8

Lamina Flexural Response

As defined in the three previous chapters, pure, uniform tension, compression, and shear loadings must be individually applied to establish the fundamental strength and stiffness properties of a composite material. A flexure test, i.e., the bending of a beam, typically induces tensile, compressive, and shear stresses simultaneously. Thus it is not usually a practical means of determining the fundamental properties of a composite material [1,2].

Nevertheless, flexure tests are popular, because of the simplicity of both specimen preparation and testing, as will be discussed subsequently. Gripping of the specimen, the need for end tabs, obtaining a pure stress state, avoiding buckling, and most of the other concerns discussed in the previous three chapters are usually nonissues when conducting a flexure test.

Flexural testing can, for example, be a simple method of monitoring quality during a structural fabrication process. The usual objective of a flexure test is to determine the flexural strength and flexural modulus of the beam material. This might be particularly relevant if the component being fabricated is to be subjected to flexural loading in service. However, because of the complex stress state present in the beam, it is typically not possible to directly relate the flexural properties obtained to the fundamental tensile, compressive, and shear properties of the material.

8.1 Testing Configurations

Figure 8.1 indicates the configuration of the ASTM D 790 three-point flexure test [3]. This standard was created in 1970 by the plastics committee within ASTM for use with unreinforced and reinforced plastics and electrical insulating materials, as its title suggests. For more than 25 years, until 1996 when it was removed, this standard also included four-point flexure. In response to demands by the composite materials community, in 1998 a new standard, ASTM D 6272, was introduced by the plastics committee for the same classes of plastics, but specifying four-point flexure [4]. That is, two standards now exist. Thus, the composites committee of ASTM is presently writing its own flexural test standard specifically for composite materials, ASTM D XXXX-02,

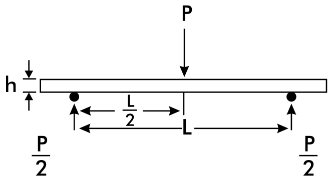

FIGURE 8.1
Three-point flexure loading configuration.

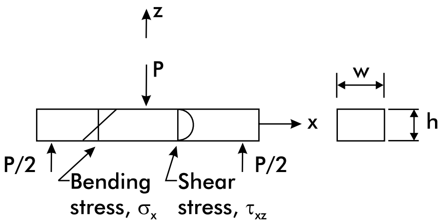

FIGURE 8.2
Stresses in a beam subjected to three-point flexure.

which includes both three-point and four-point flexure [5]. The three standards differ sufficiently in detail that it is advisable to refer to all three for guidance. It is unfortunate, but understandable because of its long existence, that many experimentalists still (incorrectly) only quote ASTM D 790 as the governing standard for all flexural testing.

Analysis of a macroscopically homogeneous beam of linearly elastic material [6] indicates that an applied bending moment is balanced by a linear distribution of normal stress, σ_x, as shown in Figure 8.2. For the three-point flexural loading shown, the top surface of the beam is in compression while the bottom surface is in tension. Assuming a beam of rectangular cross section, the midplane contains the neutral axis and is under zero bending stress. The interlaminar shear stress, τ_{xz}, is maximum on this midplane, varying parabolically from zero on the free surfaces as shown [6]. For three-point flexure, the shear stress is constant along the length of the beam, and directly proportional to the applied force P. However, the flexural stresses, in addition to being directly proportional to P, vary linearly with position along the length of the beam, and are zero at each end support and maximum at the center. Thus, the stress state is highly dependent on the support span length-to-specimen thickness ratio (L/h). Beams with small L/h ratios are dominated by shear. As discussed in Chapter 7, a short-span (L/h = 4) three-point flexure test is commonly used for interlaminar shear strength determination. Beams with long spans usually fail in tension or compression. Typically, composite materials are stronger in tension than compression. Also, the concentrated loading is applied at the point of maximum compressive stress in the beam, often inducing local stress concentrations. Thus, the composite beam usually fails in compression at the midspan loading point.

Although testing of a unidirectional lamina is the primary subject of the present chapter, laminates of various other orientations can also be tested in flexure, as will be discussed in more detail in Chapter 11.

Flexural testing of a unidirectional lamina is generally limited to beams with the fibers aligned parallel to the beam axis. That is, 0° flexural properties are determined. Beams with fibers oriented perpendicular to the beam axis almost always fail in transverse tension on the lower surface because the transverse tensile strength of most composite materials is lower than the transverse compressive strength, usually by a factor of three or more, and is lower than the interlaminar shear strength [7]. In fact, the transverse tensile flexure test has been suggested as a simple means of obtaining the transverse tensile strength of a unidirectional composite [7], although there has been no movement to standardize it as such.

As do the other two flexural test standards, ASTM D XXXX requires that a sufficiently large support span-to-specimen thickness ratio be chosen, "such that failures occur in the outer fibers of the specimens, due only to the bending moment." The standard recommends support span-to-thickness ratios of 16:1, 20:1, 32:1, 40:1, and 60:1, indicating that, "as a general rule support span-to-thickness ratios of 16:1 are satisfactory when the ratio of the tensile strength to shear strength is less than 8 to 1." High-strength unidirectional composites can have much higher strength ratios, requiring correspondingly higher support span-to-thickness ratios. For example, ASTM D XXXX suggests a ratio of 32:1 for such materials. These ratios are specified to be the same for three-point and four-point flexure.

Per ASTM D XXXX, the diameter of the loading noses and supports should be 6 mm. The other two ASTM standards have slightly different requirements. This perhaps confirms experimental observations that the test results obtained are not strongly influenced by the specific diameters used, as long as the diameters are not so small that local bearing damage of the composite material occurs [8].

It has become customary when testing in four-point flexure to use either "third-point" or "quarter-point" loading, as shown in Figure 8.3. For third-point loading, the loading points are each positioned one third of the support span length from the respective support, and hence are also one third of the support span length from each other. For quarter-point loading, the loading points are each positioned one quarter of the support span length from the respective support, and hence are one half of the support span length from each other. ASTM D XXXX includes only quarter-point loading.

Although the specimen thickness can be arbitrary (as long as the recommended span-to-thickness ratio is maintained), ASTM D XXXX suggests a 100-mm-long, 2.4-mm-thick, 13-mm-wide specimen for high-strength laminae, supported on an 76.8-mm span. This results in a specimen overhang of 11.6 mm at each end. Suggestions are given for other types of composites as well. The other two ASTM standards recommend slightly different specimen sizes and amounts of overhang. Again, this suggests that the exact specimen size is not critical.

FIGURE 8.3
Four-point flexure test configurations: (a) quarter-point loading, and (b) third-point loading.

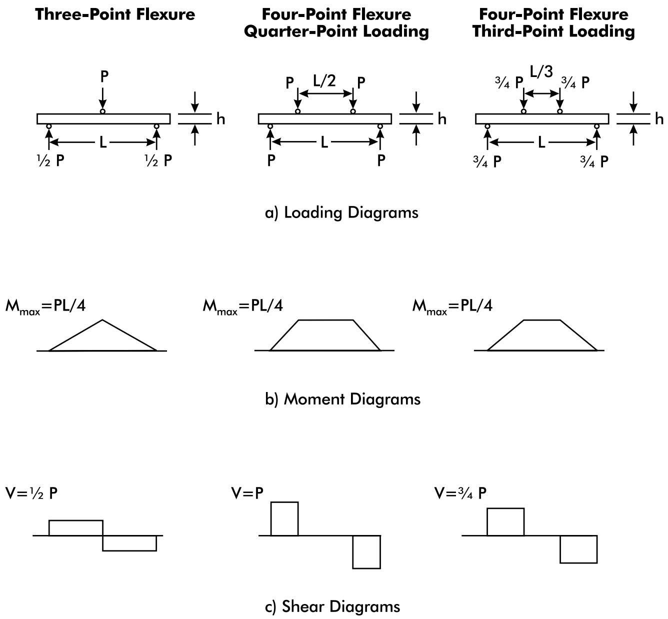

FIGURE 8.4
Required loadings for equal maximum bending moment in various beam configurations, with corresponding vertical shear force distributions: (a) loading diagrams, (b) moment diagrams, and (c) shear diagrams.

8.2 Three- Vs. Four-Point Flexure

As noted in the previous section, either three- or four-point flexure can be, and is, used. The various ASTM standards make no specific recommendations concerning when to use each test. In fact, there is no clear advantage of one test over the other, although there are significant differences. Figure 8.4 indicates the required loadings and the corresponding bending moment (M) and transverse shear force (V) distributions in the beam for each of the loadings.

For three-point flexure the maximum bending moment in the beam, and hence the location of the maximum tensile and compressive flexural stresses, is at midspan and is equal to M_{max} = PL/4. For four-point flexure with loading at the quarter points, attaining the same maximum bending moment, i.e., M_{max} = PL/4, requires twice the testing machine force, i.e., 2P; this maximum bending moment is constant over the entire span L/2 between the applied loads (Figure 8.4(b)). For four-point flexure with loading at the one-third points, attaining the same maximum bending moment, i.e., $M_{max} = \frac{3}{4} P (L/3) = PL/4$, requires 50% more testing machine force, i.e., 1.5P, than for three-point flexure. This maximum bending moment is constant over the entire span L/3 between the applied loads.

Thus, for quarter-point loading the force exerted by the testing machine must be twice as high, and for third-point loading the force must be one and one-half times as high, as for three-point flexure. Normally this is not in itself a significant factor because the loads required to fail a beam in flexure are not extremely high. What may be more significant is the magnitude of the required concentrated force at the loading point(s), and the magnitude of the corresponding induced stress concentrations, because the flexural stresses are a maximum at these locations. As noted in Figure 8.4, for both three-point flexure and four-point flexure with quarter-point loading, the maximum concentrated force on the beam is P. However, for four-point flexure with third-point loading, it is only $\frac{3}{4}$ P. This indicates an advantage for third-point loading.

There are other considerations, however. The maximum transverse shear force, V, and hence the maximum interlaminar shear stress in the beam, also varies with the type of loading, as indicated in Figure 8.4. For three-point loading the shear force is equal to $\frac{1}{2}$ P and is constant over the entire support span. For four-point flexure, quarter-point loading, it is equal to P and exists only over the end quarters of the beam. For four-point flexure, third-point loading, it is equal to $\frac{3}{4}$ P and exists only over the end thirds of the beam. As discussed in the previous section, it is desirable to keep the ratio of interlaminar shear stress to flexural stress sufficiently low to avoid shear failures at the midplane rather than tensile or compressive failures at the beam surfaces. As explained, this is normally achieved by controlling the span length-to-specimen thickness ratio. Assuming the same specimen thickness for all loadings, three-point flexure ($V = \frac{1}{2}$ P) would be preferred, because it minimizes the required support span length, followed by four-point flexure, third-point loading ($V = \frac{3}{4}$ P).

One additional consideration should be noted, although it is usually of lesser importance. As will be discussed in Section 8.4, it may also be desired to calculate flexural modulus, which is often based on the measured deflection of the beam. The shear stresses in the beam contribute to the total deformation, in proportion to the product of the shear force and the length over which it acts. Because the entire length of the beam in three-point flexure is subjected to the shear force $\frac{1}{2}$ P, but only one half of the four-point flexure, quarter-point loading beam is subjected to the shear force P, and only two thirds of the four-point flexure, third-point loading beam is subjected to the shear force $\frac{3}{4}$ P, the net shear deformation effect is the same in all three cases. This consideration is important when beam deflection is used to determine modulus.

In summary, quarter-point flexure with third-point loading appears to be a good overall choice. However, each of the three loading modes (Figure 8.4) has some individual advantages. Thus, all three are used, with three-point flexure being the most common, perhaps only because it requires the simplest test fixture. A typical test fixture, with adjustable loading and support spans such that it can be used for both three- and four-point flexure, is shown in Figure 8.5.

As an aside, note that three-point flexure is commonly (although perhaps erroneously) referred to as three-point loading, and likewise four-point flexure

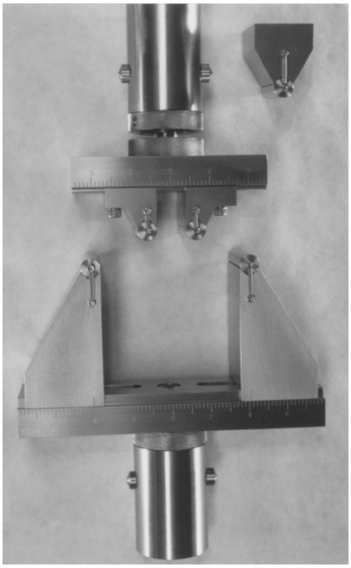

FIGURE 8.5
Photograph of a flexure test fixture with interchangeable three- and four-point loading heads and adjustable spans. (Photograph courtesy of Wyoming Test Fixtures, Inc.)

is referred to as four-point loading. As a result, this has become accepted terminology, included in all three ASTM standards.

8.3 Specimen Preparation and Flexure Test Procedure

The flexure specimen is simply a strip of test material of constant width and thickness. As noted in Section 8.1, for a unidirectional lamina subjected to longitudinal flexure, the suggested dimensions in ASTM D XXXX are support span length, 76.8 mm; specimen total length, 100 mm; specimen width, 13 mm; and specimen thickness, 2.4 mm. Suggested tolerances on these dimensions are also given in the standard.

Although all three ASTM standards specify the use of a deflection-measuring device mounted under the midspan of the specimen, occasionally a strain gage is used instead. One longitudinal strain gage can be mounted at the midspan on the tension side (bottom surface) of the specimen. The test fixture support span is to be set according to the beam thickness, specimen material properties, and fiber orientation, as discussed previously. ASTM D XXXX specifies that the specimen is to be loaded at a testing machine crosshead rate of 1 mm/min. ASTM D 790 specifies that the testing machine crosshead rate, $\dot{x}$, be selected such that the maximum strain rate (of the surface fibers) is, $\dot{\varepsilon} = 0.01$/min. This leads to [1],

$$\dot{x} = \frac{\dot{\varepsilon} L^2}{6h} \tag{8.1}$$

Commonly, a crosshead rate in the range of 1 to 5 mm/min is selected.

The beam deflection, δ, is measured using a calibrated linear variable differential transformer (LVDT) at the beam midspan. Alternatively, the beam displacement may be approximated as the travel of the testing machine crosshead if the components of this travel that are due to the machine compliance and to the indentations of the specimen at the loading and support points are subtracted out. If a strain gage is used, the specimen is to be placed in the fixture with the strain gage on the tension side of the beam and centered at midspan. The strain or displacement readings may be recorded continuously or at discrete load intervals. If discrete data are recorded, the load and strain–displacement readings should be taken at small load intervals, with at least 25 points in the linear response region (so that an accurate flexural modulus can be determined). The total number of data points should be enough to accurately describe the complete beam response to failure.

8.4 Data Reduction

From simple beam theory [6], the tensile and compressive stresses at the surfaces of the beam at any location where the bending moment is a maximum is

$$\sigma_{max} = \frac{M_{max}(h/2)}{I} \tag{8.2}$$

where M_{max} is the maximum bending moment, h is the thickness of the beam, and $I = wh^3/12$ is the moment of inertia of a beam of rectangular cross section, with w being the beam width.

For three-point flexure, for example, the bending moment is maximum, i.e., $M_{max} = PL/4$, at the midlength of the beam. Substitution into Equation (8.2) gives

$$\sigma_{max} = \frac{3PL}{2wh^2} \tag{8.3}$$

This equation enables construction of a stress–strain plot from the recorded load vs. displacement or load vs. strain data.

A reasonable approximation for most materials is that the modulus in tension is the same as in compression. If a strain gage is used, the initial slope of the flexural stress–strain plot can be obtained using a linear least-squares fit. Figures 8.6 and 8.7 show typical stress–strain curves obtained from three-point flexure tests.

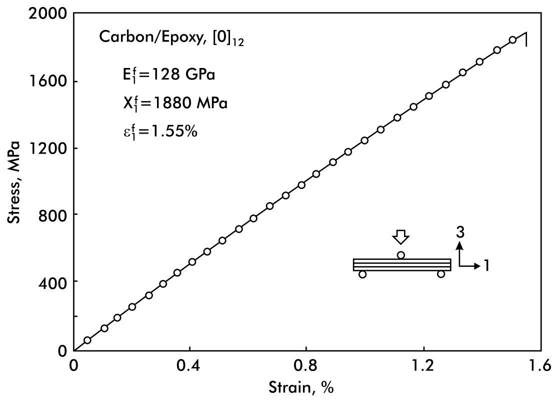

FIGURE 8.6
Flexural stress–strain response of a $[0]_{12}$ carbon/epoxy test specimen.

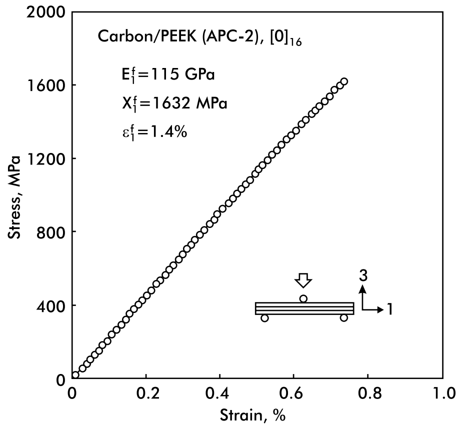

FIGURE 8.7
Flexural stress–strain response of a $[0]_{16}$ carbon/PEEK test specimen.

For the case where the three-point flexure specimen is not strain-gaged, the flexural modulus may be determined from a plot of load, P, vs. center deflection, δ, as

$$E_f = \frac{L^3}{4wh^3}\frac{\Delta P}{\Delta \delta} \tag{8.4}$$

This relation, however, assumes that shear deformation is negligible. For $[90]_n$ beams (fibers perpendicular to the beam axis), shear deformation is generally insignificant and Equation (8.4) should be accurate. On the other hand, the results of Zweben [9] for $[0]_n$ beams (fibers parallel to the beam axis) shown in Figure 8.8 illustrate that for certain unidirectional composites, large support span-to-thickness ratios, L/h, are required to minimize the influence of shear deformation and thus produce an accurate flexural modulus.

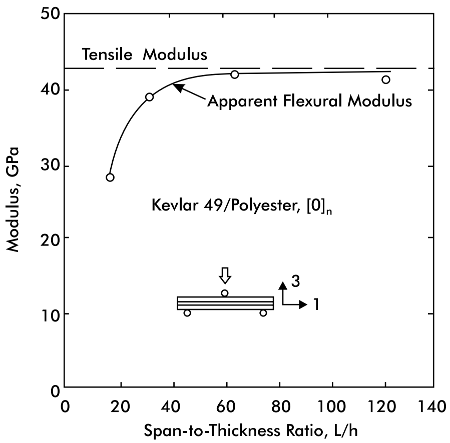

FIGURE 8.8
Apparent flexural modulus of a $[0]_n$ Kevlar 49–polyester specimen as a function of span-to-thickness ratio [9].

If a flexural test is conducted for which the shear deformation component is significant, the following equation may be used to evaluate the flexural modulus:

$$E_f = \frac{L^3}{4wh^3}\left(1 + \frac{6h^2E_f}{5L^2G_{13}}\right)\frac{\Delta P}{\Delta \delta} \tag{8.5}$$

where G_{13} is the interlaminar shear modulus. The second term in the parentheses of Equation (8.5) is a shear correction factor that may be significant for composites with a high axial modulus and a low interlaminar shear modulus. The flexural and shear moduli (E_f and G_{13}), however, may not be known prior to the test. For Equation (8.5), the modulus E_f may be replaced by the tensile modulus, E_1, and the shear modulus can be approximated as the in-plane shear modulus, G_{12}, assuming the fibers are oriented along the beam axis. If the shear correction factor is not determined, the flexural modulus can be evaluated using Equation (8.4) for tests conducted at increasing span lengths until a constant value is achieved, as suggested in Figure 8.8.

Additional details of data reduction, including for four-point flexure, can be found in the three ASTM flexural testing standards cited in this chapter.

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