Appendix A

Compliance and Stiffness Transformations and Matrix Operations

Transformation of plane stress compliance (S_{ij}) and stiffness (Q_{ij}) elements:

$$
\begin{aligned}
\bar{S}_{11} &= m^4 S_{11} + m^2 n^2 (2S_{12} + S_{66}) + n^4 S_{22} \\
\bar{S}_{12} &= m^2 n^2 (S_{11} + S_{22} - S_{66}) + S_{12}(m^4 + n^4) \\
\bar{S}_{22} &= n^4 S_{11} + m^2 n^2 (2S_{12} + S_{66}) + m^4 S_{22} \\
\bar{S}_{16} &= 2m^3 n (S_{11} - S_{12}) + 2mn^3 (S_{12} - S_{22}) - mn(m^2 - n^2) S_{66} \\
\bar{S}_{26} &= 2mn^3 (S_{11} - S_{12}) + 2m^3 n (S_{12} - S_{22}) + mn(m^2 - n^2) S_{66} \\
\bar{S}_{66} &= 4m^2 n^2 (S_{11} - S_{12}) - 4m^2 n^2 (S_{12} - S_{22}) + (m^2 - n^2) S_{66}
\end{aligned}
\tag{A.1}
$$

$$
\begin{aligned}
\bar{Q}_{11} &= m^4 Q_{11} + 2m^2 n^2 (Q_{12} + 2Q_{66}) + n^4 Q_{22} \\
\bar{Q}_{12} &= m^2 n^2 (Q_{11} + Q_{22} - 4Q_{66}) + (m^4 + n^4) Q_{12} \\
\bar{Q}_{22} &= n^4 Q_{11} + 2m^2 n^2 (Q_{12} + 2Q_{66}) + m^4 Q_{22} \\
\bar{Q}_{16} &= m^3 n (Q_{11} - Q_{12}) + mn^3 (Q_{12} - Q_{22}) - 2mn(m^2 - n^2) Q_{66} \\
\bar{Q}_{26} &= mn^3 (Q_{11} - Q_{12}) + m^3 n (Q_{12} - Q_{22}) + 2mn(m^2 - n^2) Q_{66} \\
\bar{Q}_{66} &= m^2 n^2 (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}) + (m^4 + n^4) Q_{66}
\end{aligned}
\tag{A.2}
$$

The matrices [A′], [B′], and [D′] may be determined from

$$
\begin{aligned}
[A'] &= [A^*] - [B^*][D^*]^{-1}[C^*] \\
[B'] &= [B^*][D^*]^{-1} \\
[D'] &= [D^*]^{-1}
\end{aligned}
\tag{A.3}
$$

where

$$[A^*] = [A]^{-1}$$

$$[B^*] = -[A]^{-1}[B] \tag{A.4}$$

$$[C^*] = [B][A]^{-1}$$

$$[D^*] = [D] - [B][A]^{-1}[B]$$

For symmetric laminates, [B] = [0]. For that case,

$$[A'] = [A]^{-1}$$

$$[B'] = [0] \tag{A.5}$$

$$[D'] = [D]^{-1}$$