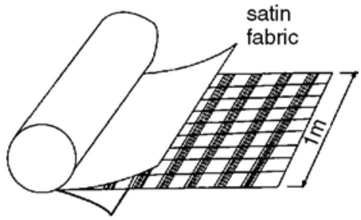
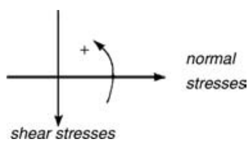


Figure 5.5 A Fabric Layer

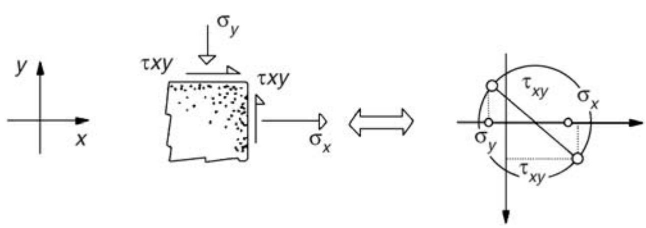
5.2.2 Importance of Ply Orientation

One of the fundamental advantages of laminates is their ability to adapt and control the orientation of fibers so that the material can best resist loadings. It is therefore important to know how the plies contribute to the laminate resistance, taking into account their relative orientation with respect to the loading direction. Figures 5.6 through 5.9 show the favorable situations and those that should be avoided.

Recall the Mohr circle:



cf. for example the stress state below and the associated Mohr circle



In Figure 5.7, the Mohr circle for stresses shows that the 45° fibers support the compression, $\sigma_1 = -\tau$ (τ is the arithmetic value of shear stress), while the resin supports the tension, $\sigma_2 = \tau$, with low fracture limit. The fibers in Figure 5.8 support the tension, $\sigma_1 = \tau$, whereas the resin supports the compression, $\sigma_2 = -\tau$. In Figure 5.9, one has deposited the fibers at 45° and –45°. Taking into account the previous remarks, one observes that the 45° fibers can support the tension $\sigma_1 = \tau$, whereas the –45° fibers can support the compression, $\sigma_2 = -\tau$. The resin is less loaded than previously.

5.2.3 Code to Represent a Laminate

5.2.3.1 Normalized Orientation

Considering the working mode of the plies as discussed in the previous section, the most frequently used orientations are represented as in Figure 5.10. The direction called "0°" corresponds to either the main loading direction, a preferred direction of the piece under consideration, or the axis of the chosen coordinates.

Note: One also finds in real applications plies with orientations ±30° and ±60°.

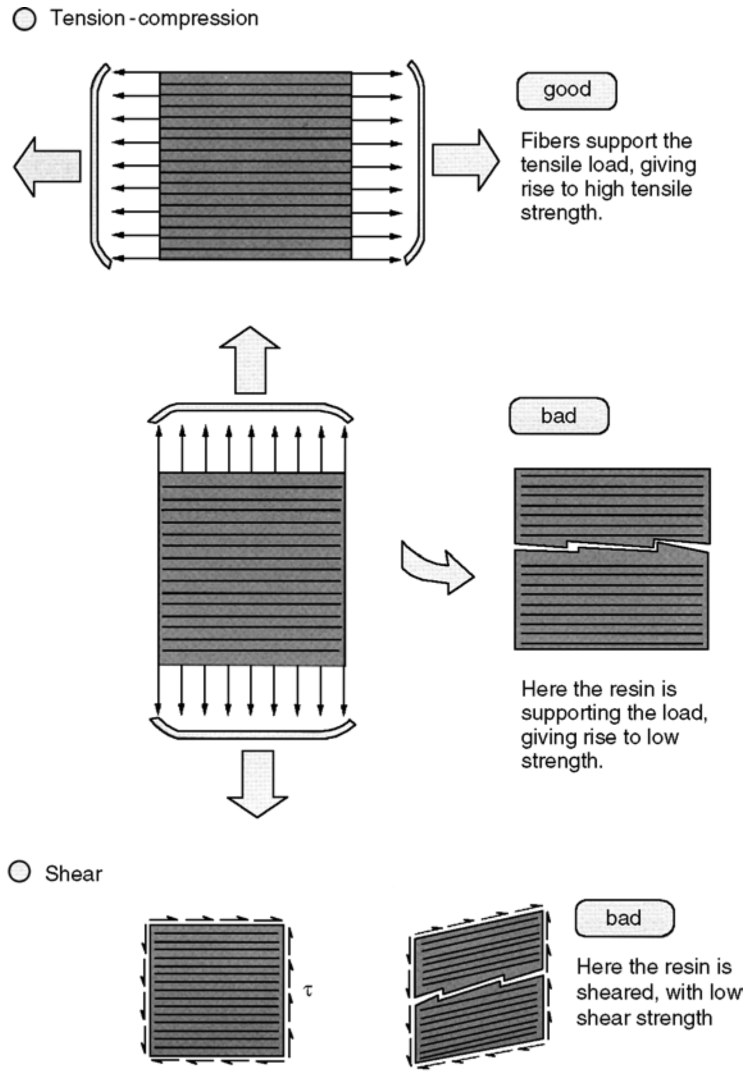


Figure 5.6 Effect of Ply Orientation

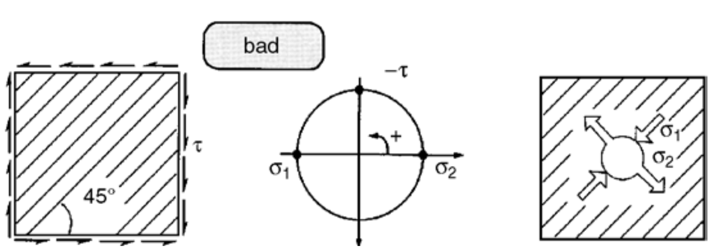


Figure 5.7 Bad Design

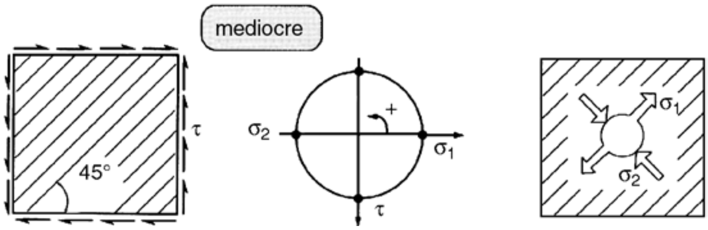


Figure 5.8 Mediocre Design

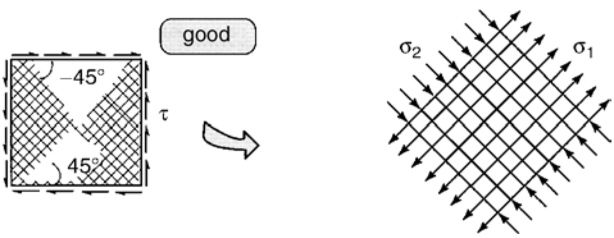


Figure 5.9 Good Design

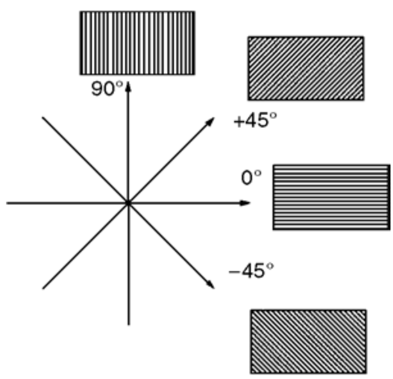


Figure 5.10 Common Orientations

5.2.3.2 Middle Plane

By definition the middle plane is the one that separates two half-thicknesses of the laminate. In Figure 5.11, the middle plane is the plane x–y. On this plane, $z = 0$.

5.2.3.3 Description of Plies

The description of plies is done by beginning with the lowest ply on the side $z < 0$ and proceeding to the uppermost ply of the side $z > 0$. In so doing,

- Each ply is noted by its orientation.
- The successive plies are separated by a slash "/".

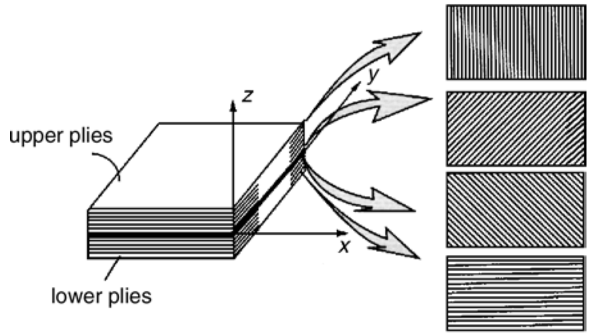


Figure 5.11 Laminate and its Middle Plane x–y

- One must avoid the grouping of too many plies of the same orientation.[2] However, when this occurs, an index number is used to indicate the number of these identical plies.

5.2.3.4 Midplane Symmetry

One notes that a laminate has midplane symmetry or is symmetric when the stacking of the plies on both sides starting from the middle plane is identical.

Example:

PLY NUMBER	ORIENTATION	CONVENTIONAL NOTATION	SYMBOL
10	90°		
9	0°		
8	0°		
7	– 45°		2
6 mid	+ 45°		2
5 plane	+ 45°	$[90/0_2/-45/45]_s$	10 → 4 (40%)
4	– 45°		2
3	0°		
2	0°		
1	90°		

Example:

PLY NUMBER	ORIENTATION	CONVENTIONAL NOTATION	SYMBOL
7	0°		
6	+45°		1
5	–45°		2
4 mid plane	90°	$[0/45/-45/\overline{90}]_s$	7 → 2 (28%)
3	–45°		2
2	+45°		
1	0°		

[2] This is to limit the interlaminar stresses (see Section 5.4.4 and Chapter 17). This precaution applies also to the fabrics (for example, no more than four consecutive fabric layers of carbon/epoxy along one direction).

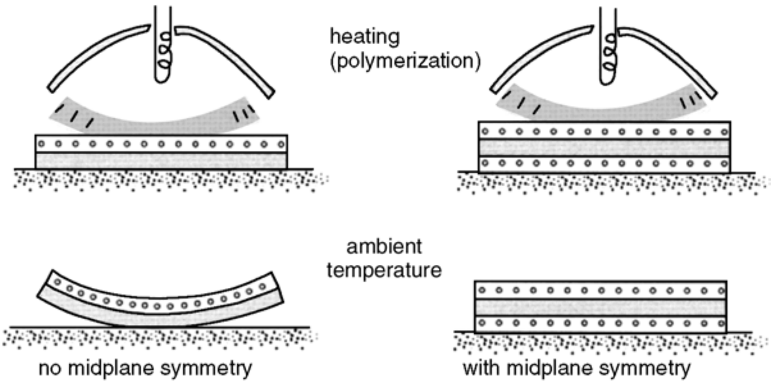


Figure 5.12 Effect of Laminate Lay-up on Deformation

5.2.3.4.1 What Is the Need of Midplane Symmetry

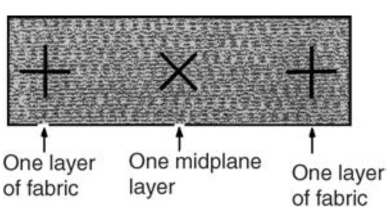
For the construction of laminated pieces, the successive impregnated plies are stacked at ambient temperature, then they are placed within an autoclave for curing. At high temperature, the extension of the whole laminate takes place without warping. However, during cooling, the plies have a tendency to contract differently depending on their orientations. From this, thermal residual stresses occur.

When midplane symmetry is utilized, it imposes the symmetry on these stresses and prevents the deformations of the whole part, for example, warping as shown in Figure 5.12.

5.2.3.5 Particular Cases of Balanced Fabrics

Some laminates are made partially or totally of layers of balanced fabric. One then needs to describe on the drawing the composition of the laminate.

Example:



The previous laminate, made up of three layers of balanced fabric, has midplane symmetry. In effect, if one considers one woven fabric layer as equivalent to two series of unidirectional layers crossed at 90°, it also has midplane symmetry.[3]

[3] If this hypothesis is to be verified for a plain weave or a taffeta (see Section 3.4.1), and even for a ribbed twill, it becomes worse as long as the pitch of the weaving machine increases (pitch of the plain weave: 2; ribbed twill: 3; 4-harness satin: 4; 5-harness satin: 5; etc.). If one supposes that this pitch is increasing towards infinity, then the woven fabric becomes the superposition of two unidirectional layers crossed at 90°. It then does not possess midplane symmetry any more. This property can be observed on a unique ply of 5-harness satin of carbon/epoxy as it is cured in an autoclave, which deforms (curved surface) on demolding (see Application 18.2.17).

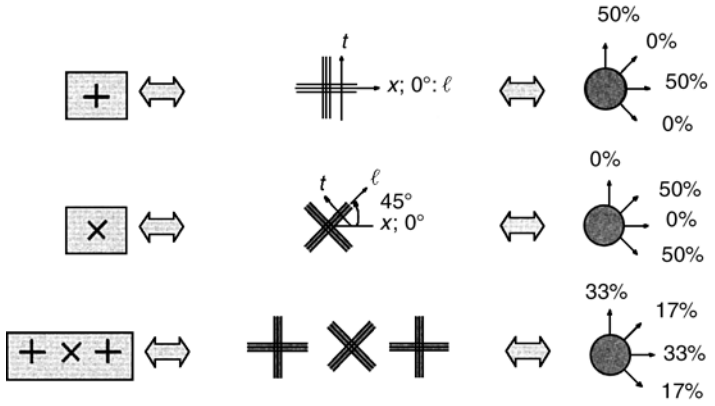


Figure 5.13 Laminate with Balanced Fabrics; Representation 1

As indicated in Section 3.4.2, one can consider the resulting laminate in two different ways[4]:

(a) Each layer of fabric is replaced by two identical plies crossed at 90°, each with thickness equal to half the thickness e of the fabric layer and each with known elastic properties. This representation is convenient for the determination of the elastic properties of the laminate. One then has the equivalencies shown in Figure 5.13.
(b) Each layer of fabric is replaced by one anisotropic ply with thickness e for which one knows the elastic properties and failure strengths. This representation is useful for the determination of the rupture stress of the laminate. One then has the equivalencies shown in Figure 5.14.

5.2.3.6 Technological Minimum

Generally one uses a minimum amount of plies (from 5 to 10%) for each direction: 0°, 90°, 45°, −45°. The minimum thickness of a laminate[5] should be of the order of one millimeter, for example, eight unidirectional layers, or three to four layers of balanced fabric of carbon/epoxy.

5.2.4 Arrangement of Plies

The proportion and the number of plies to place along each of the directions — 0°, 90°, 45°, −45°—take into account the mechanical loading that is applied to the laminate at the location under consideration. A current case consists of loading

[4] See Exercises 18.2.9 and 18.2.10.
[5] Apart from space applications, where thicknesses are very small, the skins of sandwich plates are laminates which do not have separately midplane symmetry.

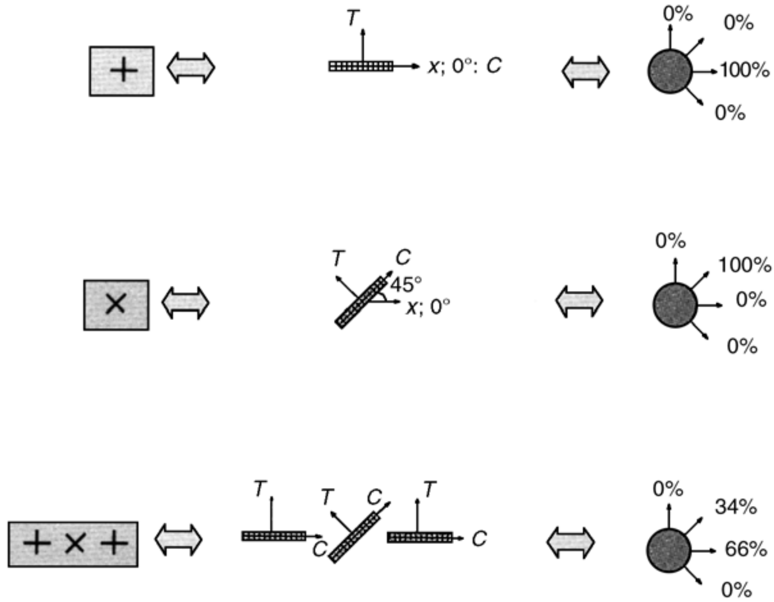


Figure 5.14 Laminate with Balanced Fabrics; Representation 2

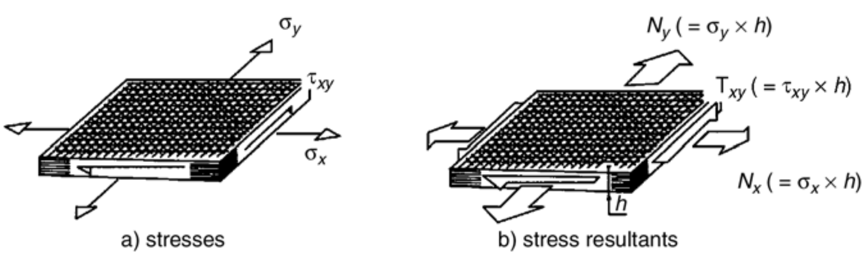


Figure 5.15 Stresses and Stress Resultants

of the laminate in its plane. This is called **membrane loading**.[6] The mechanical loadings can take the form of stresses (σ_x, σ_y, τ_{xy} in Figure 5.15a) or **stress resultants** (N_x, N_y, T_{xy} in Figure 5.15b). The stress resultants are the products of the stresses with the thickness h of the laminate.

Generally, three criteria should be considered by the designer for the ply configuration:

1. Support the loading without deterioration of the laminate
2. Limit the deformation of the loaded piece
3. Minimize the weight of the material used

These criteria do not always work together. For example, searching for minimum thickness might not be compatible with high rigidity. Searching for high rigidity

[6] The laminate can also work in bending. This is studied in Chapters 12 and 17.

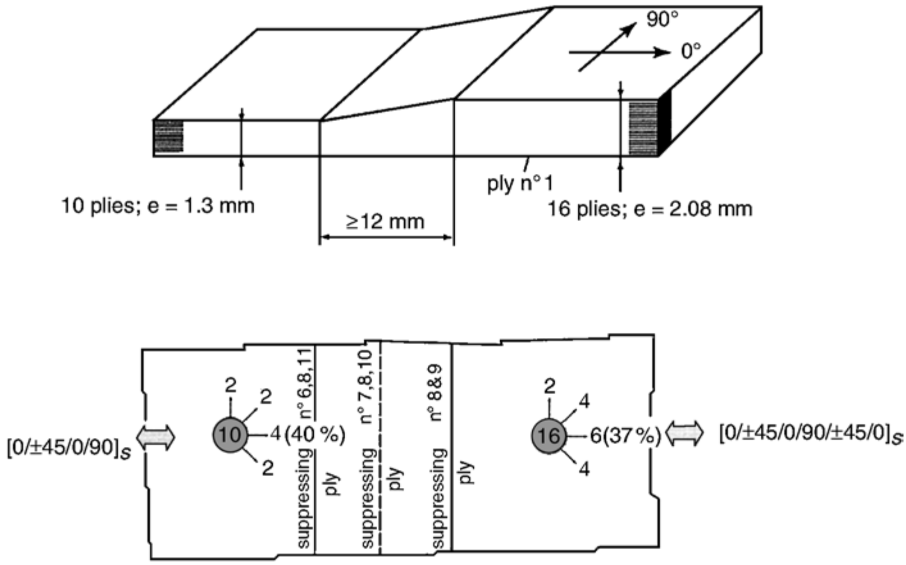


Figure 5.16 Example of Representation

might not be compatible with minimum weight. One will see in Section 5.4 guidelines for proportions values that allows a laminate with minimum laminate thickness to support specified mechanical loading without damage,.

Once a laminate is defined (number of layers and orientations), one must respect the following conditions (without forgetting the technological minimum indicated at the end of the previous paragraph) as much as possible:

- 90° plies placed on the surface, then 45° and −45° plies, when the predominant stress resultant is oriented along the 0° direction
- No more than 4 consecutive plies along the same direction

5.2.4.1 Example of Representation

The plies are progressively terminated to obtain a gradual change in thickness (maximum 2 plies for each 6 mm interval). The symbols for the composition of the laminate are shown on **plan** view (see Figure 5.16).

5.2.4.2 The Case of Sandwich Structure

The description of the sandwich material is done as in Figure 5.17.

5.3 FAILURE OF LAMINATES

5.3.1 Damages

Figure 5.18 shows schematically different types of failure leading to damage of a laminate.

The main modes of damage, when the loads exceed the critical limits, are illustrated in Figure 5.19.

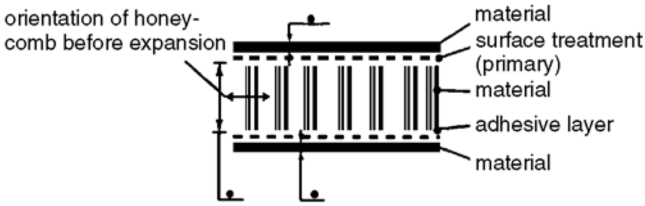


Figure 5.17 Description of a Sandwich Material

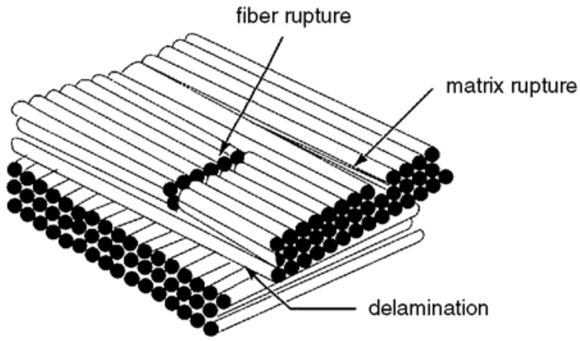


Figure 5.18 Different Modes of Failure

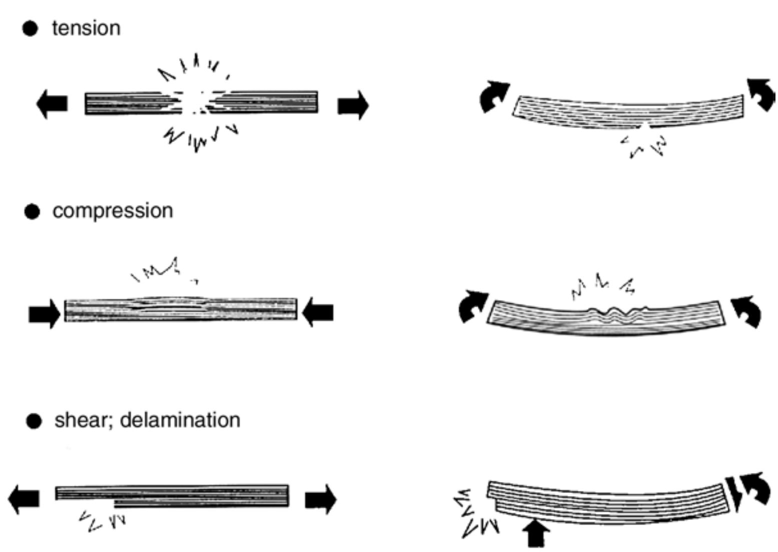


Figure 5.19 Modes of Damage

One cannot be satisfied with the classical maximum stress criterion

Figure 5.20 shows a unidirectional laminate loaded successively in two different manners. In the two cases, the maximum normal stress has the same value denoted

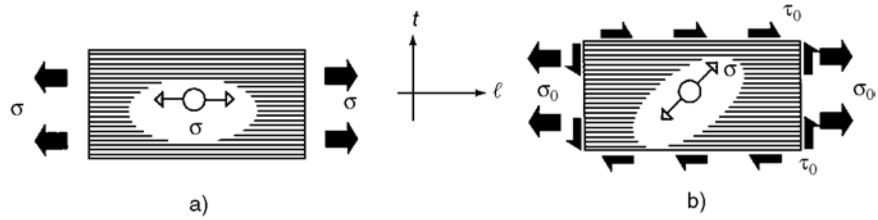


Figure 5.20 Stresses and Fiber Orientation

as σ. In the loading case (a), the unidirectional specimen will rupture when

$$\sigma > \sigma_{\text{rupture along } \ell}$$

This is the maximum stress criterion.

In the loading case (b), the maximum normal stress occurs in a direction that is different from that of the fibers (one can obtain this by tracing the Mohr's circle as discussed previously). We have seen (Section 3.3.2) that the rupture resistance decreases. It is weaker than the situation of case (a). The unidirectional laminate therefore ruptures when

$$\sigma < \sigma_{\text{rupture along } \ell}$$

This phenomenon is more evident if the unidirectional laminate is loaded in a direction transverse to the fibers t. In this case, the laminate rupture resistance is that of the matrix, which is much less than that of the fibers.

Taking into consideration the evolution of the rupture resistance with the loading direction, one can not use a simple maximum stress criterion as for the classical metallic materials.

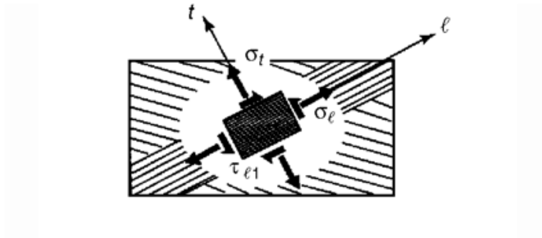
5.3.2 Most Frequently Used Criterion: Hill–Tsai Failure Criterion[7]

One can apply this criterion successively to **each ply** of the laminate, that is for each one of the orientations 0°, 90°, ±45° that have been considered. As has been discussed in Chapter 3, the axes of a unidirectional ply are denoted as ℓ for the direction along the fibers, and t for the transverse direction. The stresses are denoted as σ_ℓ in the fiber direction, σ_t in the direction transverse to the fibers, and $\tau_{\ell t}$ for the shear stress (see figure below).

One denotes the **Hill–Tsai number** (see Figure 5.21) the number α such that

- If $\alpha < 1$: no ply rupture occurs.
- If $\alpha \geq 1$: rupture occurs in the ply considered. Generally, this deterioration is due to the rupture of the resin. The mechanical properties of a broken ply become almost negligible, except for those along the fiber direction (modulus of elasticity and rupture resistance)

[7] For more detailed study of this criterion, see Chapter 14.



$$\alpha^2 = \left(\frac{\sigma_\ell}{\sigma_{\ell\,\text{rupture}}}\right)^2 + \left(\frac{\sigma_t}{\sigma_{t\,\text{rupture}}}\right)^2 - \frac{\sigma_\ell \sigma_t}{\sigma^2_{\ell\,\text{rupture}}} + \left(\frac{\tau_{\ell t}}{\tau_{\ell t\,\text{rupture}}}\right)^2$$

Figure 5.21 Hill–Tsai Number

5.3.2.1 Notes

Attention: The rupture resistance σ_{rupture} does not have the same value in tension and in compression (see, for example, Section 3.3.3). It is therefore useful to place in the denominators of the previous Hill–Tsai expression the rupture resistance values corresponding to the mode of loading (tension or compression) that appear in the numerator.

- Using this criterion, when one detects the rupture of one of the plies (more precisely the rupture of the plies along one of the four orientations), this does not necessarily lead to the rupture of the whole laminate. In most cases, the degraded laminate continues to resist the applied stress resultants. In increasing these stress resultants, one can detect which orientation can produce new rupture. This may—or may not—lead to complete rupture of the laminate. If complete rupture does not occur, one can still increase the admissible stress resultants.[8] In this way one can use a multiplication factor on the initial critical loading to indicate the ratio between the first ply rupture and the ultimate rupture.
- As a consequence of the previous remark it appears possible to work with a laminate that is partially degraded. It is up to the designer to consider the finality of the application, to decide whether the partially degraded laminate can be used.

One can make a parallel–in a gross way–with the situation of classical metallic alloys as represented in Figure 5.22.

5.3.2.2 How to Determine σ_ℓ, σ_t, $\tau_{\ell t}$ in Each Ply

Consider for example the laminate shown in Figure 5.23, consisting of identical plies. The following characteristics are known:

[8] See Exercise 18.2.7.

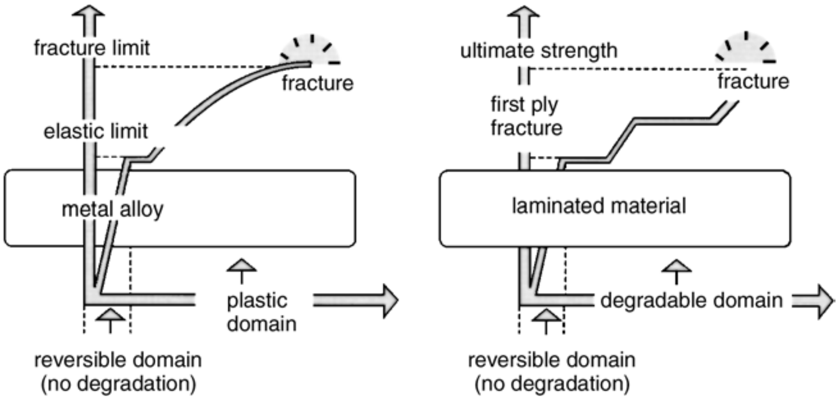


Figure 5.22 Comparison of Behavior until Failure Between Metal and Laminated Material

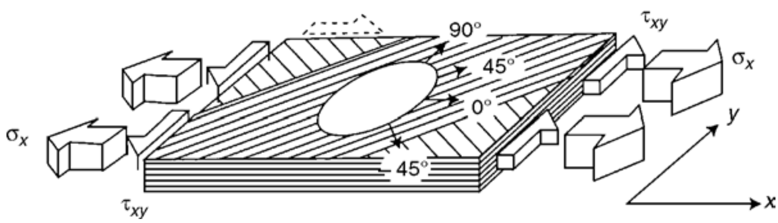


Figure 5.23 Average Stresses

- the mechanical properties of the basic ply
- the proportions (percentages) of plies in each of the directions (0°, 90°, 45°, –45°)
- the global values of the applied stresses, here, for example, σ_x and τ_{xy}

One considers this case of loading as consisting of the superposition of two simple loading cases: σ_x only, and then τ_{xy} only. For each of these cases of elementary loadings, one looks for the stresses σ_ℓ, σ_t, $\tau_{\ell t}$ in each ply. Manual calculation is usually too long.[9] It should be replaced by using a computer. Appendix 1 has tables for stress values for carbon/epoxy plies with 60% fiber volume fraction.

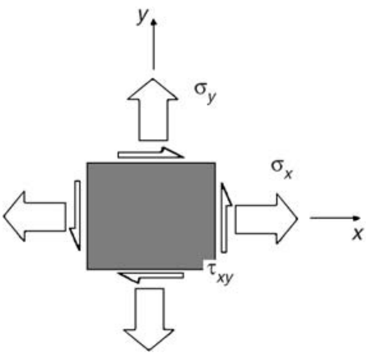
Subsequently, one finds, always **for each ply**, the sum of the stresses σ_ℓ, σ_t, and $\tau_{\ell t}$, respectively, due to each of the simple loadings σ_x and τ_{xy}. It is then possible to calculate the Hill–Tsai number to verify the integrity of each of the plies. In the Application 18.1.6, there is an example to determine the thickness of a laminate subject to this type of combined loading.

[9] The procedure for this calculation is described in Section 12.1.3.

5.4 SIZING OF THE LAMINATE

5.4.1 Modulus of Elasticity. Deformation of a Laminate

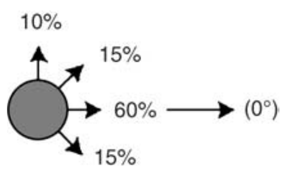
For the varied proportions of plies in the 0°, 90°, ±45°, the tables that follow allow the determination of the deformation of a laminated plate subject to the applied stresses. For this one uses a stress–strain relation similar to that described in Section 3.1 for an anisotropic plate, which is repeated below:



$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{yx}}{E_y} & 0 \\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & 0 \\ 0 & 0 & \frac{1}{G_{xy}} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \gamma_{xy} \end{Bmatrix}$$

E_x, E_y, G_{xy}, ν_{xy}, ν_{yx} are the modulus of elasticity and Poisson ratios of the laminate,[10] and ε_x, ε_y, γ_{xy} are normal and shear strains in the plane *xy*.

Example: What are the elastic moduli and thermal expansion coefficients for a glass/epoxy laminate (V_f = 60%) with the following ply configuration?



Answer: Table 5.14 indicates the following values for this laminate:

E_x = 33,100 MPa
E_y = 17,190 MPa (this value is obtained by permuting the proportions of 0° and 90°)
ν_{xy} = 0.34
ν_{yx} = 0.17

Table 5.15) shows G_{xy} = 6,980 MPa.

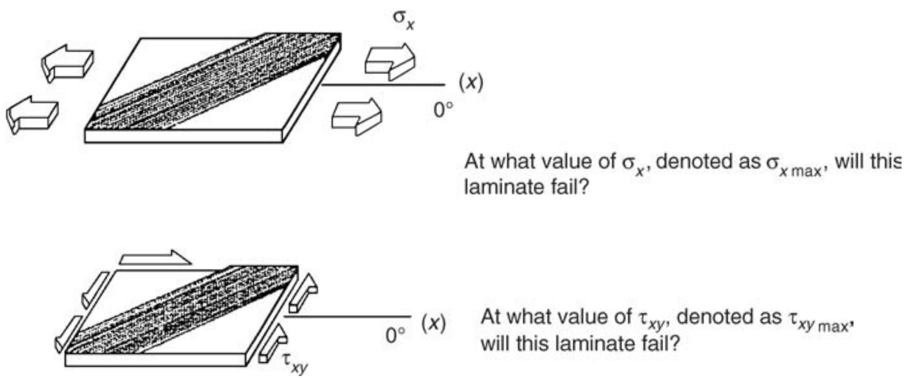
One then obtains the strains ε_x, ε_y, γ_{xy}, when the stresses are known, using the matrix relation mentioned above.

For the coefficient of thermal expansion, Table 5.14 shows: $\alpha_x = 0.64 \times 10^{-5}$ and $\alpha_y = 1.21 \times 10^{-5}$ by permuting the proportions of 0° and 90°.

[10] Recall (Sections 3.1 and 3.2) that $\upsilon_{xy}/E_x = \upsilon_{yx}/E_y$.

5.4.2 Case of Simple Loading

The laminate is subjected to only one single stress: σ_x or σ_y or τ_{xy}. Depending on the percentages of the plies in the four directions, one would like to know the order of magnitude of the stresses that can cause first ply failure in the laminate.

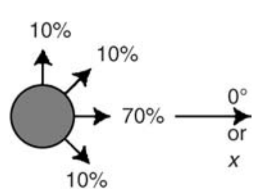


Tables 5.1 through 5.15 indicate the maximum stresses as well as the elastic characteristics and the coefficients of thermal expansion for the laminates having the following characteristics:

- Materials include **carbon, Kevlar, glass/epoxy** with V_f = 60% fiber volume fraction.
- All have identical plies (same unidirectionals, same thickness).
- The laminate is balanced (same number of 45° and −45° plies). The midplane symmetry is realized.
- The percentages of plies along the 4 directions 0°, 90°, ±45° vary in increments of 10%.

Calculation of the maximum stresses $\sigma_{x\,\max}$, $\sigma_{y\,\max}$, $\tau_{xy\,\max}$ is done based on the Hill–Tsai failure criterion.[11]

Example of how to use the tables:



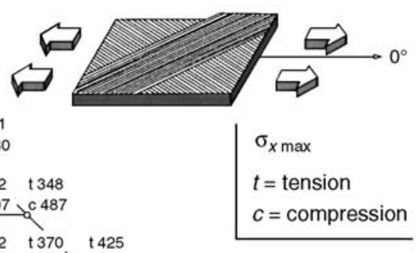
Which maximum tensile stress along the 0° direction can be applied to a Kevlar/epoxy laminate containing 60% fiber volume with the orientation distribution as shown in the above figure?

Answer: Table 5.6 indicates the maximum stress in the 0° direction (or x). For the percentages given, one has:

$$\sigma_{x\,\max(\text{tension})} = 308 \text{ MPa}$$

[11] See Application 18.2.2.

Table 5.1 Carbon/Epoxy Laminate: V_f = 60%, Ply Thickness = 0.13 mm

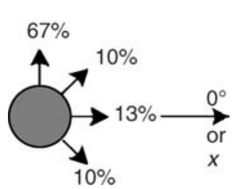


Percentage of 90° plies \ Percentage of 0° plies	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
100%	t 42 c 141										
90%	t 61 c 108	t 118 c 166									
80%	t 80 c 139	t 138 c 192	t 195 c 273								
70%	t 97 c 168	t 157 c 217	t 215 c 300	t 271 c 380							
60%	t 113 c 193	t 174 c 240	t 234 c 324	t 292 c 407	t 348 c 487						
50%	t 127 c 213	t 190 c 260	t 252 c 346	t 312 c 431	t 370 c 514	t 425 c 595					
40%	t 137 c 226	t 203 c 275	t 268 c 364	t 330 c 451	t 391 c 537	t 448 c 621	t 502 c 702				
30%	t 143 c 229	t 212 c 283	t 279 c 375	t 345 c 466	t 409 c 555	t 470 c 643	t 527 c 728	t 580 c 809			
20%	t 141 c 217	t 213 c 276	t 284 c 372	t 354 c 468	t 422 c 564	t 488 c 657	t 550 c 747	t 608 c 834	t 659 c 917		
10%	t 126 c 185	t 201 c 246	t 275 c 341	t 349 c 438	t 422 c 538	t 495 c 640	t 565 c 744	t 632 c 846	t 692 c 939	t 741 c 1024	
0%	t 123 c 125	t 223 c 182	t 324 c 265	t 426 c 351	t 529 c 438	t 633 c 530	t 740 c 627	t 848 c 733	t 957 c 851	t 1059 c 990	t 1270 c 1130

Percentage of ±45° plies (diagonals, from 0% of 0° plies): 100%, 90%, 80%, 70%, 60%, 50%, 40%, 30%, 20%, 10%, 0%

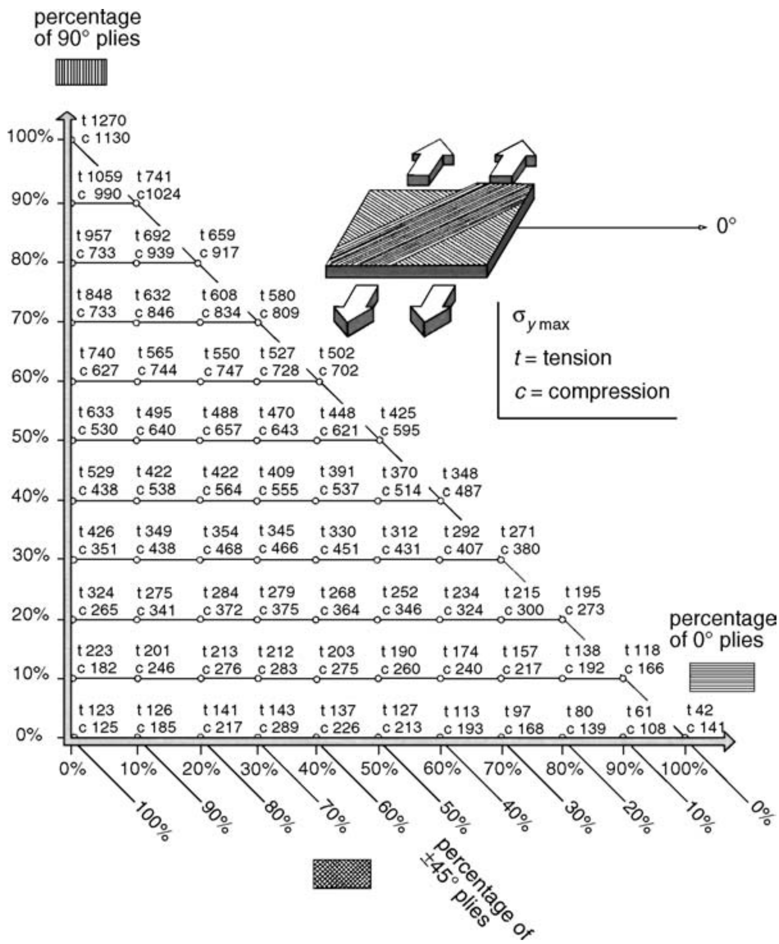
Maximum stress $\sigma_{x\,max}$ (MPa) as a function of the ply percentages in the directions 0°, 90°, +45°, –45°.
(More information on modulus and strength of a basic ply: see Section 3.3.3)

Example:



Which maximum compression stress along the 90°direction (or y) can be applied to a carbon/epoxy laminate containing 60% fiber volume fraction with the orientation distribution as shown in the above figure?

Table 5.2 Carbon/Epoxy Laminate: V_f = 60%, Ply Thickness = 0.13 mm



Maximum stress σ_{ymax} (MPa) as a function of the ply percentages in the directions 0°, 90°, +45°, −45°.
(More information on modulus and strength of a basic ply: see Section 3.3.3)

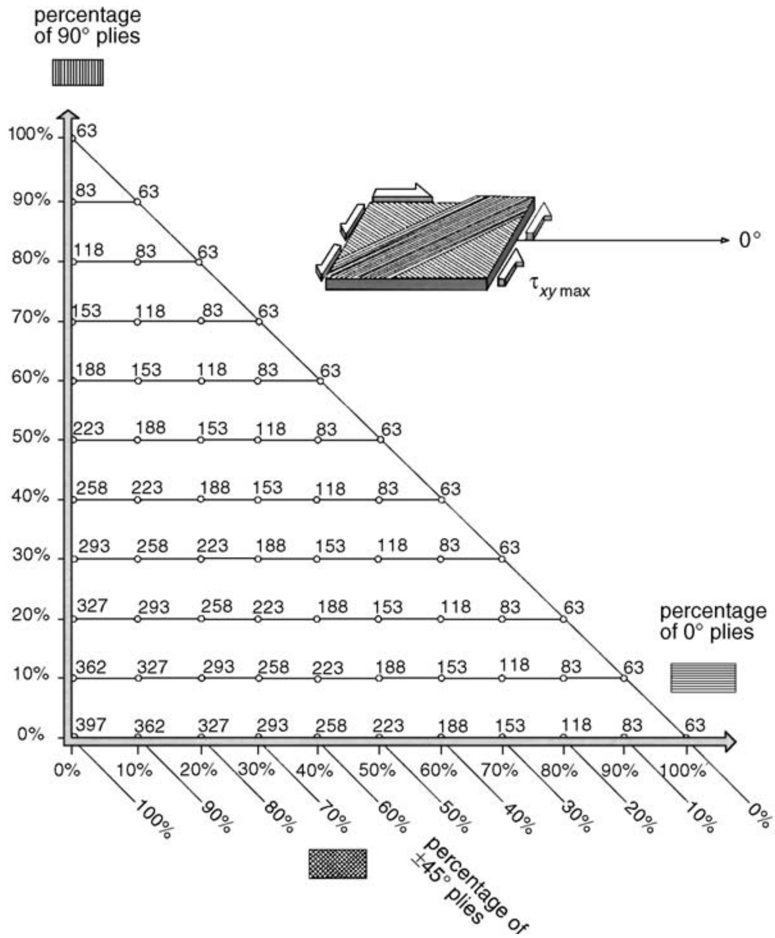
Answer: Table 5.2 shows the maximum stresses in the 90° direction. For this configuration, one has

$$\sigma_{y\max} = \sigma_{13/67/10/10} = \sigma_{10/60/15/15} + \Delta\sigma = 744 + \Delta\sigma$$

Denoting $p^{0°}$ and $p^{90°}$ as the proportions of the plies along the 0° and 90° directions, one has

$$\Delta\sigma = \frac{\partial\sigma}{\partial p^{0°}} \times \Delta p^{0°} + \frac{\partial\sigma}{\partial p^{90°}} \times \Delta p^{90°}$$

Table 5.3 Carbon/Epoxy Laminate: V_f = 60%, Ply Thickness = 0.13 mm



Maximum stress τ_{xymax} (MPa) as a function of the ply percentages in the directions 0°, 90°, +45°, −45°.
(More information on modulus and strength of a basic ply: see Section 3.3.3)

One obtains by linear interpolation:

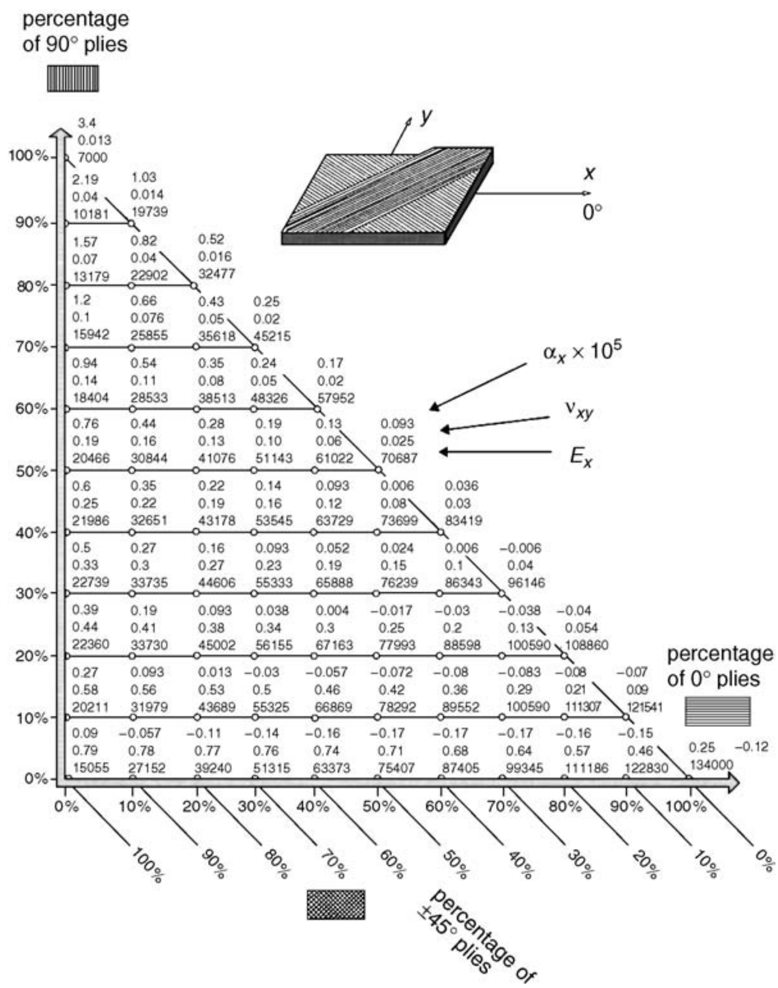
$$\Delta\sigma = (747 - 744) \times \frac{3}{10} + (846 - 744) \times \frac{7}{10} = 72 \text{ MPa}$$

Therefore,

$$\sigma_{y\max} = 744 + 72 = 816 \text{ MPa}$$

Remark: The plates that show the maximum stresses are not usable for the balanced fabrics. In effect, the compression strength values of a layer of balanced

Table 5.4 Carbon/Epoxy Laminate: V_f = 60%, Ply Thickness = 0.13 mm



Modulus E_x (MPa), Poisson ratio υ_{xy} and coefficient of thermal expansion α_x as a function of the ply percentages in the directions 0°, 90°, +45°, −45°.

(More information on modulus and strength of a basic ply: see Section 3.3.3)

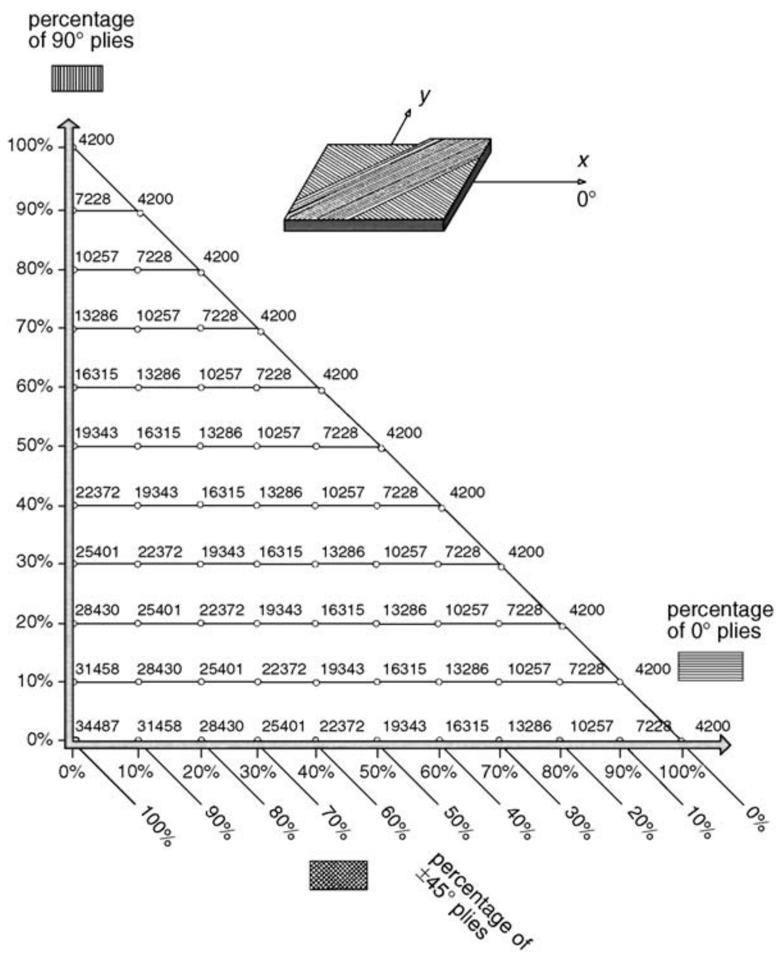
fabric are smaller than what is obtained when one superimposes the unidirectional plies crossed at 0° and 90° in equal quantities in these two directions.[12]

5.4.3 Case of Complex Loading—Approximate Orientation Distribution of a Laminate

When the normal and tangential loadings are applied **simultaneously** onto the laminate, the previous tables are not valid because they were established for the

[12] Also see remarks in Section 3.4.2.

Table 5.5 Carbon/Epoxy Laminate: V_f = 60%, Ply Thickness = 0.13 mm



Shear modulus G_{xy} (MPa) as a function of the ply percentages in the directions 0°, 90°, +45°, −45°.

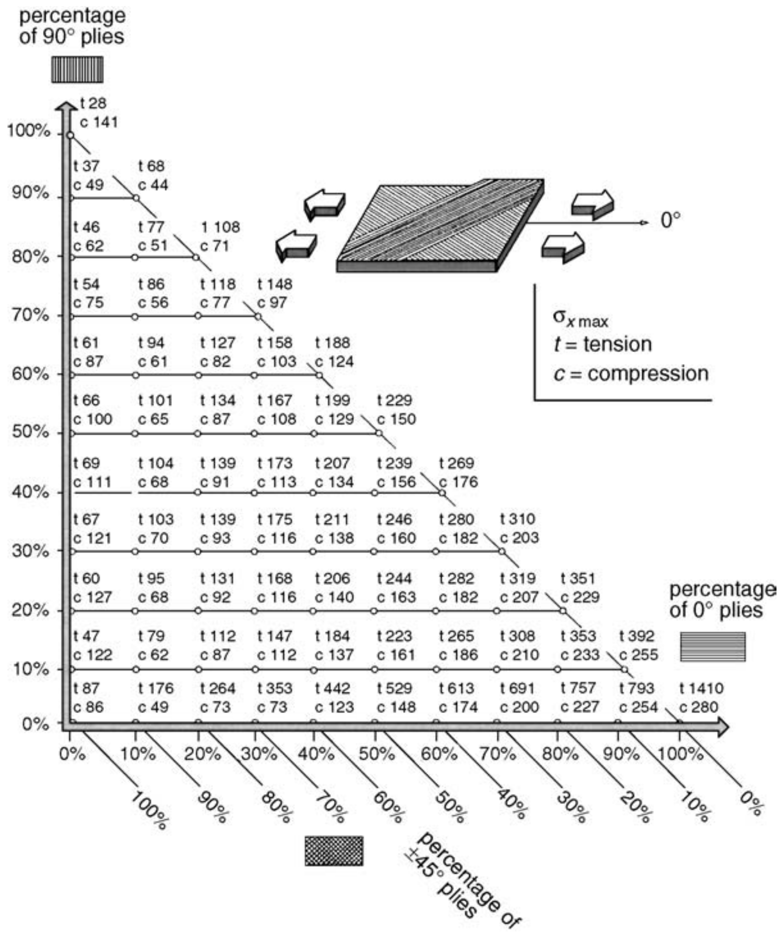
(More information on modulus and strength of a basic ply: see Section 3.3.3)

cases of simple stress states. However, one can still use them to effectively obtain a first estimate of the proportions of plies along the four orientations.[13]

The principle is as follows: Consider the case of complex loading and replacing the stresses with the stress resultants N_x, N_y, T_{xy} which were defined in Section 5.2.4. In general these stress resultants constitute the initial numerical data that are given by some previous studies. One then can assume that each one of the three stress resultants is associated with an appropriate orientation of the plies following the remarks made in Section 5.2.2.

[13] Attention: What follows is for the determination of *proportions*, and not *thicknesses*.

Table 5.6 Kevlar/Epoxy Laminate: V_f = 60%, Ply Thickness = 0.13 mm



Maximum stress $\sigma_{x\max}$ (MPa) as a function of the ply percentages in the directions 0°, 90°, +45°,−45°.
(More information on modulus and strength of a basic ply: see Section 3.3.3)

Using this hypothesis, N_x, assumed to be supported by the 0° plies (or along x), requires a thickness e_x for these plies such that:

$$e_x = \frac{N_x}{\sigma_{\ell\ \text{rupture}}}$$

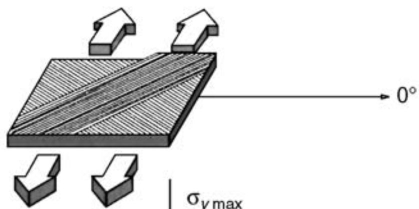
where $\sigma_{\ell\ \text{rupture}}$ is the rupture stress of a unidirectional ply in the long direction. In the same manner, N_y is supposed to be supported by the 90° plies (or along y), and requires a thickness for these plies of

$$e_y = \frac{N_y}{\sigma_{\ell\ \text{rupture}}}$$

Table 5.7 Kevlar/Epoxy Laminate: V_f = 60%, Ply Thickness = 0.13 mm

percentage of 90° plies \ percentage of 0° plies	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
100%	t 1410 c 280										
90%	t 793 c 254	t 392 c 255									
80%	t 757 c 227	t 353 c 233	t 351 c 229								
70%	t 691 c 200	t 308 c 210	t 319 c 207	t 310 c 203							
60%	t 613 c 174	t 265 c 186	t 282 c 185	t 280 c 182	t 269 c 150						
50%	t 529 c 148	t 223 c 161	t 244 c 163	t 246 c 160	t 239 c 156	t 229 c 150					
40%	t 442 c 123	t 184 c 137	t 206 c 140	t 211 c 138	t 207 c 134	t 199 c 129	t 148 c 124				
30%	t 353 c 98	t 147 c 112	t 168 c 116	t 175 c 116	t 173 c 113	t 167 c 108	t 158 c 103	t 148 c 97			
20%	t 264 c 73	t 112 c 87	t 131 c 92	t 139 c 93	t 139 c 91	t 134 c 87	t 127 c 82	t 118 c 77	t 108 c 71		
10%	t 176 c 49	t 79 c 62	t 95 c 68	t 103 c 70	t 104 c 70	t 101 c 65	t 94 c 61	t 86 c 56	t 77 c 51	t 68 c 44	
0%	t 87 c 86	t 47 c 122	t 60 c 127	t 67 c 121	t 69 c 111	t 66 c 100	t 61 c 87	t 54 c 75	t 46 c 62	t 37 c 49	t 28 c 141

Percentage of ±45° plies (diagonals, from the 0% column of 0° plies onward): 100%, 90%, 80%, 70%, 60%, 50%, 40%, 30%, 20%, 10%, 0%



$\sigma_{y\,max}$
t = tension
c = compression

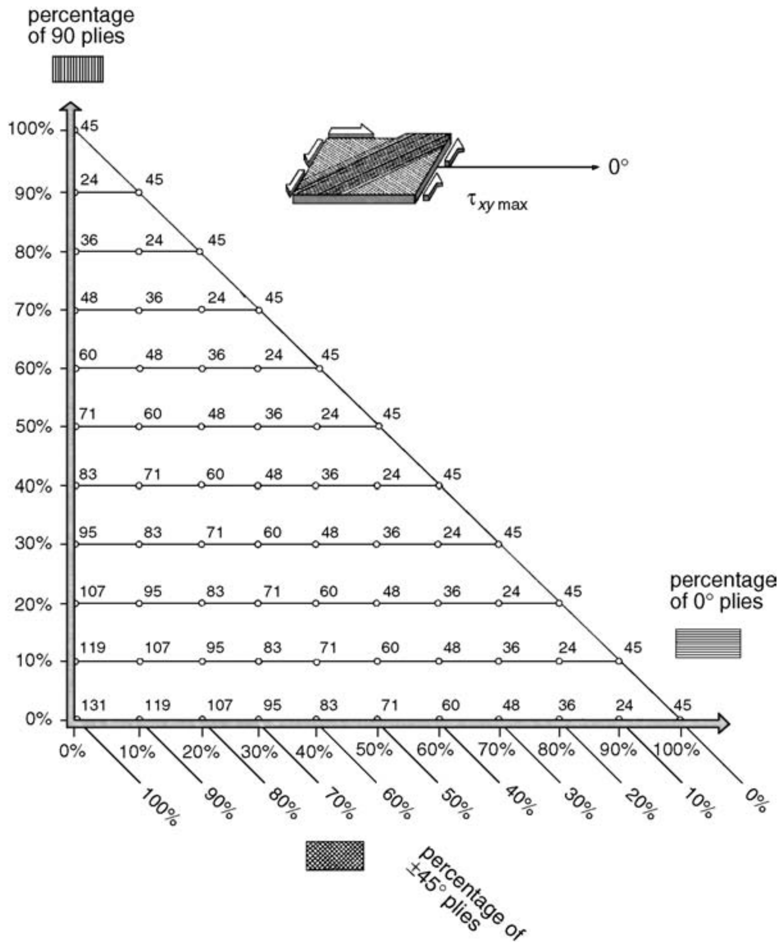
Maximum stress $\sigma_{y\max}$ (MPa) as a function of the ply percentages in the directions 0°, 90°, +45°, −45°.
(More information on modulus and strength of a basic ply: see Section 3.3.3)

Finally, T_{xy} is assumed to be supported by the ±45° plies and requires a thickness for these plies of

$$e_{xy} = \frac{T_{xy}}{\tau_{\text{rupture}}}$$

where τ_{rupture} is the maximum stress that a ±45° laminate can support.

Table 5.8 Kevlar/Epoxy Laminate: V_f = 60%, Ply Thickness = 0.13 mm



Maximum stress $\tau_{xy\,max}$ (MPa) as a function of the ply percentages in the directions 0°, 90°, +45°, −45°.
(More information on modulus and strength of a basic ply: see Section 3.3.3)

One then can retain for the complete laminate the proportions indicated below.

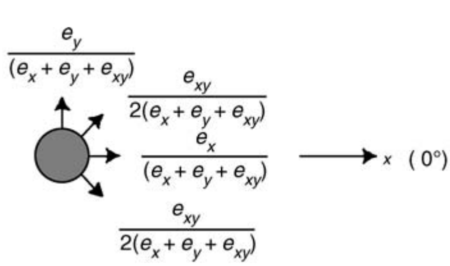
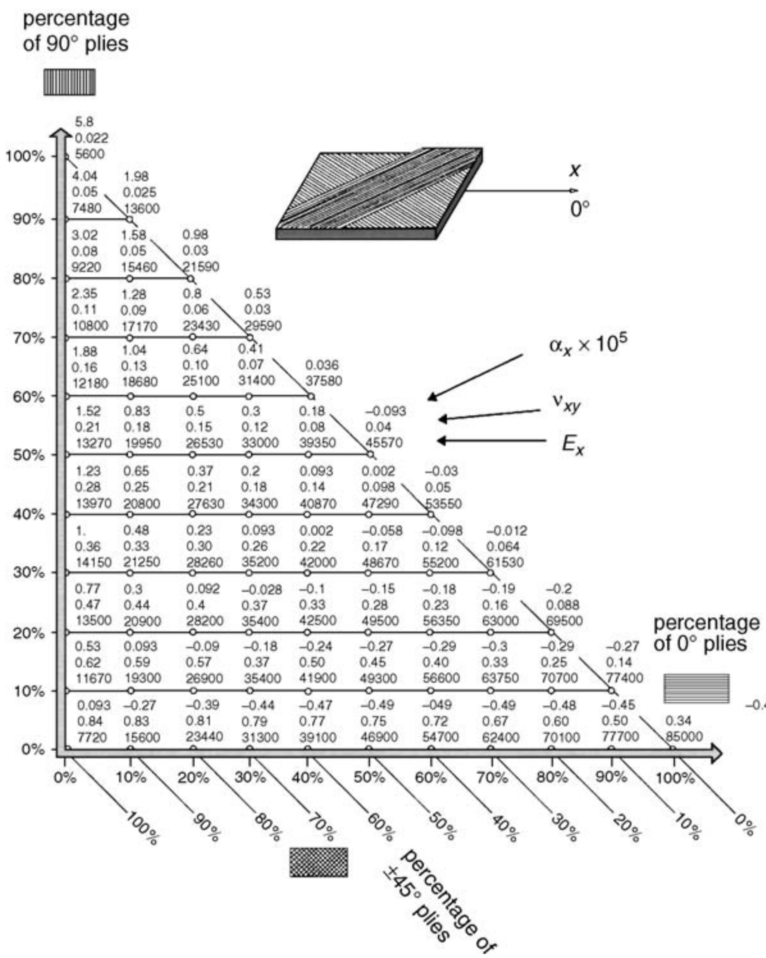


Table 5.9 Kevlar/Epoxy Laminate: V_f = 60%, Ply Thickness = 0.13 mm

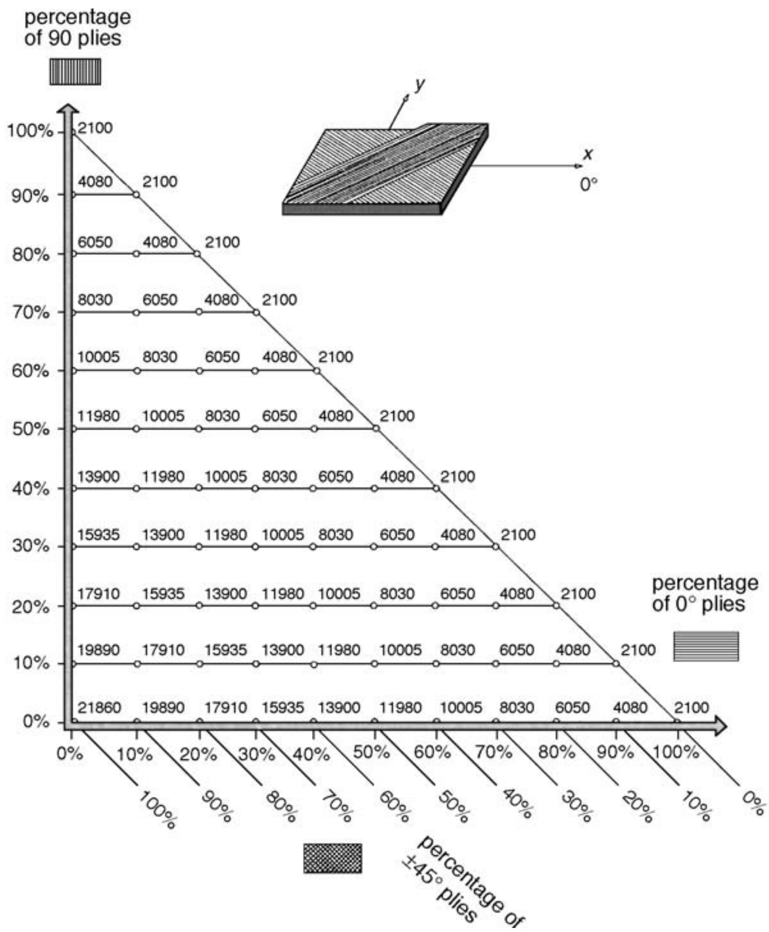


Longitudinal modulus E_x (MPa), Poisson ratio υ_{xy} and coefficient of thermal expansion α_x as a function of the ply percentages in the directions 0°, 90°, +45°, −45°.
(More information on modulus and strength of a basic ply: see Section 3.3.3)

Example: Determine the composition of a laminate made up of unidirectional plies of carbon/epoxy (V_f = 60%) to support the stress resultants N_x = –800 N/mm, N_y = −900 N/mm, T_{xy} = −300 N/mm. The compression strength $\sigma_{\ell\ \text{rupture}}$ is 1,130 MPa (see Section 3.3.3, or Table 5.1 for 100% of 0° plies). Then:

$$e_x = \frac{800}{1130} = 0.71 \text{ mm}; \quad e_y = \frac{900}{1130} = 0.8 \text{ mm}$$

Table 5.10 Kevlar/Epoxy Laminate: V_f = 60%, Ply Thickness = 0.13 mm



Shear modulus G_{xy} (MPa) as a function of the ply percentages in the directions 0°, 90°, +45°, −45°.
(More information on modulus and strength of a basic ply: see Section 3.3.3)

The optimum shear strength $\tau_{rupture}$ is given in Table 5.3 for 100% ±45°, then:

$$\tau_{rupture} = 397 \text{ MPa}$$

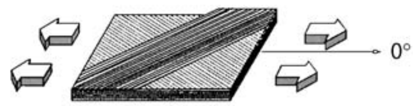
from which:

$$e_{xy} = \frac{340}{397} = 0.86 \text{ mm}$$

Table 5.11 Glass/Epoxy Laminate: V_f = 60%, Ply Thickness = 0.13 mm

percentage of 90° plies \ percentage of 0° plies	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
100%	t 35 c 141										
90%	t 37 c 128	t 45 c 156									
80%	t 39 c 134	t 47 c 162	t 54 c 190								
70%	t 41 c 139	t 49 c 167	t 57 c 195	t 64 c 223							
60%	t 43 c 143	t 51 c 171	t 59 c 200	t 67 c 228	t 75 c 257						
50%	t 45 c 145	t 53 c 174	t 61 c 203	t 69 c 232	t 77 c 261	t 85 c 290					
40%	t 46 c 146	t 55 c 175	t 64 c 204	t 72 c 234	t 80 c 263	t 88 c 293	t 95 c 322				
30%	t 48 c 144	t 57 c 174	t 66 c 203	t 74 c 233	t 82 c 263	t 90 c 293	t 98 c 323	t 106 c 354			
20%	t 48 c 140	t 58 c 170	t 67 c 200	t 76 c 230	t 85 c 260	t 93 c 290	t 101 c 321	t 109 c 352	t 116 c 384		
10%	t 48 c 133	t 58 c 162	t 68 c 192	t 77 c 222	t 87 c 252	t 96 c 283	t 104 c 314	t 112 c 346	t 120 c 378	t 128 c 411	
0%	t 94 c 122	t 114 c 150	t 134 c 179	t 152 c 209	t 170 c 239	t 186 c 269	t 201 c 300	t 214 c 332	t 226 c 365	t 235 c 399	t 1250 c 600

percentage of ±45° plies: 100%, 90%, 80%, 70%, 60%, 50%, 40%, 30%, 20%, 10%, 0%



$\sigma_{x\,max}$

t = tension

c = compression

Maximum stress $\sigma_{x\,max}$ (MPa) as a function of the ply percentages in the directions 0°, 90°, +45°, −45°.

(More information on modulus and strength of a basic ply: see Section 3.3.3)

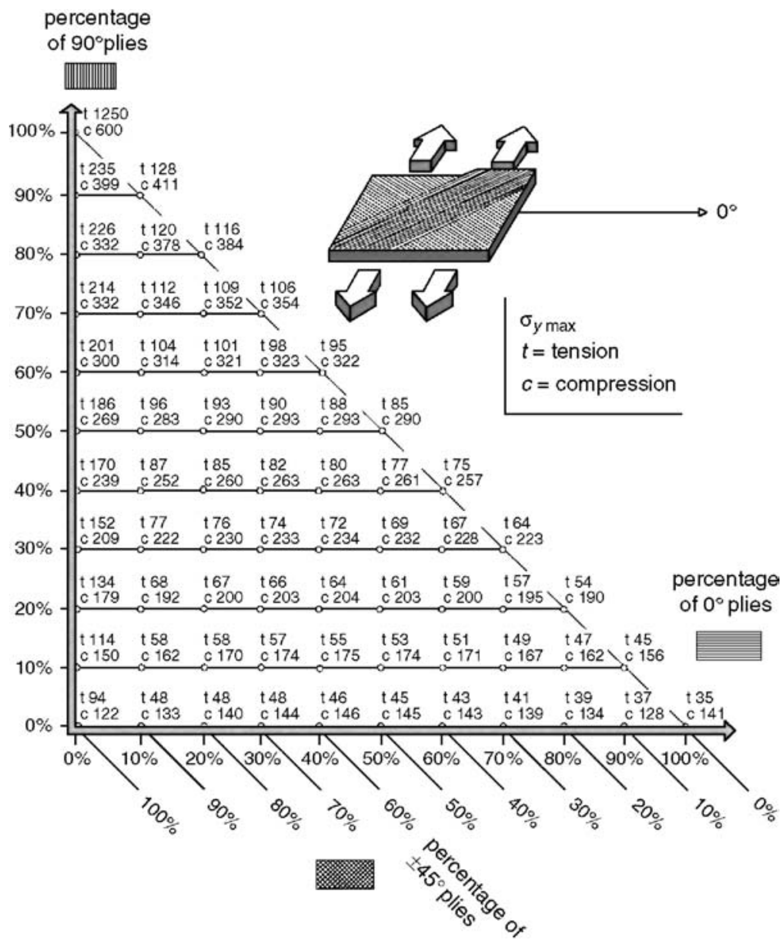
One obtains for the proportions at

$$0°: \quad \frac{e_x}{e_x + e_y + e_{xy}} = 0.3$$

$$90°: \quad \frac{e_y}{e_x + e_y + e_{xy}} = 0.34$$

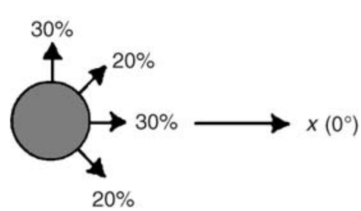
$$\pm 45°: \quad \frac{e_{xy}}{e_x + e_y + e_{xy}} = 0.36$$

Table 5.12 Glass/Epoxy Laminate: V_f = 60%, Ply Thickness = 0.13 mm



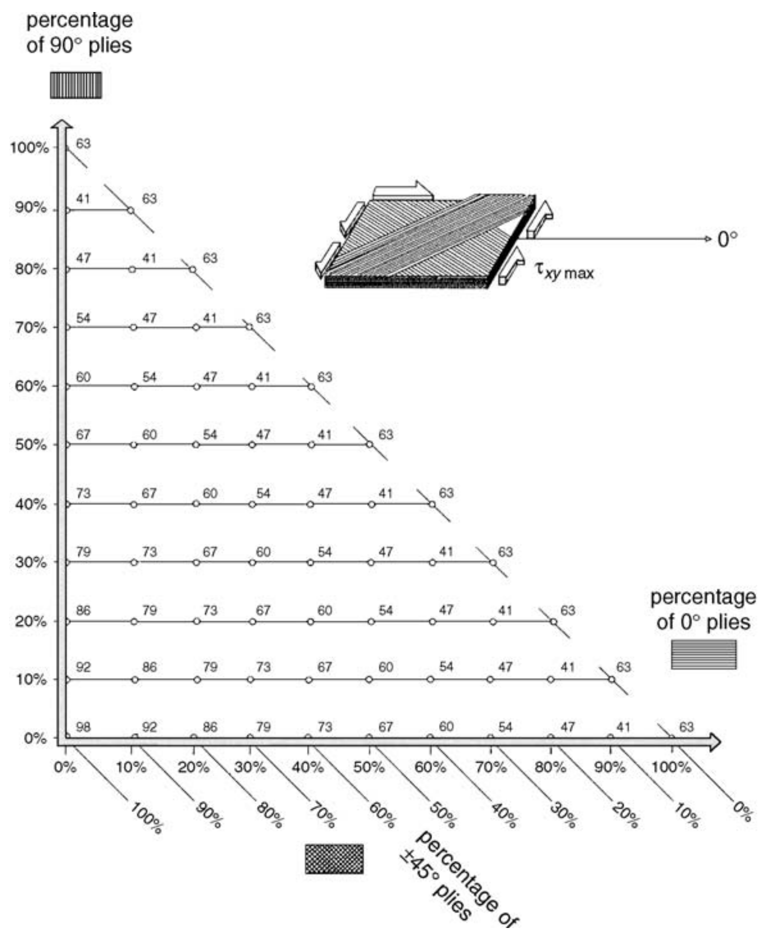
Maximum stress $\sigma_{y\ max}$ (MPa) as a function of the ply percentages in the directions 0°, 90°, +45°, −45°.
(More information on modulus and strength of a basic ply: see Section 3.3.3)

One can then retain for the composition of the laminate the following approximate values:



Remark: The thicknesses e_x, e_y, e_{xy} evaluated above only serve to determine the proportions. After that, they **should not be kept**. In effect each orientation really

Table 5.13 Glass/Epoxy Laminate: V_f = 60%, Ply Thickness = 0.13 mm

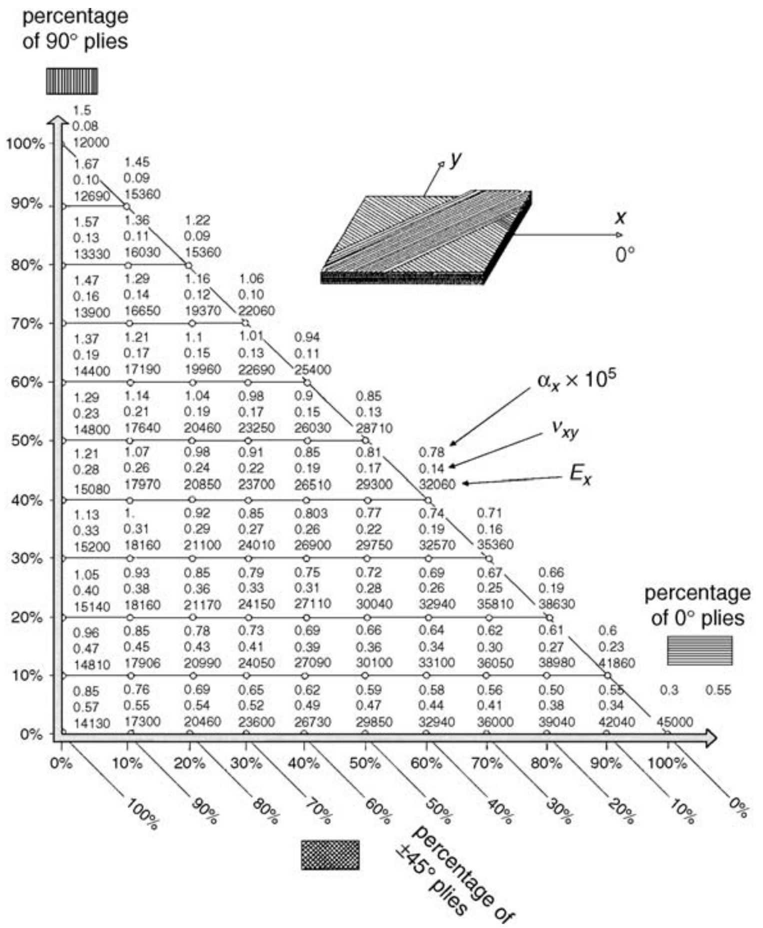


Maximum stress $\tau_{xy\,\max}$ (*MPa*) as a function of the ply percentages in the directions 0°, 90°, +45°, −45°.

(More information on modulus and strength of a basic ply: see Section 3.3.3)

supports a part of each stress resultant. For example, the 0° plies cover the major part of stress resultant N_x, but they also support a part of stress resultant N_y and a part of stress resultant T_{xy}. This then results in a more unfavorable situation for each orientation as compared with what has been assumed previously. The minimum necessary thickness of the laminate will in fact be larger than the previous result ($e_x + e_y + e_{xy}$), which therefore appears to be **dangerously optimistic**. The practical determination of the minimum thickness of the laminate is determined based on the Hill–Tsai failure criterion, as indicated at the end of Section 5.3.2, and explained in details in Application 18.1.6. Also, with the same stress resultants and proportions as in the previous example, one finds a minimum thickness of 2.64 mm (see Application 18.1.6, in Chapter 18), whereas the previous sum ($e_x + e_y + e_{xy}$) gives a thickness of 2.37 mm, 10% lower than the required minimum thickness (2.64 mm).

Table 5.14 Glass/Epoxy Laminate: V_f = 60%, Ply Thickness = 0.13 mm



Longitudinal modulus E_x (MPa), Poisson ratio υ_{xy}, and coefficient of thermal expansion α_x as a function of the ply percentages in the directions 0°, 90°, +45°, −45°.

(More information on modulus and strength of a basic ply: see Section 3.3.3)

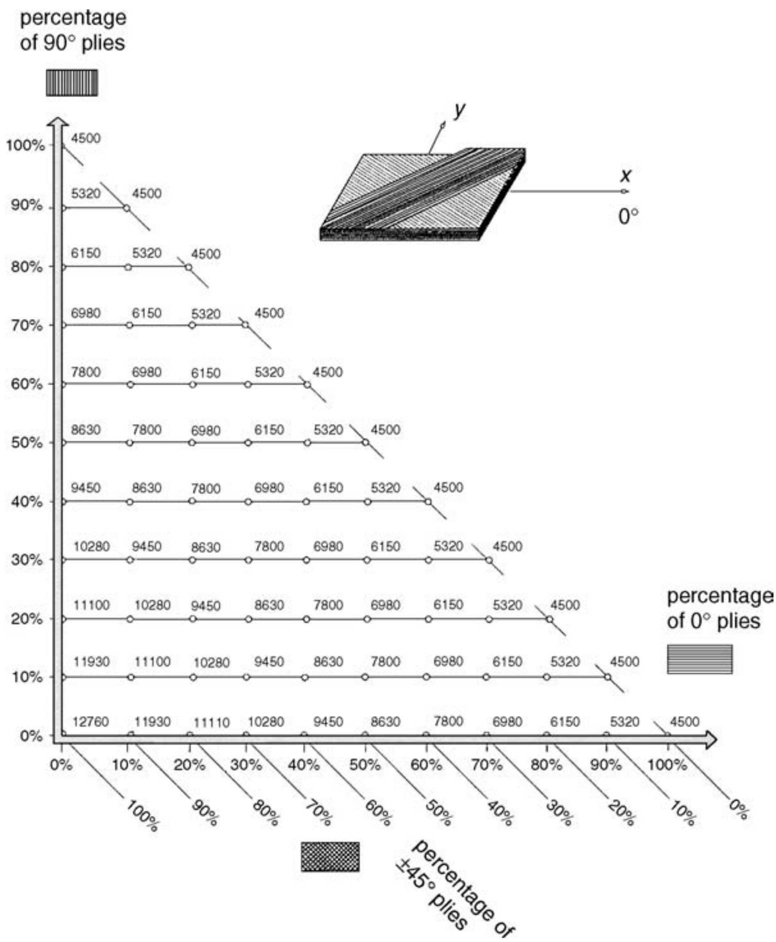
5.4.4 Case of Complex Loading: Optimum Composition of a Laminate

Estimation of the proportions in the previous paragraph does not lead to an optimum laminate in general. An optimum laminate is the one with the smallest thickness among all laminates of different compositions that can support the given combined stress resultants N_x, N_y, T_{xy}.

Tables 5.15 through 5.19, based on Hill-Tsai criterion,[14] give the optimum compositions of laminates based on unidirectionals of carbon/epoxy for the various

[14] See Section 5.3.2.

Table 5.15 Glass/Epoxy Laminates. V_f = 60%. Ply Thickness = 0.13 mm



Shear modulus G_{xy} as a function of percentages of plies in directions 0°, 90°, +45°, −45°.
(More information on modulus and strength of a basic ply, see Section 3.3.3).

stress resultants N_x, N_y, T_{xy}. The indicated compositions correspond to laminates that are capable of supporting the specified stress resultants and at the same time keeping a minimum thickness. One can see this number, in millimeters, within the circles, when the arithmetic sum of the stress resultants is equal to 100 N/mm.

Also represented in the tables are

- The direction along which the first ply failure will occur.
- The multiplication factor for the stress resultants in order to go from first ply failure to ultimate fracture of the laminate.
- The two compositions that are closest to the optimum composition, obtained by varying the indicated composition along the direction of the arrows.

First, the arrows in increasing solid line or decreasing solid line denote the increase or decrease of 5% in terms of the proportions marked. Next, the arrows in increasing broken line or decreasing broken line denote the increase or decrease of 5% in terms of the proportions marked.

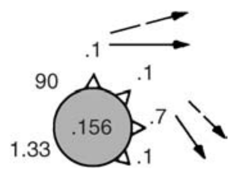
Example: Given the stress resultants:

$$N_x = 720 \text{ N/mm}; \qquad N_y = 0; \qquad T_{xy} = 80 \text{ N/mm}$$

one can then deduce values of the reduced stress resultants:

$$\bar{N}_x = 720/(720+80) = 0.9; \qquad \bar{N}_y = 0; \qquad \bar{T}_{xy} = 80/(720+80) = 0.1$$

One then uses Table 5.16 (all stress resultants are positive), where one can obtain for these values of reduced stress resultants the following figure:



This can be interpreted as follows:

- Optimal composition of the laminate
  - 70% of 0° plies (along x direction)
  - 10% of 90° plies
  - 10% of plies in 45°, 10% of plies in –45°
- Critical thickness of the laminate: 0.156 mm when the arithmetic sum of the 3 stress resultants is equal to 100 N/mm. For this thickness, the first ply failure occurs in the 90° plies. However, one can continue to load this laminate until it reaches 1.33 times the critical load, as:

$$N_x = 1.33 \times 720 = 957 \text{ N/mm}; \qquad N_y = 0$$

$$T_{xy} = 1.33 \times 80 = 106 \text{ N/mm}$$

At this point, there is complete rupture of the laminate.

Returning to our example, the arithmetic sum of the stress resultants is equal to 720 + 80 = 800 N/mm. Then, the thickness of the laminate has to be more than:

$$8 \times 0.156 = 1.25 \text{ mm}$$

Neighboring compositions: The second smallest thickness in the vicinity is obtained by modifying the indicated composition in the direction specified by the arrows in **solid** line, as

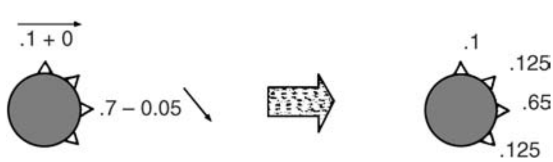
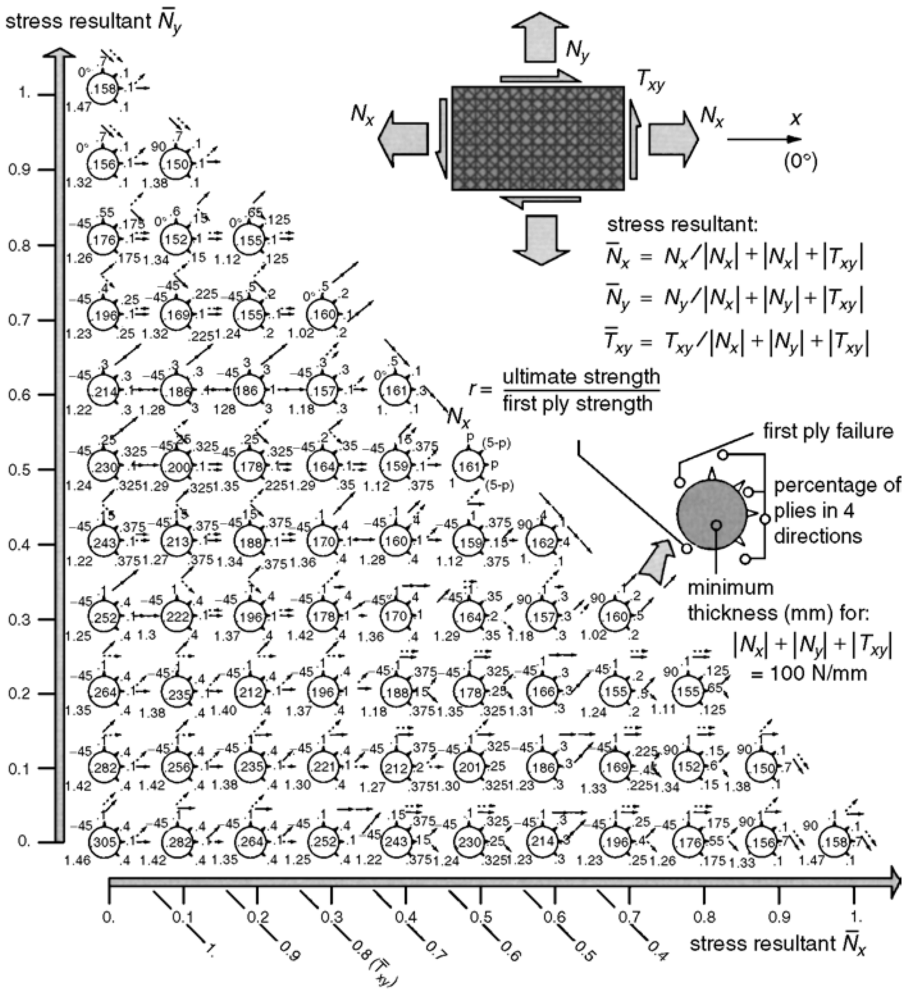


Table 5.16 Optimum Composition of a Carbon/Epoxy Laminate



$V_f = 0.6$, 10% minimum in each direction of 0°, 90°, +45°, −45°. (Ply characteristics: see Appendix 1 or Section 3.3.3).

One then obtains (not shown on the plate) a thickness of 0.160 mm (increase of 2.5% relative to the previous value) and a multiplication factor for the loading equal to 1.35.

Continuing in the direction of increasing thickness, the third smallest thickness in the immediate vicinity is obtained by modifying the indicated composition in the direction specified by the arrows in **broken** line, as:

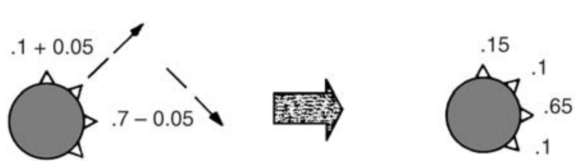
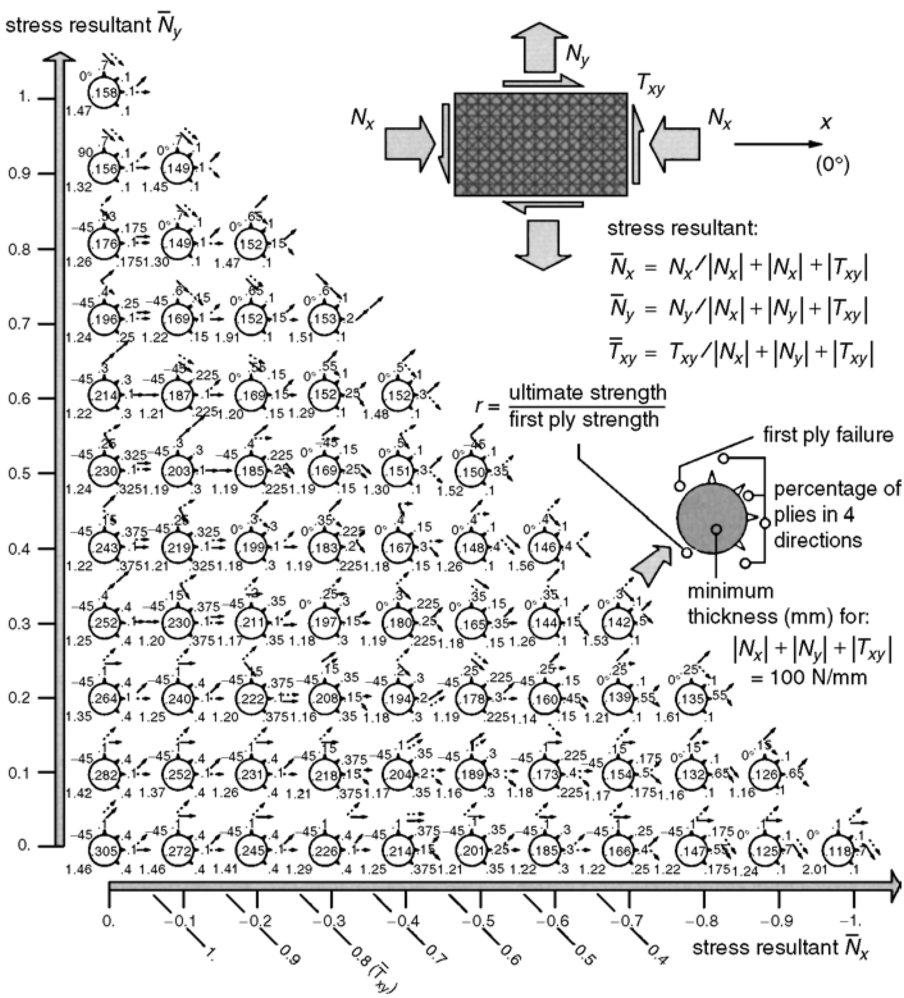


Table 5.17 Optimum Composition of a Carbon/Epoxy Laminate



V_f = 0.6, 10% minimum in each direction of 0°, 90°, +45°, −45°. (Ply characteristics: see Appendix 1 or Section 3.3.3).

One then obtains a thickness (not shown on the plate) of 0.165 mm (increase of 6%) and a multiplication factor of 1.3 for the load.

Example: Given the stress resultants

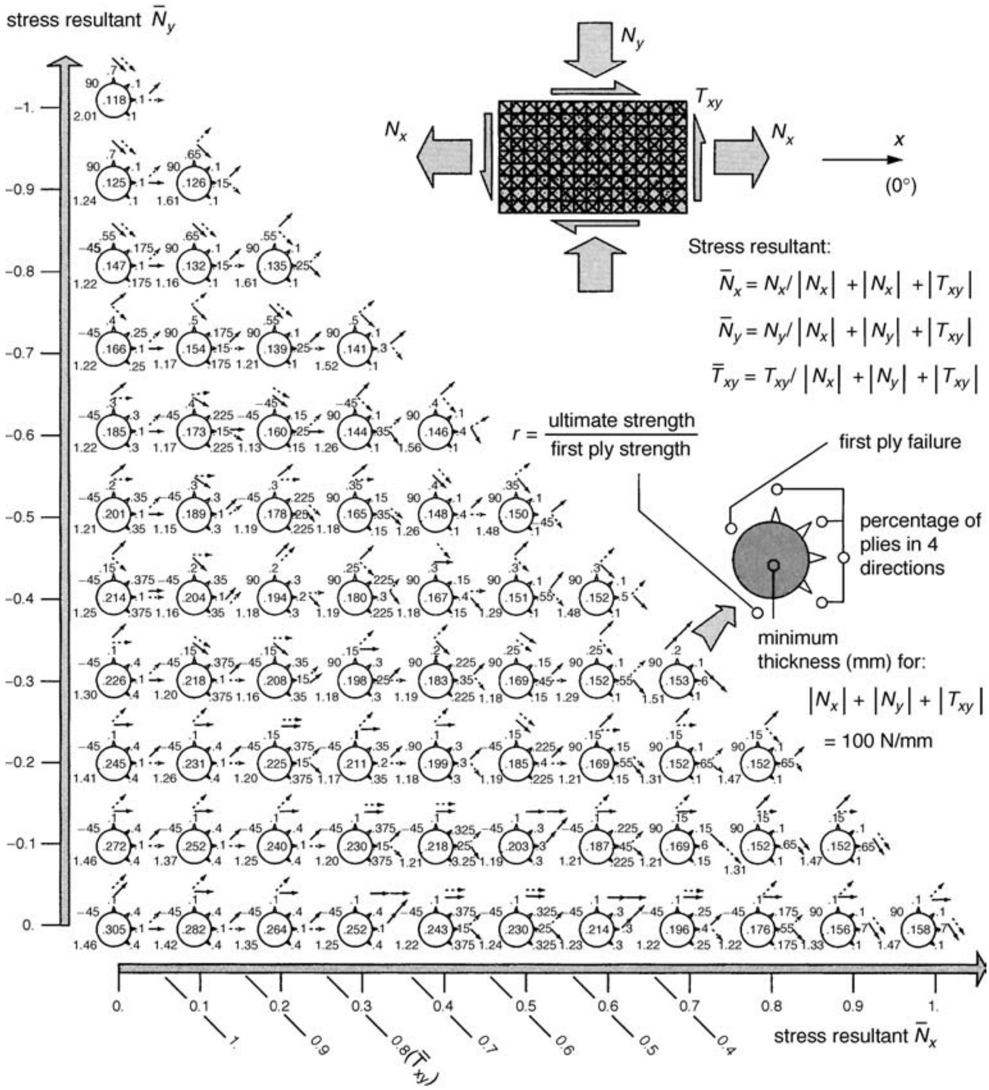
$$N_x = 600 \text{ N/mm}; \; N_y = -300 \text{ N/mm}; \; T_{xy} = 100 \text{ N/mm}$$

The corresponding reduced stress resultants are

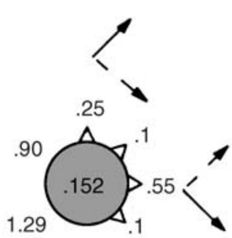
$$\bar{N}_x = .6; \; \bar{N}_y = -.3; \; \bar{T}_{xy} = 0.1 \text{ N/mm}$$

One obtains from Table 5.18:

Table 5.18 Optimum Composition of a Carbon/Epoxy Laminate



V_f = 0.6, 10% minimum in each direction of 0°, 90°, +45°, −45°. (Ply characteristics: see Appendix 1 or Section 3.3.3).



one can extract the following:

- optimal composition of the laminate:

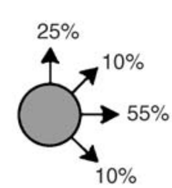
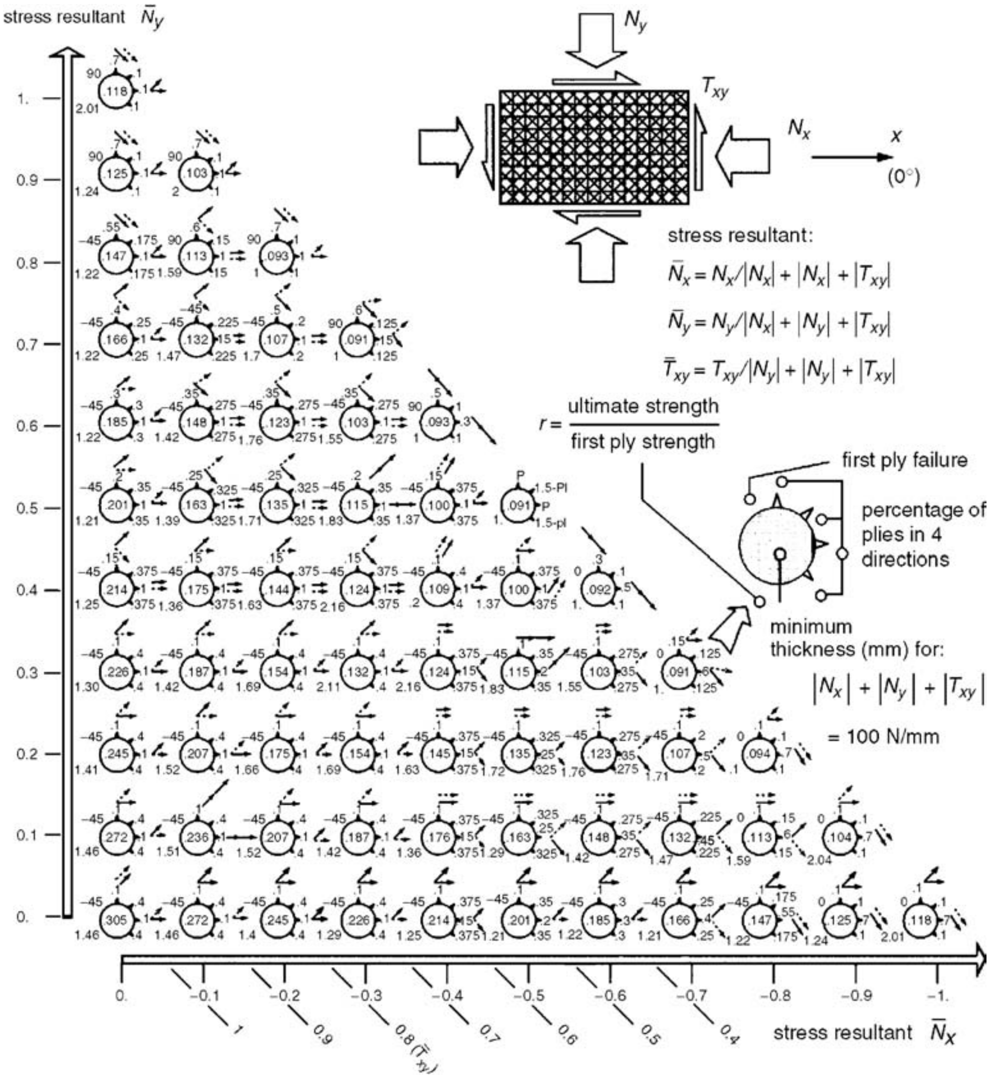


Table 5.19 Optimum Composition of a Carbon/Epoxy Laminate



V_f = 0.6, 10% minimum in each direction of 0°, 90°, +45°, −45°. (Ply characteristics: see Appendix 1 or Section 3.3.3).

where

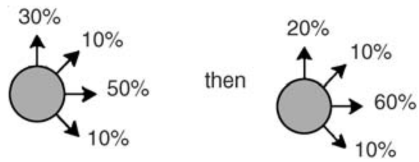
- Critical thickness is $10 \times 0.152 = 1.52$ mm
- These are the 90° plies that fail first.
- Complete rupture of the laminate occurs when:

$$N_x = 1.29 \times 600 = 774 \text{ N/mm}$$

$$N_y = 1.29 \times -300 = -387 \text{ N/mm}$$

$$T_{xy} = 1.29 \times 100 = 129 \text{ N/mm}$$

- The closest critical thicknesses (in increasing order) are obtained with the following successive compositions:



Remarks: A few loading cases can lead to several distinct optimum compositions, but with identical thicknesses. For example the reduced stress resultants:

$$\bar{N}_x = \bar{N}_y = 0.5; \; \bar{T}_{xy} = 0$$

This is a case of isotropic loading, the Mohr circle is reduced to one point (see figure below).

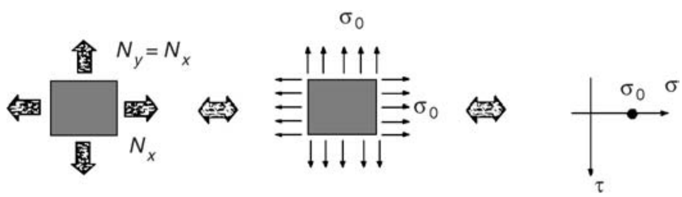
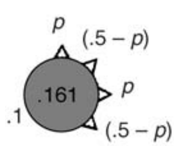
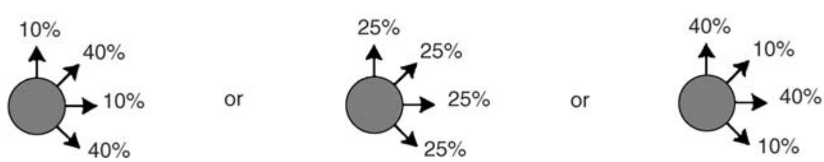


Table 5.16 indicates



meaning that all values of p less than or equal to 0.5 apply, for example:



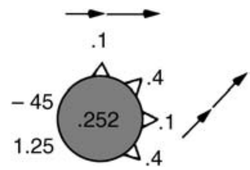
One obtains in this case a unique critical thickness of 0.161 mm (corresponding to a sum $N_x + N_y = 100$ N/mm) **independent** of the proportion p.[15] The isotropic composition (25%, 25%, 25%, 25%) in the directions 0°, 90°, +45°, –45°, which appears intuitive, can in fact be replaced by compositions that present the values of modulus of elasticity that are varied and adaptable to the designer in the directions 0°, 90° 45°, or –45°,[16] or even α, $\alpha + \pi/2$ with a certain α.

[15] See Section 18.2.8.
[16] See Section 5.4.2, Table 5.4.

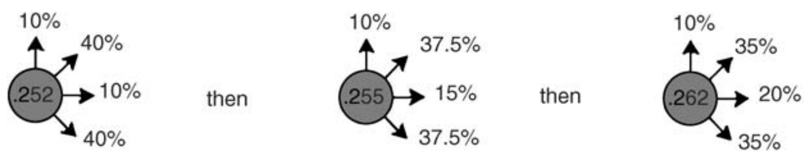
In some loading cases, one finds from the table only arrows in a solid line. For example, for the following reduced stress resultants:

$$\bar{N}_x = 0.3; \qquad \bar{N}_y = 0; \qquad \bar{T}_{xy} = 0.7$$

one finds from Table 5.16 the following figure:



The three neighboring optimum compositions in increasing order are



(The thicknesses of 0.255 mm and 0.262 mm are not indicated on the plate). The third composition, characterized by an increase in thickness of 0.252 to 0.262 mm, or 6%, leads to an increase in modulus of elasticity in the x (0°) by 36% (see Section 5.4.2, Table 5.4).

One can note finally that for the majority of cases, the optimum compositions indicated in Tables 5.16 to 5.19 are not easy to postulate using intuition.[17]

5.4.5 Practical Remarks: Particularities of the Behavior of Laminates

- The **fabrics** are able to cover the left surfaces[18] due to pushing action in warp and fill directions.
- The radii of the mold must not be too small. This concerns particularly the inner radius R_i as shown in Figure 5.24. The graph gives an idea for the minimum value required for the inner and outer radii.
- The **thickness** of a polymerized ply is not more than **0.8** to **0.85** times that of a ply before polymerization. This value of the final thickness must also take into account a margin of uncertainty on the order of 15%.
- When one unidirectional sheet **does not cover** the whole surface required to constitute a ply, it is necessary to take precautions when cutting the different pieces of the sheet. A few examples of wrapping are given in Figure 5.25.
- The unidirectional sheets do not fit well into sharp corners in the fiber direction. The schematic in Figure 5.26 shows the dispositions to accommodate sudden changes in draping directions.

[17] See Exercise 18.1.6.

[18] It is more difficult for the plain weave than for the satins, due to the mode of weaving (See Section 3.4.1).

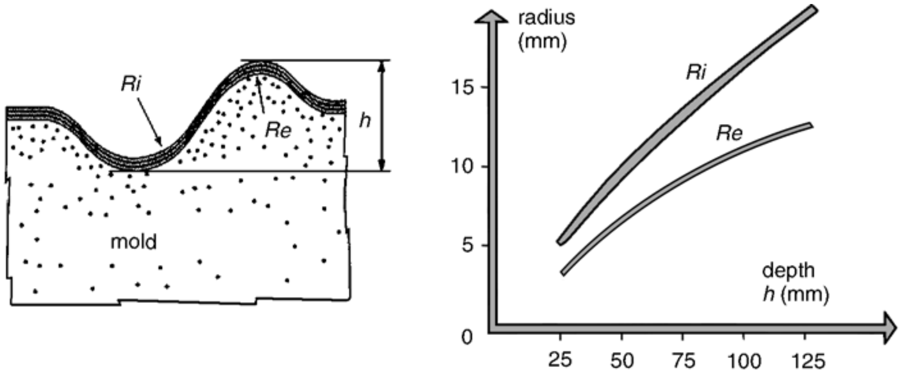


Figure 5.24 Minimum Required for Inner and Outer Radii of Mold

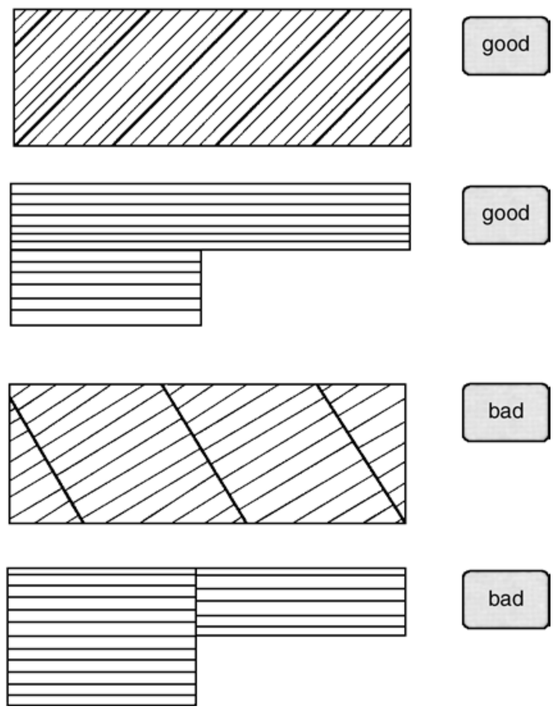


Figure 5.25 Recommended Arrangement for Cutting

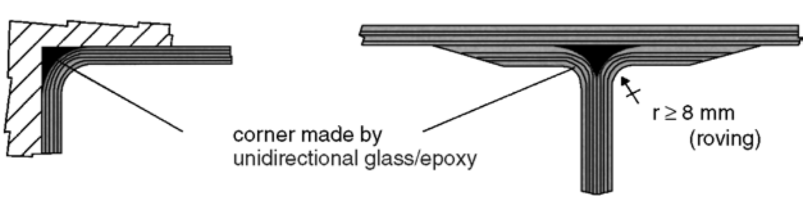


Figure 5.26 Method to Lay up at Corner

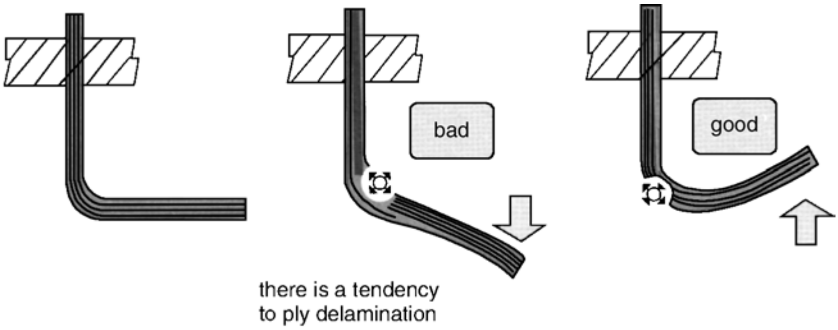


Figure 5.27 Corner Situations

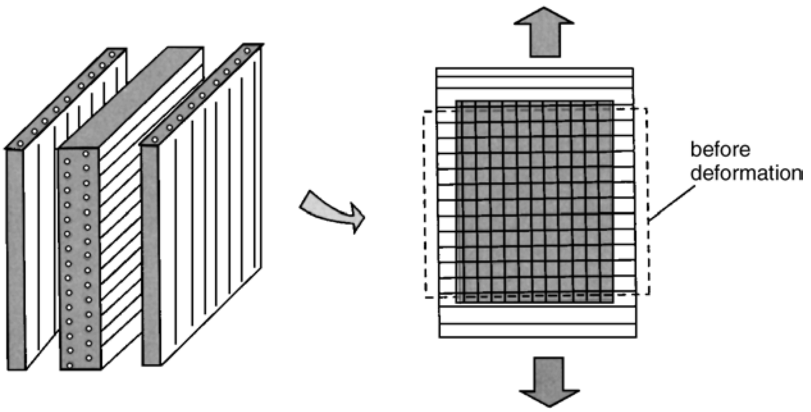


Figure 5.28a Three Plies in Separate Positions

Delaminations: When the plies making up the laminate separate from each other, it is called **delamination**. Many causes are susceptible to provoke this deterioration:

- An **impact** that does not leave apparent traces on the surface and can lead to internal delaminations.
- A mode of loading leads to the disbond of the plies (tension over the interface) as shown in Figure 5.27.
- **Shear** stresses at the interfaces between different plies, very near the edges of the laminates, which one can make evident as follows (with a three-ply laminate):
  1. Consider the three plies in Figure 5.28(a), separated. Under the effect of loading (right-hand-side figure), they are deformed independently and do not fit with each other when they are put together.
  2. Now the plies constitute a balanced laminate. Under the same loading, they deform together, without distorsion, as shown in Figure 5.28(b).
  3. This is because interlaminar stresses occur on the bonded faces. One can show that these stresses are located very close to the edges of the laminate, as illustrated in Figure 5.28(c).

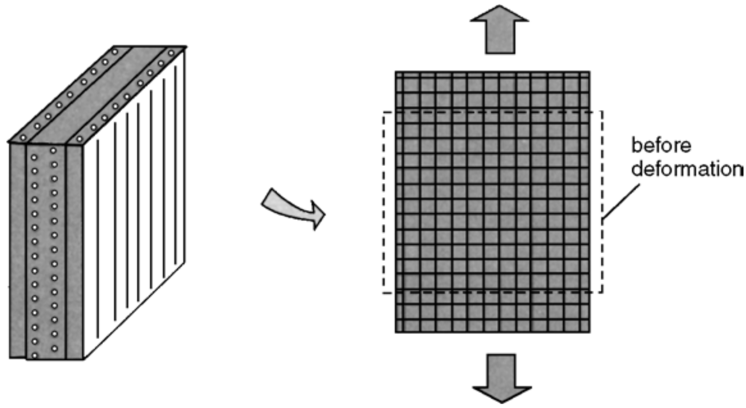


Figure 5.28b Three Plies, Together

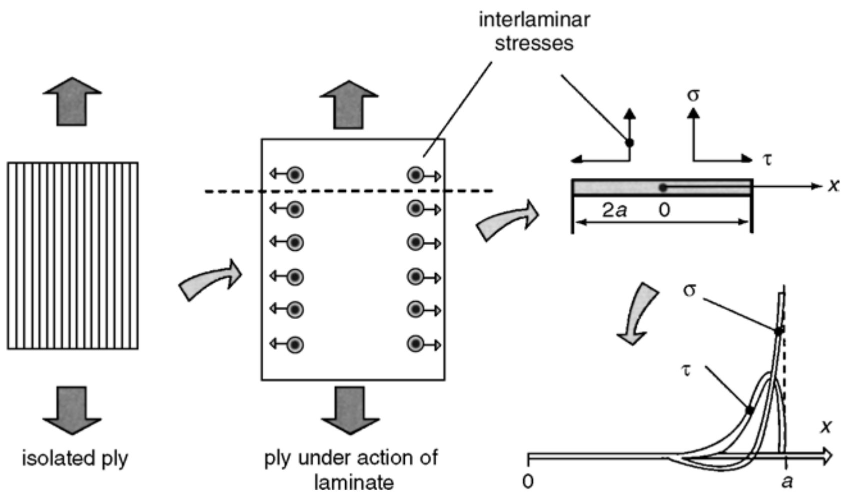


Figure 5.28c Stresses at Free Edge

- A complex state of stresses at the interface, due to local buckling, for example (see Figure 5.29).

Practical as well as theoretical studies of these interlaminar stresses are difficult, and the phenomenon is still not well understood.

Why is Fatigue Resistance so Good?

Paradox: Glass is a very brittle material (no plastic deformation). The resin is also often a brittle material. (for example, epoxy). However, the association of reinforcement/matrix constituted of these two materials opposes to the propagation of cracks and makes the resultant composite to endure fatigue remarkably.

Explanation: When the cracks propagate, for example, in the unidirectional layer shown schematically in Figure 5.30 in the form of alternating of fibers and

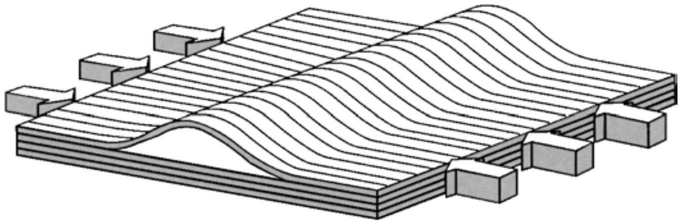


Figure 5.29 Delamination Due to Buckling at Interface

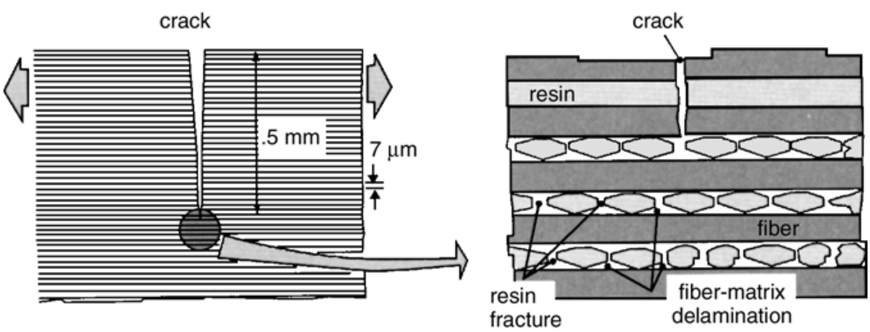


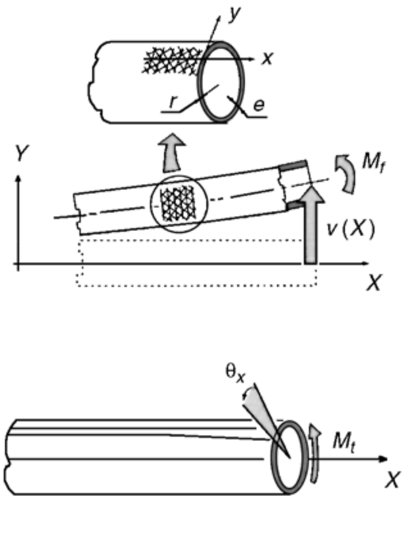
Figure 5.30 Crack Propagation in Composites

resin, the initial stress concentration at the end of the crack leads to failure in the resin. The fibers, then debonded, benefit from a relaxation of the stresses. There is no more stress concentrations as in a homogeneous material.

Laminated tubes can be obtained by winding of filaments, rubans of unidirectionals or fabrics. In a first approximation, one can[19] estimate the strains and stresses in flexure and in torsion from the relations in Figure 5.31 in which

E_x and G_{xy} are the moduli of elasticity in the tangent plane x,y.
I and I_o are respectively the quadratic moment of inertia and polar moment of inertia ($I_o = 2I$).
Y is the coordinate of a point in the section (in the undeformed position) in the X, Y, Z coordinates.
r is the average radius of the tube.

[19] For a complete study of the flexure and torsion of the composite beams, See Section 15 and 16.



flexure

$$E_x I \frac{d^2 v}{dX^2} = M_f$$

$$\sigma_x = -\frac{M_f}{I} \times Y$$

torsion

$$G_{xy} I_0 \frac{d\theta_X}{dX} = M_t$$

$$\tau_{xy} = \frac{M_t}{I_0} \times r$$

Figure 5.31 Composite Tube Relations