12

Laminate Thermoelastic Response

The thermoelastic response of a general laminate may be very complex [1]. For the particular case of unsymmetric laminates, bending–extension coupling, Equations (2.44) and (2.45) indicate the existence of out-of-plane deflections for a laminate subject to a temperature change. Hyer [2] and Dang and Hyer [3] have performed very detailed experiments and analysis of warping deformations of unsymmetric composite laminates during cooling from elevated (cure) temperatures. For symmetric laminates, however, it can be shown that the bending–extension coupling disappears, $[B] = [0]$. For a balanced laminate, $A_{16} = A_{26} = 0$. Hence, a symmetric and balanced laminate behaves as a homogeneous orthotropic material in a macroscopic sense. Typical balanced symmetric laminates are $[0/\pm 45/90]_s$, $[0_2/\pm 45]_s$, and $[0_2/90_2]_s$.

For a symmetric and balanced laminate, Equation (2.51) yields

$$\begin{bmatrix} \varepsilon^\circ \\ \kappa \end{bmatrix} = \begin{bmatrix} A' & 0 \\ 0 & D' \end{bmatrix} \begin{bmatrix} N^T \\ M^T \end{bmatrix} \tag{12.1}$$

where $[N^T]$ and $[M^T]$ are given by Equations (2.37) and (2.38). It may also be shown that the thermal moment resultants vanish, $[M^T] = [0]$, and the thermal in-plane shear force resultant $N_{xy}^T = 0$. Equations (12.1) then yield

$$\left[\varepsilon^0\right] = \left[A'\right]\left[N^T\right] \tag{12.2a}$$

$$[\kappa] = [0] \tag{12.2b}$$

Hence, a symmetric laminate does not bend due to a temperature change (Equation (12.2b)). The expanded form of Equation (12.2a) is

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} A'_{11} & A'_{12} & 0 \\ A'_{12} & A'_{22} & 0 \\ 0 & 0 & A'_{66} \end{bmatrix} \begin{bmatrix} N_x^T \\ N_y^T \\ 0 \end{bmatrix} \tag{12.3}$$

Consequently, $\gamma_{xy} = 0$, which shows that a balanced laminate will not deform in shear due to the temperature change.

The thermal expansion coefficients of the laminate, α_x and α_y, are obtained from

$$\alpha_x = \frac{\varepsilon_x}{\Delta T} \tag{12.4a}$$

$$\alpha_y = \frac{\varepsilon_y}{\Delta T} \tag{12.4b}$$

where ΔT is the temperature change from the reference state. Combining Equations (12.3) and (12.4) yields

$$\alpha_x = \left(A'_{11} N_x^T + A'_{12} N_y^T\right) / \Delta T \tag{12.5a}$$

$$\alpha_y = \left(A'_{12} N_x^T + A'_{22} N_y^T\right) / \Delta T \tag{12.5b}$$

where the compliance elements, A'_{ij}, are

$$A'_{11} = \frac{A_{22}}{A_{11}A_{22} - A_{12}^2} \tag{12.6a}$$

$$A'_{12} = \frac{-A_{12}}{A_{11}A_{22} - A_{12}^2} \tag{12.6b}$$

$$A'_{22} = \frac{A_{11}}{A_{11}A_{22} - A_{12}^2} \tag{12.6c}$$

To determine the laminate thermal expansion coefficients, the effective thermal forces $[N_x^T, N_y^T]$ and the stiffnesses A_{11}, A_{12}, and A_{22} are calculated using Equations (2.37) and (2.35a), respectively. Such calculation requires knowledge of the basic ply (lamina) mechanical properties and thermal expansion coefficients. The calculation of the laminate thermal expansions is quite involved. It is recommended that a computer code be used.

12.1 Preparation of Test Specimens and Measurement of Thermal Expansion

The test specimen used for determining thermal expansion coefficients should be a representative 50×50 mm flat panel of the laminate. Apply two strain gages and one temperature sensor (or thermocouple) according to the procedure outlined in Chapter 10. Align the gages with the principal directions of the laminate. Follow the procedures outlined in Chapter 10 when measuring the laminate thermoelastic response.

12.2 Data Reduction

Plot the laminate expansional strains ε_x and ε_y vs. temperature or temperature change, $\Delta T = T - T_0$, where T_0 is the initial (reference) temperature of the laminate. Figure 12.1 shows typical results for a quasi-isotropic $[0/\pm 45/90]_s$ carbon/epoxy laminate. To determine the thermal expansion coefficient in the actual temperature range, evaluate the slope of the strain vs. temperature plot. Hysteresis will be noted at higher temperatures in Figure 12.1b. However, at lower temperatures, the heating and cooling slopes are consistent within experimental error.

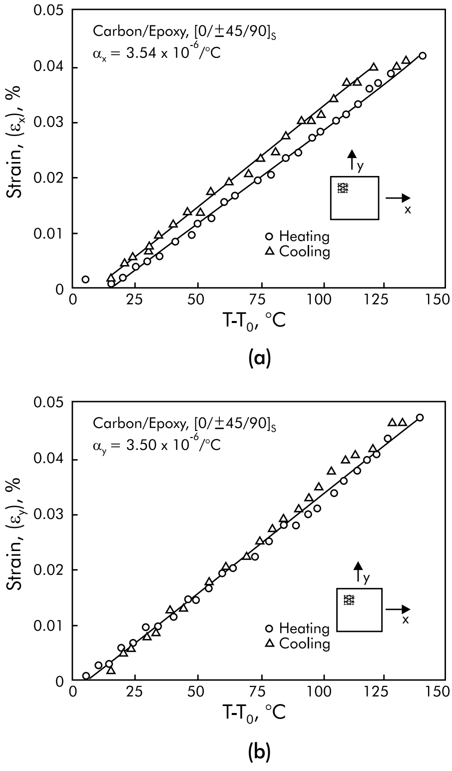

FIGURE 12.1
Thermal expansion strains for a quasi-isotropic $[0/\pm 45/90]_s$ carbon/epoxy laminate: (a) ε_x, and (b) ε_y.

12.3 Analysis of Thermoelastic Response

The following thermal expansion coefficients were evaluated from the thermal expansion strain data shown in Figure 12.1 for the quasi-isotropic $[0/\pm 45/90]_s$ laminate: $\alpha_x = 3.54 \times 10^{-6}/°C$, and $\alpha_y = 3.50 \times 10^{-6}/°C$. The following set of ply properties were independently measured:

$$E_1 = 140 \text{ GPa} \qquad \alpha_1 = -0.7 \times 10^{-6}/°C$$

$$E_2 = 10.3 \text{ GPa} \qquad \alpha_2 = 31.2 \times 10^{-6}/°C$$

$$\nu_{12} = 0.29 \qquad G_{12} = 5.15 \text{ GPa}$$

Calculation of the thermal expansion coefficients for the quasi-isotropic $[0/\pm 45/90]_s$ laminate from Equations (12.5) gives

$$\alpha_x = \alpha_y = 3.30 \times 10^{-6}/°C$$

This value is in reasonable good agreement with those experimentally observed.

Table 12.1 displays thermal expansion data for a carbon/epoxy $[0/\pm 60/0]_s$ laminate with the following ply properties:

$$E_1 = 160 \text{ GPa} \qquad \alpha_1 = 0.64 \times 10^{-6}/°C$$

$$E_2 = 9.2 \text{ GPa} \qquad \alpha_2 = 28.1 \times 10^{-6}/°C$$

$$\nu_{12} = 0.33 \qquad G_{12} = 5.24 \text{ GPa}$$

The experimental and predicted coefficients of thermal expansion listed in Table 12.1 can be compared. It is observed that the data are reasonably consistent upon heating and cooling. The analysis somewhat underpredicts α_x and overpredicts α_y. Note also the significant anisotropy in thermal expansion for this lay-up.

TABLE 12.1

Measured and Predicted Thermal Expansion Coefficients (in units of $10^{-6}/°C$ for a $[0/\pm 60/0]_s$ carbon/epoxy [IM6/3501-6] laminate)

CTE	Measured	Predicted
α_x	1.97 (H)[a] 2.04 (C)[a]	1.68
α_y	3.15 (H) 3.41 (C)	3.94

[a] H and C represent heating and cooling, respectively.

References

1. M.F. Hyer, *Stress Analysis of Fiber-Reinforced Composite Materials*, WCB/McGraw-Hill, Boston, 1998.
2. M.W. Hyer, Calculations of the room temperature shape of unsymmetric laminates, *J. Compos. Mater.*, 15, 296–310, 1981.
3. M.-L. Dang and M.W. Hyer, Thermally-induced deformation behavior of unsymmetric laminates, *Int. J. Solids Struct.*, 35, 2101–2120, 1998.