APPENDIX A

Cross-Sectional Properties of Thin-Walled Composite Beams

In the following tables a_{11} and d_{11} are the elements of the compliance matrix of symmetrical laminates (Eqs. 3.29 and 3.30) evaluated at the midsurface; α_{ij}, β_{ij}, δ_{ij} are the elements of the compliance matrix of nonsymmetrical laminates (Eq. 3.23). These properties are evaluated at each wall segment's "neutral" plane. In the main text these elements are identified by the superscript ϱ. To simplify the notation, in this Appendix the superscript ϱ is omitted.

The subscripts f1, f2, and w refer to flange 1, flange 2 and the web, respectively.

The modified material properties are

$$\overline{\alpha}_{11} = \left(\alpha_{11} - \frac{(\beta_{16})^2}{\delta_{66}}\right) \qquad \widehat{\alpha}_{11} = \left(\alpha_{11} - \frac{\beta_{12}^2}{\delta_{22}}\right)$$

$$\overline{\beta}_{11} = \left(\beta_{11} - \frac{\beta_{16}\delta_{16}}{\delta_{66}}\right) \qquad \widehat{\beta}_{11} = \left(\beta_{11} - \frac{\beta_{12}\delta_{12}}{\delta_{22}}\right)$$

$$\overline{\delta}_{11} = \left(\delta_{11} - \frac{(\delta_{16})^2}{\delta_{66}}\right) \qquad \widehat{\delta}_{11} = \left(\delta_{11} - \frac{\delta_{12}^2}{\delta_{22}}\right).$$

The location of the "neutral" plane with respect to the midplane is

$$\widetilde{\varrho} = -\frac{\beta_{11}^{\mathrm{MP}}}{\delta_{11}^{\mathrm{MP}}} \qquad \begin{array}{c}\text{open section}\\ \text{orthotropic unsymmetrical layup}\\ \text{arbitrary cross section}\end{array} \tag{A.1}$$

$$\overline{\varrho} = -\frac{\overline{\beta}_{11}^{\mathrm{MP}}}{\overline{\delta}_{11}^{\mathrm{MP}}} \qquad \begin{array}{c}\text{open section}\\ \text{arbitrary layup}\\ \text{symmetrical cross section}\end{array} \tag{A.2}$$

$$\widehat{\varrho} = -\frac{\widehat{\beta}_{11}^{\mathrm{MP}}}{\widehat{\delta}_{11}^{\mathrm{MP}}} \qquad \begin{array}{c}\text{closed section}\\ \text{orthotropic unsymmetrical layup}\\ \text{arbitrary cross section.}\end{array} \tag{A.3}$$

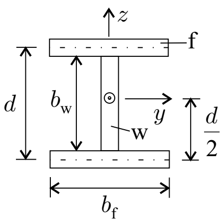
β_{11}^{MP} and $\delta_{11}^{\mathrm{MP}}$ are evaluated at the midplanes.

The stiffness α_{66}^{ν} is (Eq. 6.196)

$$\alpha_{66}^{\nu} = \alpha_{66} - \frac{\beta_{66}^2}{\delta_{66}}, \tag{A.4}$$

where α_{66}, β_{66}, and δ_{66} are evaluated at the "neutral" plane.

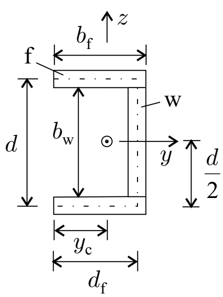
Table A.1. The tensile and bending stiffnesses and the coordinates of the centroid. The layup of each wall segment is orthotropic and symmetrical. The properties are evaluated at each wall segment's "neutral" plane, which is at the midplane.



$$\widehat{EA} = \frac{2b_f}{(a_{11})_f} + \frac{b_w}{(a_{11})_w}$$

$$\widehat{EI}_{yy} = \frac{b_f}{(a_{11})_f}\frac{d^2}{2} + \frac{2b_f}{(d_{11})_f} + \frac{b_w^3}{12(a_{11})_w}$$

$$\widehat{EI}_{zz} = \frac{b_w}{(d_{11})_w} + \frac{2b_f^3}{12(a_{11})_f}$$

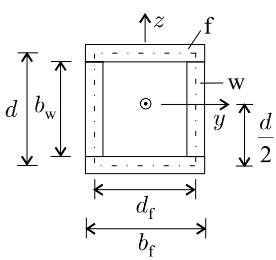


$$\widehat{EA} = \frac{2b_f}{(a_{11})_f} + \frac{b_w}{(a_{11})_w}$$

$$y_c = \frac{1}{\widehat{EA}}\left(\frac{2b_f}{(a_{11})_f}\frac{b_f}{2} + \frac{b_w}{(a_{11})_w}d_f\right)$$

$$\widehat{EI}_{yy} = \frac{b_f}{(a_{11})_f}\frac{d^2}{2} + \frac{2b_f}{(d_{11})_f} + \frac{b_w^3}{12(a_{11})_w}$$

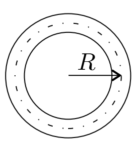
$$\widehat{EI}_{zz} = \frac{b_w}{(a_{11})_w}(d_f - y_c)^2 + \frac{b_w}{(d_{11})_w} + \frac{2}{(a_{11})_f}\left(\frac{y_c^3}{3} + \frac{(b_f - y_c)^3}{3}\right)$$



$$\widehat{EA} = \frac{2b_f}{(a_{11})_f} + \frac{2b_w}{(a_{11})_w}$$

$$\widehat{EI}_{yy} = \frac{b_f}{(a_{11})_f}\frac{d^2}{2} + \frac{2b_f}{(d_{11})_f} + \frac{2b_w^3}{12(a_{11})_w}$$

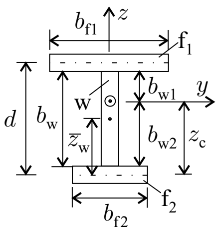
$$\widehat{EI}_{zz} = \frac{b_w}{(a_{11})_w}\frac{d_f^2}{2} + \frac{2b_w}{(d_{11})_w} + \frac{2b_f^3}{12(a_{11})_f}$$



$$\widehat{EA} = \frac{2R\pi}{a_{11}}$$

$$\widehat{EI}_{yy} = \widehat{EI}_{zz} = \pi\left(\frac{R^3}{a_{11}} + \frac{R}{d_{11}}\right)$$

Table A.2. The tensile and bending stiffnesses and the coordinates of the centroid. The layup of each wall segment is orthotropic and unsymmetrical. The properties α_{11} and δ_{11} are evaluated at each wall segment's "neutral" plane, which is at $\widetilde{\varrho}$.

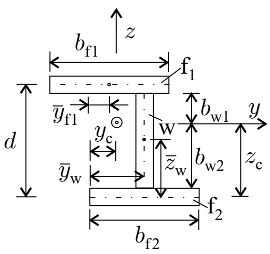


$$\widehat{EA} = \frac{b_{f1}}{(\alpha_{11})_{f1}} + \frac{b_{f2}}{(\alpha_{11})_{f2}} + \frac{b_w}{(a_{11})_w}$$

$$z_c = \frac{1}{\widehat{EA}}\left(\frac{b_{f1}}{(\alpha_{11})_{f1}} d + \frac{b_w}{(a_{11})_w}\overline{z}_w\right)$$

$$\widehat{EI}_{zz} = \frac{b_w}{(d_{11})_w} + \frac{1}{(\alpha_{11})_{f1}}\frac{b_{f1}^3}{12} + \frac{1}{(\alpha_{11})_{f2}}\frac{b_{f2}^3}{12}$$

$$\widehat{EI}_{yy} = \frac{b_{f1}}{(\alpha_{11})_{f1}}(d - z_c)^2 + \frac{b_{f2}}{(\alpha_{11})_{f2}} z_c^2 + \frac{b_{f1}}{(\delta_{11})_{f1}} + \frac{b_{f2}}{(\delta_{11})_{f2}} + \frac{1}{(a_{11})_w}\left(\frac{b_{w1}^3 + b_{w2}^3}{3}\right)$$



$$\widehat{EA} = \frac{b_{f1}}{(\alpha_{11})_{f1}} + \frac{b_{f2}}{(\alpha_{11})_{f2}} + \frac{b_w}{(\alpha_{11})_w}$$

$$z_c = \frac{1}{\widehat{EA}}\left(\frac{b_{f1}}{(\alpha_{11})_{f1}} d + \frac{b_w}{(\alpha_{11})_w}\overline{z}_w\right)$$

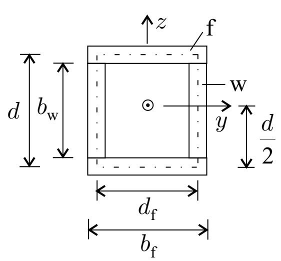
$$y_c = \frac{1}{\widehat{EA}}\left(\frac{b_{f1}}{(\alpha_{11})_{f1}}\overline{y}_{f1} + \frac{b_{f2}}{(\alpha_{11})_{f2}}\frac{b_{f2}}{2} + \frac{b_w}{(\alpha_{11})_w}\overline{y}_w\right)$$

$$\widehat{EI}_{yy} = \frac{b_{f1}}{(\alpha_{11})_{f1}}(d - z_c)^2 + \frac{b_{f2}}{(\alpha_{11})_{f2}} z_c^2 + \frac{b_{f1}}{(\delta_{11})_{f1}} + \frac{b_{f2}}{(\delta_{11})_{f2}} + \frac{1}{(\alpha_{11})_w}\left(\frac{b_{w1}^3 + b_{w2}^3}{3}\right)$$

$$\widehat{EI}_{zz} = \frac{b_w}{(\alpha_{11})_w}(\overline{y}_w - y_c)^2 + \frac{b_w}{(\delta_{11})_w} + \frac{1}{(\alpha_{11})_{f1}}\frac{b_{f1}^3}{12} + \frac{1}{(\alpha_{11})_{f2}}\frac{b_{f2}^3}{12} + \frac{b_{f1}}{(\alpha_{11})_{f1}}(y_c - \overline{y}_{f1})^2 + \frac{b_{f2}}{(\alpha_{11})_{f2}}\left(y_c - \frac{b_{f2}}{2}\right)^2$$

$$\widehat{EI}_{yz} = \frac{b_{f1}}{(\alpha_{11})_{f1}}(\overline{y}_{f1} - y_c)(d - z_c) - \frac{b_{f2}}{(\alpha_{11})_{f2}}\left(\frac{b_{f2}}{2} - y_c\right) z_c + \frac{b_w}{(\alpha_{11})_w}(\overline{y}_w - y_c)\left(\frac{b_w}{2} - z_c\right)$$

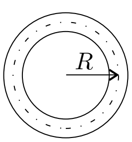
Table A.3. The tensile and bending stiffnesses. The layup of each wall segment is orthotropic and unsymmetrical. Doubly symmmetrical cross section. The property $\widehat{\alpha}_{11}$ is evaluated at each wall segment's "neutral" plane, which is at $\widehat{\varrho}$. The properties α_{11} and δ_{11} are evaluated at each wall segment's "neutral" plane, which is at $\widetilde{\varrho}$.



$$\widehat{EA} = \frac{2b_f}{(\widehat{\alpha}_{11})_f} + \frac{2b_w}{(\widehat{\alpha}_{11})_w}$$

$$\widehat{EI}_{yy} = \frac{b_f}{(\alpha_{11})_f}\frac{d^2}{2} + \frac{2b_f}{(\delta_{11})_f} + \frac{2b_w^3}{12(\alpha_{11})_w}$$

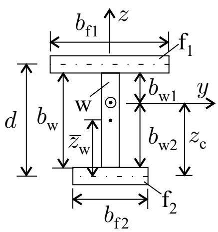
$$\widehat{EI}_{zz} = \frac{b_w}{(\alpha_{11})_w}\frac{d_f^2}{2} + \frac{2b_w}{(\delta_{11})_w} + \frac{2b_f^3}{12(\alpha_{11})_f}$$



$$\widehat{EA} = 2R\pi\frac{1}{\widehat{\alpha}_{11}}$$

$$\widehat{EI}_{yy} = \widehat{EI}_{zz} = \pi\left(R^3\frac{1}{\alpha_{11}} + R\frac{1}{\delta_{11}}\right)$$

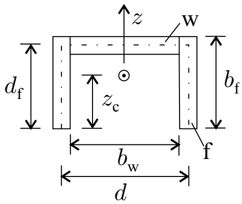
Table A.4. The tensile and bending stiffnesses and the coordinates of the centroid. The cross section is symmetrical about the *z*-axis. The layup of each wall segment is arbitrary. The properties $\overline{\alpha}_{11}$ and $\overline{\delta}_{11}$ are evaluated at each wall segment's "neutral" plane, which is at $\overline{\varrho}$.



$$\widehat{EA} = \frac{b_{f1}}{(\overline{\alpha}_{11})_{f1}} + \frac{b_{f2}}{(\overline{\alpha}_{11})_{f2}} + \frac{b_w}{(a_{11})_w}$$

$$z_c = \frac{1}{\widehat{EA}}\left(\frac{b_{f1}}{(\overline{\alpha}_{11})_{f1}} d + \frac{b_w}{(a_{11})_w}\overline{z}_w\right)$$

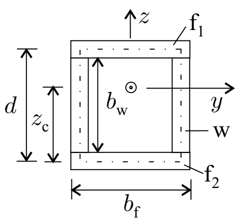
$$\widehat{EI}_{yy} = \frac{b_{f1}}{(\overline{\alpha}_{11})_{f1}}(d - z_c)^2 + \frac{b_{f2}}{(\overline{\alpha}_{11})_{f2}} z_c^2 + \frac{b_{f1}}{(\overline{\delta}_{11})_{f1}} + \frac{b_{f2}}{(\overline{\delta}_{11})_{f2}} + \frac{1}{(a_{11})_w}\left(\frac{b_{w1}^3 + b_{w2}^3}{3}\right)$$



$$\widehat{EA} = \frac{2b_f}{(\overline{\alpha}_{11})_f} + \frac{b_w}{(\overline{\alpha}_{11})_w}$$

$$z_c = \frac{1}{\widehat{EA}}\left(\frac{2b_f}{(\overline{\alpha}_{11})_f}\frac{b_f}{2} + \frac{b_w}{(\overline{\alpha}_{11})_w} d_f\right)$$

$$\widehat{EI}_{yy} = \frac{2}{(\overline{\alpha}_{11})_f}\frac{b_f^3}{12} + \frac{2}{(\overline{\alpha}_{11})_f} b_f\left(\frac{b_f}{2} - z_c\right)^2 + \frac{b_w}{(\overline{\alpha}_{11})_w}(d_f - z_c)^2 + \frac{b_w}{(\overline{\delta}_{11})_w}$$

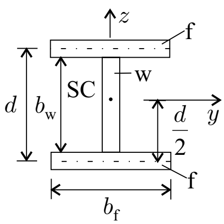


$$\widehat{EA} \approx \frac{b_{f1}}{(\overline{\alpha}_{11})_{f1}} + \frac{b_{f2}}{(\overline{\alpha}_{11})_{f2}} + \frac{2b_w}{(\overline{\alpha}_{11})_w}$$

$$z_c \approx \frac{1}{\widehat{EA}}\left(\frac{b_f}{(\overline{\alpha}_{11})_{f1}} d + \frac{2b_w}{(\overline{\alpha}_{11})_w}\frac{d}{2}\right)$$

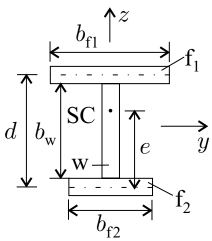
$$\widehat{EI}_{yy} \approx \frac{b_{f1}}{(\overline{\delta}_{11})_{f1}} + \frac{b_{f2}}{(\overline{\delta}_{11})_{f2}} + \frac{2b_w^3}{12(\overline{\alpha}_{11})_w}$$
$$+ \frac{b_{f1}}{(\overline{\alpha}_{11})_{f1}}(d - z_c)^2 + \frac{b_{f2}}{(\overline{\alpha}_{11})_{f2}} z_c^2 + \frac{2b_w}{(\overline{\alpha}_{11})_w}\left(\frac{d}{2} - z_c\right)^2$$

Table A.5. The warping and torsional stiffnesses and the location of the shear center. The layup of each wall segment is orthotropic and unsymmetrical. The properties δ_{66} and δ_{11} are evaluated at each wall segment's "neutral" plane, which is at $\widetilde{\varrho}$.



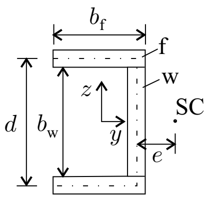
$$\widehat{GI}_{\mathrm{t}} = 4\left(\frac{2b_{\mathrm{f}}}{(\delta_{66})_{\mathrm{f}}} + \frac{b_{\mathrm{w}}}{(\delta_{66})_{\mathrm{w}}}\right)$$

$$\widehat{EI}_{\omega} = \frac{1}{(\alpha_{11})_{\mathrm{f}}}\frac{d^2 b_{\mathrm{f}}^3}{24}$$



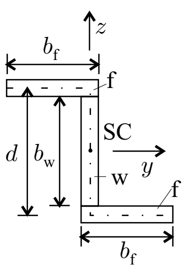
$$\widehat{GI}_{\mathrm{t}} = 4\left(\frac{b_{\mathrm{f1}}}{(\delta_{66})_{\mathrm{f1}}} + \frac{b_{\mathrm{f2}}}{(\delta_{66})_{\mathrm{f2}}} + \frac{b_{\mathrm{w}}}{(\delta_{66})_{\mathrm{w}}}\right)$$

$$\widehat{EI}_{\omega} = \frac{\frac{b_{\mathrm{f2}}^3}{(\alpha_{11})_{\mathrm{f2}}}}{12}ed \qquad e = d\frac{\frac{b_{\mathrm{f1}}^3}{(\alpha_{11})_{\mathrm{f1}}}}{\frac{b_{\mathrm{f1}}^3}{(\alpha_{11})_{\mathrm{f1}}}+\frac{b_{\mathrm{f2}}^3}{(\alpha_{11})_{\mathrm{f2}}}}$$



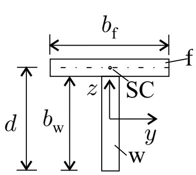
$$\widehat{GI}_{\mathrm{t}} = 4\left(\frac{2b_{\mathrm{f}}}{(\delta_{66})_{\mathrm{f}}} + \frac{b_{\mathrm{w}}}{(\delta_{66})_{\mathrm{w}}}\right) \qquad e = \frac{\frac{3b_{\mathrm{f}}^2}{(\alpha_{11})_{\mathrm{f}}}}{\frac{6b_{\mathrm{f}}}{(\alpha_{11})_{\mathrm{f}}}+\frac{d}{(\alpha_{11})_{\mathrm{w}}}}$$

$$\widehat{EI}_{\omega} = \frac{b_{\mathrm{f}}^3 d^2 \frac{1}{(\alpha_{11})_{\mathrm{f}}}}{12}\;\frac{\frac{3b_{\mathrm{f}}}{(\alpha_{11})_{\mathrm{f}}}+\frac{2d}{(\alpha_{11})_{\mathrm{w}}}}{\frac{6b_{\mathrm{f}}}{(\alpha_{11})_{\mathrm{f}}}+\frac{d}{(\alpha_{11})_{\mathrm{w}}}}$$



$$\widehat{GI}_{\mathrm{t}} = 4\left(\frac{2b_{\mathrm{f}}}{(\delta_{66})_{\mathrm{f}}} + \frac{b_{\mathrm{w}}}{(\delta_{66})_{\mathrm{w}}}\right)$$

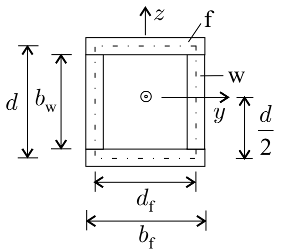
$$\widehat{EI}_{\omega} = \frac{b_{\mathrm{f}}^3 d^2}{12(2b_{\mathrm{f}}+d)^2}\left(2\frac{\left(b_{\mathrm{f}}^2+b_{\mathrm{f}}d+d^2\right)}{(\alpha_{11})_{\mathrm{f}}} + 3\frac{b_{\mathrm{f}}d}{(\alpha_{11})_{\mathrm{w}}}\right)$$



$$\widehat{GI}_{\mathrm{t}} = 4\left(\frac{b_{\mathrm{f}}}{(\delta_{66})_{\mathrm{f}}} + \frac{b_{\mathrm{w}}}{(\delta_{66})_{\mathrm{w}}}\right)$$

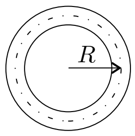
$$\widehat{EI}_{\omega} = 0$$

Table A.6. The warping and torsional stiffnesses of closed-section beams. The layup of each wall segment is orthotropic and unsymmetrical. α_{66}^{ν} is given by Eq. (A.4).



$$\widehat{GI}_{\mathrm{t}} = \frac{2d_{\mathrm{f}}^2 d^2}{\left(\alpha_{66}^{\nu}\right)_{\mathrm{f}} d_{\mathrm{f}} + \left(\alpha_{66}^{\nu}\right)_{\mathrm{w}} d}$$

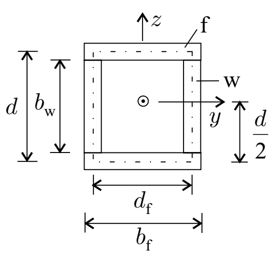
$$\widehat{EI}_{\omega} \approx 0$$



$$\widehat{GI}_{\mathrm{t}} = \frac{2R^3\pi}{\alpha_{66}^{\nu}}$$

$$\widehat{EI}_{\omega} = 0$$

Table A.7. Shear compliances of closed-section beams. The layup of each wall segment is orthotropic and unsymmetrical. The properties $\widetilde{\delta}_{66}$ and $\widetilde{\delta}_{11}$ are evaluated at each wall segment's "neutral" plane, which is at $\widetilde{\varrho}$. α_{66}^{ν} is given by Eq. (A.4).

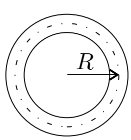


$$\widehat{s}_{zz} = \frac{\left(\alpha_{66}^{\nu}\right)_{\mathrm{w}}}{2d} + \frac{1}{6}\frac{\left(\alpha_{66}^{\nu}\right)_{\mathrm{f}} d_{\mathrm{f}}}{d^2 \gamma_z^2}$$

$$\widehat{s}_{yy} = \frac{\left(\alpha_{66}^{\nu}\right)_{\mathrm{f}}}{2d_{\mathrm{f}}} + \frac{1}{6}\frac{\left(\alpha_{66}^{\nu}\right)_{\mathrm{w}} d}{d_{\mathrm{f}}^2 \gamma_y^2}$$

$$\widehat{s}_{yz} = 0$$

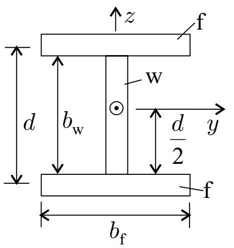
$$\gamma_z = 1 + \frac{1}{3}\frac{(\alpha_{11})_{\mathrm{f}}}{(\alpha_{11})_{\mathrm{w}}}\frac{d}{d_{\mathrm{f}}} \qquad \gamma_y = 1 + \frac{1}{3}\frac{(\alpha_{11})_{\mathrm{w}}}{(\alpha_{11})_{\mathrm{f}}}\frac{d_{\mathrm{f}}}{d}$$



$$\widehat{s}_{zz} = \widehat{s}_{yy} = \frac{\alpha_{66}^{\nu}}{R\pi}$$

$$\widehat{s}_{yz} = 0$$

Table A.8. Shear compliances. The layup of each wall segment is orthotropic and unsymmetrical. The coordinates of the centroid y_c, z_c are given in Tables A.1 through A.4, and the coordinate of the shear center e in Table A.5. The properties are evaluated at each wall segment's "neutral" plane, which is at $\widetilde{\varrho}$. α_{66}^{ν} is given by Eq. (A.4).



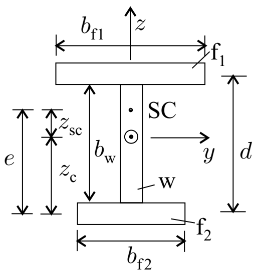
$$\widehat{s}_{yy} = 1.2\frac{\left(\alpha_{66}^{\nu}\right)_{\rm f}}{2b_{\rm f}}$$

$$\widehat{s}_{zz} = \frac{\left(\alpha_{66}^{\nu}\right)_{\rm w}}{d} + \frac{1}{6}\frac{\left(\alpha_{66}^{\nu}\right)_{\rm f} b_{\rm f}}{d^2\gamma^2}$$

$$\widehat{s}_{\omega\omega} = \frac{2.4}{d^2}\frac{\left(\alpha_{66}^{\nu}\right)_{\rm f}}{b_{\rm f}}$$

$$\widehat{s}_{yz} = \widehat{s}_{y\omega} = \widehat{s}_{z\omega} = 0$$

$$\text{where} \quad \gamma = 1 + \frac{1}{6}\frac{(\alpha_{11})_{\rm f}}{(\alpha_{11})_{\rm w}}\frac{d}{b_{\rm f}}$$



$$\widehat{s}_{yy} = 1.2\left(\frac{\left(\alpha_{66}^{\nu}\right)_{\rm f1}}{b_{\rm f1}(1+\delta_{\rm sc})^2} + \frac{\left(\alpha_{66}^{\nu}\right)_{\rm f2}}{b_{\rm f2}\left(1+\frac{1}{\delta_{\rm sc}}\right)^2}\right)$$

$$\widehat{s}_{zz} = \frac{\left(\alpha_{66}^{\nu}\right)_{\rm w}}{d} + \frac{1}{12}\frac{\left(\alpha_{66}^{\nu}\right)_{\rm f1} b_{\rm f1}}{d^2\gamma_1^2} + \frac{1}{12}\frac{\left(\alpha_{66}^{\nu}\right)_{\rm f2} b_{\rm f2}}{d^2\gamma_2^2}$$

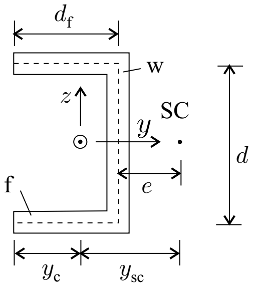
$$\widehat{s}_{\omega\omega} = \frac{1.2}{d^2}\left(\frac{\left(\alpha_{66}^{\nu}\right)_{\rm f1}}{b_{\rm f1}} + \frac{\left(\alpha_{66}^{\nu}\right)_{\rm f2}}{b_{\rm f2}}\right)$$

$$\widehat{s}_{y\omega} = \frac{1.2}{d}\left(-\frac{\left(\alpha_{66}^{\nu}\right)_{\rm f1}}{b_{\rm f1}(1+\delta_{\rm sc})} + \frac{\left(\alpha_{66}^{\nu}\right)_{\rm f2}}{b_{\rm f2}\left(1+\frac{1}{\delta_{\rm sc}}\right)}\right)$$

$$\widehat{s}_{yz} = \widehat{s}_{z\omega} = 0$$

$$\text{where} \quad \delta_{\rm c} = \frac{d-z_{\rm c}}{z_{\rm c}} \qquad \delta_{\rm sc} = \frac{d-e}{e}$$

$$\gamma_1 = 1 + \frac{1}{3}\frac{(\alpha_{11})_{\rm f1}}{(\alpha_{11})_{\rm w}}\frac{d}{b_{\rm f1}\left(1+\frac{1}{\delta_{\rm c}}\right)} \qquad \gamma_2 = 1 + \frac{1}{3}\frac{(\alpha_{11})_{\rm f2}}{(\alpha_{11})_{\rm w}}\frac{d}{b_{\rm f2}(1+\delta_{\rm c})}$$



$$\widehat{s}_{yy} = \frac{\rho_y}{2}\frac{\left(\alpha_{66}^{\nu}\right)_{\rm f}}{d_{\rm f}} + d\frac{1}{3}q_{yu}^2\left(\alpha_{66}^{\nu}\right)_{\rm w}$$

$$\widehat{s}_{zz} = \frac{\left(\alpha_{66}^{\nu}\right)_{\rm w}}{d} + \frac{2}{3}\frac{\left(\alpha_{66}^{\nu}\right)_{\rm f} d_{\rm f}}{d^2\gamma^2}$$

$$\widehat{s}_{\omega\omega} = 2\rho_\omega\frac{1}{d^2}\frac{\left(\alpha_{66}^{\nu}\right)_{\rm f}}{d_{\rm f}} + d\frac{1}{5}q_{\omega u}^2\left(\alpha_{66}^{\nu}\right)_{\rm w}$$

$$\widehat{s}_{z\omega} = \frac{\left(\alpha_{66}^{\nu}\right)_{\rm f}}{d^2}\frac{2(5-3\delta_\omega)}{\gamma(8-4\delta_\omega)}$$

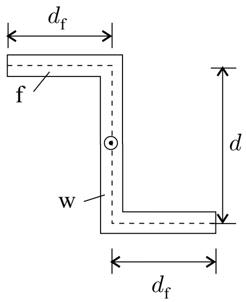
$$\widehat{s}_{yz} = \widehat{s}_{y\omega} = 0$$

$$\text{where} \quad \gamma = 1 + \frac{1}{6}\frac{(\alpha_{11})_{\rm f}}{(\alpha_{11})_{\rm w}}\frac{d}{d_{\rm f}}$$

$$\delta_y = \frac{d_{\rm f}-y_{\rm c}}{y_{\rm c}} \qquad \delta_\omega = \frac{1}{\frac{d_{\rm f}}{e}-1} \qquad \rho_y = \frac{3}{5}\frac{8-9\delta_y+3\delta_y^2}{(2-\delta_y)^2} \approx 1.2$$

$$q_{yu} = \frac{1}{2d_{\rm f}}\frac{3-3\delta_y}{2-\delta_y} \qquad q_{\omega u} = \frac{1}{dd_{\rm f}}\frac{3-3\delta_\omega}{2-\delta_\omega} \qquad \rho_\omega = \frac{3}{5}\frac{8-9\delta_\omega+3\delta_\omega^2}{(2-\delta_\omega)^2} \approx 1.2$$

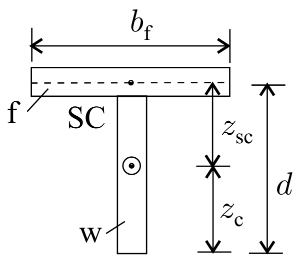
Table A.9. Shear compliances. The layup of each wall segment is orthotropic and unsymmetrical. The coordinate of the centroid z_c is given in Table A.2. The properties are evaluated at each wall segment's "neutral" plane, which is at $\widetilde{\varrho}$. α_{66}^{ν} is given by Eq. (A.4).



$$\widehat{s}_{yy} = 0.6\frac{\left(\alpha_{66}^{\nu}\right)_f}{d_f} + d\frac{1}{5}q_{yz}^2\left(\alpha_{66}^{\nu}\right)_w$$
$$\widehat{s}_{zz} = \frac{\left(\alpha_{66}^{\nu}\right)_w}{d} + \frac{4}{15}\frac{\left(\alpha_{66}^{\nu}\right)_f d_f}{d^2\beta^2}$$
$$\widehat{s}_{\omega\omega} = 2.4\frac{1}{d^2}\frac{\left(\alpha_{66}^{\nu}\right)_f}{d_f} + d\frac{1}{3}q_{\omega z}^2\left(\alpha_{66}^{\nu}\right)_w$$
$$\widehat{s}_{yz} = -\frac{\left(\alpha_{66}^{\nu}\right)_f}{d\beta}\frac{2-3\delta_y}{20-10\delta_y} + \frac{\left(\alpha_{66}^{\nu}\right)_w}{5}\left(1-\frac{1}{\beta}\right)q_{yz}$$
$$\widehat{s}_{y\omega} = \widehat{s}_{z\omega} = 0$$

where

$$\gamma = 1 + \frac{1}{6}\frac{(\alpha_{11})_f}{(\alpha_{11})_w}\frac{d}{d_f} \qquad \beta = 4\gamma - 3$$
$$q_{yz} = \frac{3}{4d_f}\left(1-\frac{1}{\beta}\right) \qquad \delta_\omega = \frac{1}{1+\frac{(\alpha_{11})_f}{(\alpha_{11})_w}\frac{d}{d_f}}$$
$$q_{\omega z} = \frac{1}{dd_f}\frac{3-3\delta_\omega}{2-\delta_\omega} \qquad \delta_y = \frac{3-4q_{yz}d_f}{3-2q_{yz}d_f}$$



$$\widehat{s}_{yy} = 1.2\frac{\left(\alpha_{66}^{\nu}\right)_f}{b_f}$$
$$\widehat{s}_{zz} = \rho\frac{\left(\alpha_{66}^{\nu}\right)_w}{b_w} + \frac{1}{12}\left(\alpha_{66}^{\nu}\right)_f b_f q_{zt}^2$$
$$\widehat{s}_{\omega\omega} = \infty$$
$$\widehat{s}_{yz} = \widehat{s}_{y\omega} = \widehat{s}_{z\omega} = 0$$

where

$$q_{zt} = \frac{1}{d}\frac{3-3\delta_z}{2-\delta_z} \qquad \delta_z = \frac{d-z_c}{z_c} \qquad \rho = \frac{3}{5}\frac{8-9\delta_z+3\delta_z^2}{(2-\delta_z)^2} \approx 1.2$$