13

Open-Hole Tensile and Compressive Strengths of Laminates

Experiments have shown that the tensile and compressive strengths of a composite laminate containing a hole or notch depend on hole or notch size. Because of the complexity of the fracture process in notched laminates, most strength models are semiempirical. In this chapter some of the more commonly accepted and computationally simple strength models, i.e., the point and average stress criteria developed by Whitney and Nuismer [1] will be discussed. In addition, a modification of the point stress criterion, proposed by Pipes et al. [2], will be introduced.

Reasons for the substantial tensile and compressive strength reductions of composites because of holes and notches are the brittleness of the material and the large stress concentration factors brought about by the anisotropy of the material. These strength reductions are not necessarily the same for tensile and compressive loading because the failure modes are typically different.

As discussed in Chapter 2, the stress concentration factor for a plate containing a circular hole of radius, R (Figure 13.1) is

$$K = \frac{\sigma_x(R,0)}{\bar{\sigma}_x} \tag{13.1}$$

where R is the hole radius, and $\bar{\sigma}_x$ is the average normal stress applied on the horizontal boundaries of the plate (Figure 13.1). For an infinite plate, i.e., where L,w $\rightarrow \infty$, Lekhnitski [3] derived the following expression

$$K_\infty = 1 + \sqrt{2\left(\sqrt{E_x/E_y} - \nu_{xy} + E_x/\left(2G_{xy}\right)\right)} \tag{13.2}$$

where E_x, E_y, ν_{yx}, and G_{xy} are the effective engineering constants of the plate. Note that the x-axis is oriented along the loading direction, and the y-axis is oriented transverse to the loading direction.

It is observed from Equation (13.2) that the stress concentration factor for an infinite plate is independent of hole radius. For an ideally brittle infinite plate, the notched strength would thus be

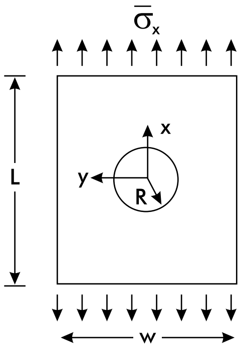

FIGURE 13.1
Finite-size plate containing a hole of diameter D = 2R subject to uniaxial tension.

$$\sigma_N = \sigma_0 / K_\infty \tag{13.3}$$

where σ_0 is the strength of the plate without a hole, i.e., the unnotched strength. Experiments, however, show that the strength of composite plates containing large holes is much less than that observed for small holes [1,2]. Such a difference for large plates cannot be explained by a net area reduction. Consequently, there must be factors other than the stress concentration factor controlling the notched strength. Consideration of the normal stress distribution across the ligaments of the plate adjacent to the hole reveals some interesting features. The approximate stress distribution in an infinite plate containing a circular hole is [4]

$$\sigma_x(y,0) = \frac{\sigma_x(\infty)}{2}\left[2 + \xi^2 + 3\xi^4 - (K_\infty - 3)(5\xi^6 - 7\xi^8)\right] \tag{13.4}$$

where $\xi = y/R$, and $\sigma_x(\infty)$ is the far-field normal stress. Figure 13.2 shows the stress, $\sigma_x(y,0)/\sigma_x(\infty)$, across a ligament for isotropic plates containing holes of two sizes (R/R_0 = 0.1 and 1.0), where R_0 is a reference radius. It is observed that the volume of material subject to a high stress is much more localized for the plate with a smaller hole, thus leading to a greater opportunity for stress redistribution to occur, explaining the increased notched strength with decreased hole size.

13.1 Point and Average Stress Criteria

The point and average stress criteria [1] incorporate the hole size effect in computationally simple fracture criteria where failure of the notched laminate is assumed to occur when the stress, σ_x, at a certain distance d_0 ahead

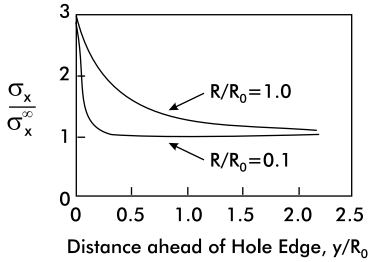

FIGURE 13.2
Normal stress distributions ahead of the hole edge for isotropic plates containing holes of two sizes.

of the notch reaches the unnotched strength, σ_0 (point stress criterion [PSC]), or the stress, σ_x, averaged over a certain distance across the ligament reaches the unnotched strength (average stress criterion [ASC]). Mathematically, these criteria can be expressed as

$$\text{PSC: } \sigma_x(R + d_0, 0) = \sigma_0 \tag{13.5a}$$

$$\text{ASC: } \frac{1}{a_0}\int_R^{R+a_0} \sigma_x(y,0)\,dy = \sigma_0 \tag{13.5b}$$

13.1.1 Point Stress Criterion (PSC)

Combination of the PSC (Equation (13.5a)) and the expression for the stress distribution (Equation (13.4)) yields

$$\frac{\sigma_N}{\sigma_0} = \frac{2}{2 + \lambda^2 + 3\lambda^4 - (K_\infty - 3)(5\lambda^6 - 7\lambda^8)} \tag{13.6}$$

where

$$\lambda = \frac{R}{R + d_0} \tag{13.7}$$

Note that for very large holes, d_0 is small compared with R, and Equation (13.6) gives

$$\frac{\sigma_N}{\sigma_0} = 1/K_\infty \tag{13.8}$$

Consequently, the notched strength ratio for a large hole is given by the inverse of the stress concentration factor. Furthermore, a notch-insensitive

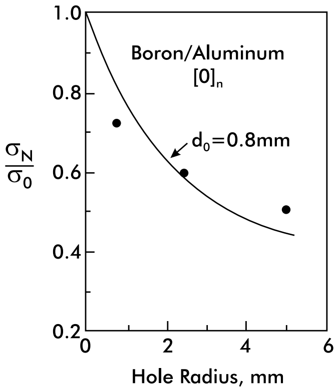

FIGURE 13.3
Experimental data on notched strength of a boron–aluminum composite and predictions based on the point stress criterion. (From R.F. Karlak, *Proceedings of a Conference on Failure Models in Composites (III)*, American Society for Metals, Chicago, 1977. With permission.)

laminate is characterized by a large d_0 in comparison to R. For that case, $\lambda \approx 0$ in Equation (13.6) and $\sigma_N/\sigma_0 \approx 1.0$.

The PSC thus contains two parameters (d_0, σ_0) that have to be determined by experiment. Having established d_0 and σ_0, the PSC allows for strength predictions of laminates containing holes of arbitrary size. Figure 13.3 shows σ_N/σ_0 plotted vs. hole size for a unidirectional $[0]_n$ boron/aluminum composite [3]. Reasonable agreement with experimental data is observed.

13.1.2 Average Stress Criterion (ASC)

Substitution of the stress distribution (Equation (13.4)) into the ASC (Equation (13.5b)) yields, after integration, the following expression for the notched laminate strength

$$\frac{\sigma_N}{\sigma_0} = \frac{2}{(1+\delta)\left(2+\delta^2+(K_\infty-3)\delta^6\right)} \tag{13.9}$$

with

$$\delta = \frac{R}{R+a_0} \tag{13.10}$$

Figure 13.4 shows experimental strength data for a $[0_2/\pm 45]_s$ carbon/epoxy laminate [5]. Experimental results are in good agreement with the ASC with a_0 = 5 mm.

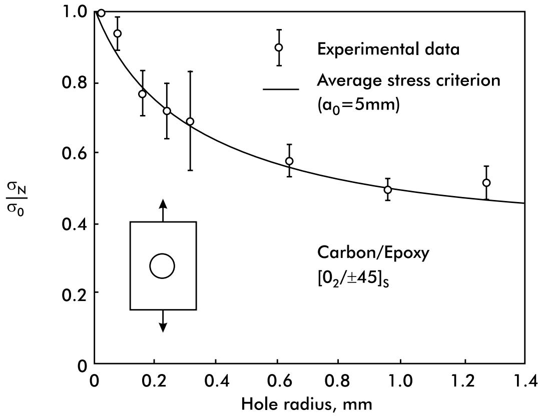

FIGURE 13.4
Notched strength data and predictions based on the average stress criterion for a notched $[0_2/\pm 45]_s$ carbon/epoxy laminate [4].

13.1.3 Modification of PSC

To improve the accuracy of notched strength predictions using the PSC, Pipes et al. [2], following Karlak's modification [3], let the characteristic distance, d_0 (Equation (13.6)) become a power function of hole radius

$$d_0 = (R/R_0)^m/C \tag{13.11}$$

where m is an exponential parameter, R_0 is a reference radius, and C is the notch sensitivity factor. In essence, this model adds one more parameter (the exponential parameter) to the PSC. The reference radius may arbitrarily be chosen as R_0 = 1 mm. The parameter λ (Equation (13.7)) then becomes

$$\lambda = 1/(1 + R^{m-1}C^{-1}) \tag{13.12}$$

Figures 13.5 and 13.6 display the influences on notched strength, σ_N/σ_0, of the parameters m and C. Figure 13.5 shows that the exponential parameter affects the slope of the notch sensitivity curve, while Figure 13.6 shows that the notch sensitivity factor shifts the curves along the log R axis without affecting the shape of the curves. The admissible ranges for the parameters are $0 \le m < 1$ and $C \ge 0$. A notch-insensitive laminate is characterized by a large d_0 in comparison to R. This corresponds to $m \to 1$ and $C \to 0$.

Figure 13.7 shows notched strength vs. hole radius for two quasi-isotropic carbon/epoxy laminates with $[\pm 45/0/90]_s$ and $[90/0/\pm 45]_s$ lay-ups and the magnitudes of the corresponding fitting parameters m and C determined as outlined in Section 13.3.

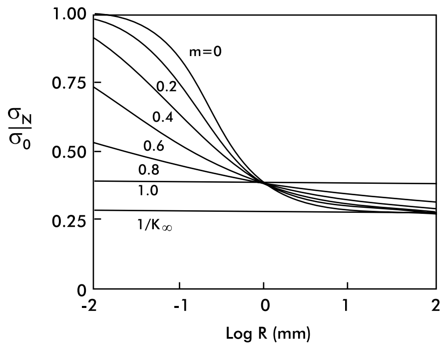

FIGURE 13.5
Influence of exponential parameter on notched strength, C = 10.0 mm^{-1} [2].

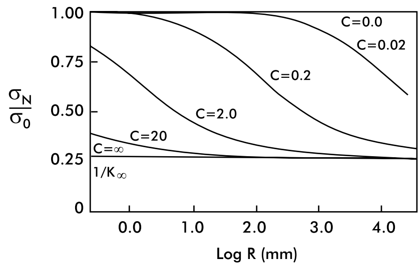

FIGURE 13.6
Influence of notch sensitivity factor on notched strength, m = 0.5, unit of C is mm^{-1} [2].

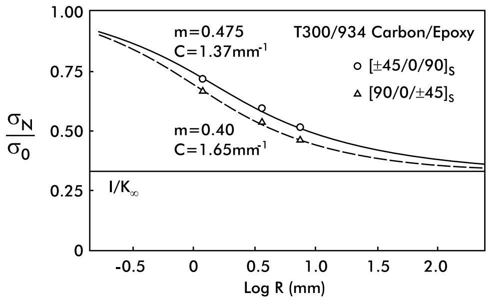

FIGURE 13.7
Notched strength data for [±45/0/90]$_s$ and [90/0/±45]$_s$ carbon/epoxy laminates [2].

13.2 Test Specimen Preparation

Although any laminate configuration can be used, most commonly a [0/±45/90]$_{ns}$ (quasi-isotropic) laminate is selected. Laminates with higher percentages of 0° plies are tested when of specific interest to the intended design application.

Often the same specimen configuration is used for both tensile and compressive open-hole tests. One commonly used specimen size is 305 mm long and 38 mm wide. Another standard open-hole compression test method utilizes a specimen only 75 mm long and 25 mm wide, as will be discussed later. If the test facilities permit, it is strongly recommended that wide specimens be used to accommodate a large range of hole sizes and better approximate an infinitely wide specimen. Daniel [5], for example, used 127-mm-wide laminates and hole diameters ranging from 6.4 to 25.4 mm. Specimen thickness is not critical and is somewhat dependent upon the specific laminate configuration to be tested. A specimen thickness on the order of 2.5 to 5 mm is commonly used. The diameter of the hole in the specimen, which is to be centered at the midlength of the specimen, can also be arbitrarily selected. However, as discussed at the beginning of this chapter, the ratio of specimen width to hole diameter influences the magnitude of the stress concentration induced. A hole diameter of 6.4 mm has become a commonly used size.

Unless a laminate with a high percentage of 0° plies is to be tested, tabs are not usually necessary. If aggressively serrated tensile wedge grips are used it may be necessary to protect the open-hole tension specimen surfaces with one or more layers of emery cloth, an (unbonded) layer of plastic sheet material (approximately 1 to 2 mm thick), or similar padding material. The open-hole compression test methods typically involve the use of some type of special fixture to prevent specimen buckling, as will be discussed. These fixtures are usually designed for use with an untabbed specimen.

Measure the cross-sectional dimensions (average six measurements) and check for parallelism of the edges and of the end-tab surfaces if used (see Chapter 4 for typical specimen tolerances).

If a series of tests are to be conducted for various hole sizes, divide the specimens into groups by hole size. Note also that one of these groups should be specimens without holes to determine the unnotched strength, σ_0. At least three specimens should be assigned to each group, although a minimum of five specimens is more common. At least three hole diameters should be investigated; for example, D = 3, 5, and 7 mm. Machine the holes as specified in Section 4.2.

13.3 Tensile Test Procedure and Data Reduction

The specimens should be mounted and tested in a properly aligned and calibrated testing machine with mechanical wedge action or hydraulic grips. Set the crosshead rate at about 0.5 to 1 mm/min. Record the load vs. crosshead displacement to detect the ultimate load and any anomalous load-displacement behavior. If a strain gage is used, place it midway between the

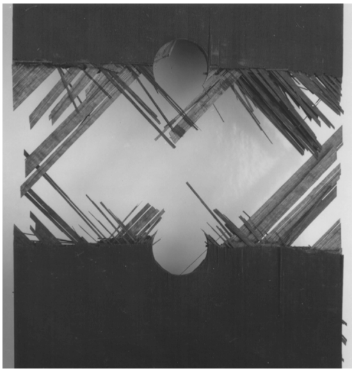

FIGURE 13.8
Carbon–epoxy open-hole tensile specimen tested to failure.

TABLE 13.1
Unnotched and Notched Strength Data for $[0/\pm 45/90]_s$ Carbon/Epoxy Coupons

Notch Radius (mm)	Strength (MPa)
0	607 (σ_0)
1.6	437
2.5	376
3.3	348

hole and the end tab. Make sure eyes are protected in the test area. Load all specimens to failure. Figure 13.8 shows an open-hole tension carbon/epoxy specimen after testing.

Notched strength, σ_N, is calculated based on the gross cross-sectional area (A = wh). A typical set of unnotched and notched strength data for $[0/45/90]_s$ carbon/epoxy coupons is given in Table 13.1. Because the strength model discussed here is restricted to a plate with an infinite width-to-hole diameter ratio, a comparison between experimental data and the notch strength model requires correction for the finite width of the specimen. A common way to correct the data is to multiply the experimental notched strength with a correction factor, K/K_∞, where K is the stress concentration factor for an orthotropic plate of finite width; i.e.,

$$\sigma_N(\infty) = \sigma_N(w)\frac{K}{K_\infty} \tag{13.13}$$

where $\sigma_N(w)$ is the experimental strength for a plate of width w, and $\sigma_N(\infty)$ is the corresponding strength for an infinite plate. A closed-form expression

TABLE 13.2

Finite Width Correction Factors for Various Carbon/Epoxy (AS4/3501-6) Lay-ups [8] E_1 = 125 GPa, E_2 = 9.9 GPa, ν_{12} = 0.28, and G_{12} = 5.5 GPa

Lay-Up	K_∞[a]	w/D = 2	3	4	6	8	10
$[0/\pm45/90]_s$	3.00	1.4340	1.1495	1.0736	1.0260	1.0107	1.0037
$[0_2/\pm45]_s$	3.48	1.3725	1.1291	1.0632	1.0216	1.0093	1.0031
$[0_4/\pm45]_s$	4.07	1.3226	1.1109	1.0577	1.0172	1.0095	1.0041
$[0_6/\pm45]_s$	4.44	1.2992	1.1006	1.0472	1.0152	1.0102	1.0051
$[\pm45]_s$	2.06	1.6425	1.2379	1.1215	1.0442	1.0180	1.0062
Equation (13.14)		1.417	1.148	1.076	1.031	1.017	1.011

[a] Determined from Equation (13.2).

for K, however, does not exist, and K has to be determined using the boundary collocation method [6] or the finite element method [7]. Table 13.2 gives finite width correction factors as a function of width-to-hole diameter ratio (w/D) for various carbon/epoxy lay-ups [8]. Note that K/K_∞ is >1, which means that finite-width specimens exhibit larger stress concentrations than infinitely wide specimens ($w/D \geq 8$). Consequently, it is expected that a hole in a finite-width specimen will be more detrimental in terms of strength than a hole in an infinitely wide specimen. A common approximation, which is also reasonably accurate for composite laminates with w/D > 4 [8] (see also Table 13.2) is to use an isotropic expression [9,10] for K/K_∞

$$\frac{K}{K_\infty} = \frac{2 + \left(1 - (D/w)\right)^3}{3\left(1 - (D/w)\right)} \tag{13.14}$$

To enable comparison of the data with the PSC (Equation (13.6)), first correct the experimental data (Table 13.1) for the finite size, using the approximate expression for the stress concentration factor (Equation (13.14)). Then solve for the parameter λ in Equation (13.6) using an interactive method such as Muller's method [11] or Newton-Raphson's method [12,13]. From the definition of λ (Equation (13.12)) it is observed that only the root between zero and one is required. For illustrative purposes, the notched strength data listed in Table 13.1 were corrected according to Equation (13.14), and the corresponding λ values were determined with the Newton-Raphson method, and are listed in Table 13.3.

To obtain the parameters m and C, Equation (13.12) may be written as

$$-\log(1/\lambda - 1) = \log C + (1 - m)\log R \tag{13.15}$$

By plotting $-\log(1/\lambda - 1)$ vs. log R, the slope and the intercept at log R = 0 can be obtained by the least-squares method. The slope is equal to 1 – m, and the intercept is equal to log C. Figure 13.9 shows $-\log(1/\lambda - 1)$ plotted vs. log R for the data of Table 13.3.

TABLE 13.3

Corrected Notched Strength Data (Table 13.1) and Values of λ Determined Using the Newton-Raphson Method

R (mm)	σ_N/σ_0[a]	λ
1.6	0.73	0.5998
2.5	0.64	0.6867
3.3	0.62	0.7052

[a] Corrected using Equation (13.14).

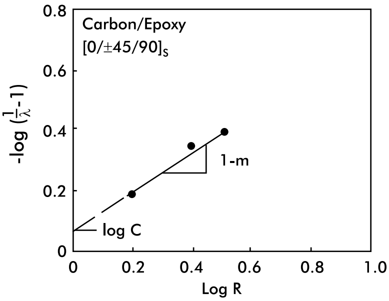

FIGURE 13.9
Determination of the parameters m and C for $[0/\pm45/90]_s$ carbon/epoxy specimens, m = 0.36, and C = 1.16 mm^{-1}.

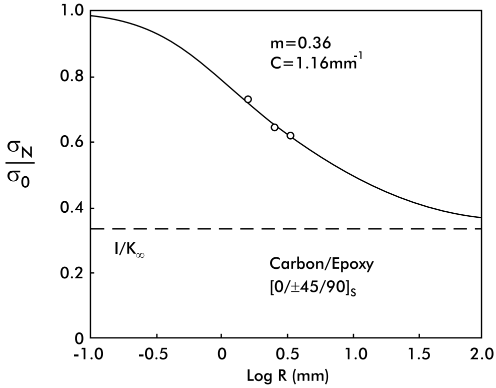

FIGURE 13.10
Theoretical and experimental notch sensitivity for $[0/\pm45/90]_s$ carbon/epoxy specimens.

As an illustration of the goodness of the fit for the parameters m and C, the theoretical curve of σ_N/σ_0 is plotted vs. log R along with the experimental values in Figure 13.10. Within the limited range of experimental data, excellent agreement is observed. Once the parameters m and C are established, it is possible to predict the notched strength for any hole size and coupon width (within reasonable limits) using the above methodology.

13.4 Standardized Open-Hole Tension Test Method

Rather than utilize the PSC and ASC developed by Whitney and Nuismer [1] discussed in the previous sections of this chapter, it has become common to simply test a specimen of one specific configuration and then use the measured strength as a comparative measure. That is, if a common specimen configuration is used, open-hole tensile strength results for different materials can be directly compared. The specimen configuration most commonly used is 305 mm long, 38 mm wide, and containing a centrally located 6.4-mm-diameter hole at the midlength of the specimen. This configuration was developed by the Boeing Company [14] and has now been standardized by both SACMA [15] and ASTM [16].

The laminate configuration and specimen thickness are somewhat arbitrary, but of course the results obtained will be dependent on these parameters as well as the type of material being tested. Although any laminate configuration can be used, most commonly a $[0/\pm45/90]_{ns}$ (quasi-isotropic) laminate is selected. Laminates with higher percentages of 0° plies are tested when of specific interest to the intended design application.

Lower strength laminate orientations such as the quasi-isotropic lay-up can normally be tested without specimen tabs. When laminates with higher percentages of 0° plies are to be tested, tabs may be necessary. However, because a strength-reducing hole is present, it may be possible to successfully test an open-hole tension specimen without tabs, whereas the corresponding laminate without a hole could not be tested.

Standard mechanical or hydraulic grips, as described in Section 13.2, can be used for this standardized test. That is, unless a laminate with a high percentage of 0° plies is to be tested, tabs are not usually necessary. If aggressively serrated tensile wedge grips are used, it may be necessary to protect the open-hole tension specimen surfaces with one or more layers of emery cloth, an (unbonded) layer of plastic sheet material (approximately 1 to 2 mm thick), or similar padding material.

Although applied load vs. crosshead displacement is usually monitored, as a means of detecting any testing anomalies, only the maximum applied load, P_{max}, to cause failure is used directly. That is, the test results are quantified for comparison purposes by calculating an open-hole tensile strength

$$\sigma_N = P_{max}/A \tag{13.16}$$

where P_{max} is the maximum load applied to fail the specimen, and A is the cross-sectional area of the specimen (A = wh), where w and h are the specimen width and thickness, respectively. Note that the open-hole strength is based on the gross cross-sectional area of the specimen (disregarding the hole) and not the net cross-sectional area.

13.5 Standardized Open-Hole Compression Test Methods

There are two standardized open-hole compression test methods in use, e.g., the so-called Boeing Open-Hole Compression [14] and the Northrop Open-Hole Compression [17] test methods. The Boeing method was first standardized by SACMA [18] and more recently by ASTM [19], as was its tensile loading counterpart. Although the Northrop method presently is still an individual company standard, it is frequently used by other groups also.

13.5.1 Boeing Open-Hole Compression Test Method

The specimen configuration is the same as that of the Boeing Open-Hole Tension test method (Section 13.4). That is, the specimen is 305 mm long and 38 mm wide, and contains a 6.4-mm-diameter hole centered at the midlength of the specimen. Again, the laminate configuration and specimen thickness are somewhat arbitrary, and the results obtained are dependent on these parameters as well as the material being tested.

A special fixture has been designed to load the specimen in compression while preventing gross (Euler) buckling, as shown in Figure 13.11. The 305-mm-long specimen is installed such that its ends are flush with the outer ends of the fixture halves. Thus, essentially the entire length of the specimen is supported against buckling, with only a small gap existing between the fixture halves themselves so that they do not come into contact when the compressive loading is applied. The standards specify that the ends of the fixture be installed in hydraulic grips. Because of the thickness (approximately 35 mm with a specimen installed) and width (76 mm) of the fixture, this requires large hydraulic wedge grips. For example, commercially available hydraulic grips of 250 kN capacity or greater are required (e.g., see Reference [19] for descriptions of such grips). There is no need for grips of this loading capacity in many testing laboratories. These large grips are

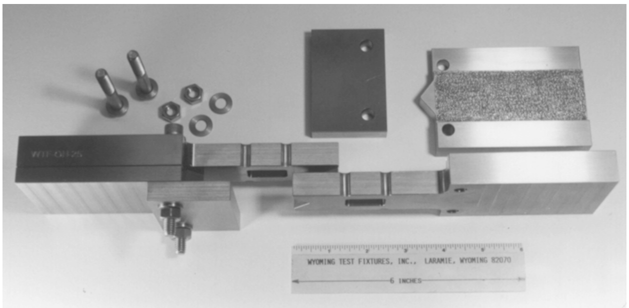

FIGURE 13.11
Boeing open-hole compression test fixture (ASTM D 6484-99). (Photograph courtesy of Wyoming Test Fixtures, Inc.)

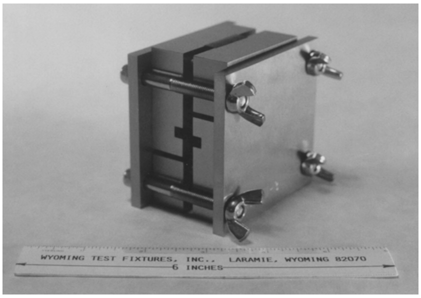

FIGURE 13.12
Northrop open-hole compression test fixture (Northrop Specification NAI-1504). (Photograph courtesy of Wyoming Test Fixtures, Inc.)

relatively expensive, and are very massive and thus difficult to handle during installation and removal from the testing machine. For example, each of the 250 kN hydraulic grips (MTS Systems Corporation, Eden Prairie, MN) weighs about 125 kg [20]. Because of this, the fixture of Figure 13.11 is often loaded directly on its ends between compression platens. The faces of the fixture in contact with the specimen are thermal-sprayed with tungsten carbide particles, as discussed in Chapters 5 and 6. Thus, if the fixture is clamped tightly to the specimen, the applied end loading will become a combined end loading and shear loading, just as for the Wyoming combined loading compression test fixture described in Section 6.3 of Chapter 6; it is hoped that end crushing of the specimen will be avoided. This success is particularly likely because, as noted above, the strength of an open-hole compression specimen is typically not very high. However, if direct end loading is to be used, care must be taken to secure the fixture so that it does not slip out from between the platens when the loading is applied and potentially cause injury to nearby personnel. For example, restraint boxes attached directly to the testing machine can be used instead of platens [21,22]; the ends of the fixture slip into these restraint boxes as shown in Figure 13.13.

Although strain gages can be used, the ASTM standard has eliminated their use because they do not provide necessary information. As for the open-hole tension testing, only the compressive strength is calculated, using Equation (13.16).

13.5.2 Northrop Open-Hole Compression Test Method

The Northrop open-hole compression test method [17] was developed at about the same time as the Boeing method. The Northrop open-hole compression test specimen is only 76 mm long and 25 mm wide, i.e., only one fourth as long and two thirds as wide as the Boeing specimen. It contains a 6.4-mm-diameter hole centered at its midlength. That is, it requires only one sixth as much material per specimen — a significant savings. However, as discussed at the beginning of this chapter, it may be desirable to use a wider

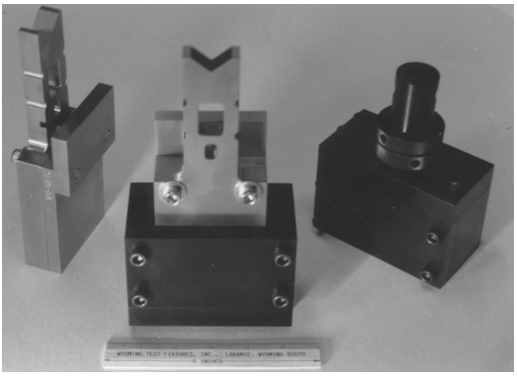

FIGURE 13.13
Restraint boxes for use with the Boeing open-hole compression test fixture when applying loading directly through the specimen ends (lower half of fixture shown in a box; both boxes to be attached directly to the testing machine). (Photograph courtesy of Wyoming Test Fixtures, Inc.)

specimen to reduce edge effects. Thus, a modification of the Northrop fixture to accommodate 38-mm-wide specimens has also been utilized [21]. It has been shown to produce compressive strengths comparable to the standard Boeing test, while reducing the volume of specimen material required to one fourth — still a significant savings.

The Northrop test fixture is shown in Figure 13.12. It is designed such that the untabbed specimen is installed flush with the ends of the fixture and is directly end loaded. Although a moderate clamping force is exerted on the specimen when the fixture screws are tightened to the recommended 6.8 N·m torque, the faces of the fixture are smooth and not intended to transfer a shear loading to the specimen. That is, the force applied to the specimen is essentially all end loading. As for the Boeing fixture, the specimen is supported over almost its full length, although a small gap is maintained between the fixture halves to prevent them from coming into contact with each other when the compressive loading is applied. Because the fixture is compact, with a base comparable in dimensions to its height, there is little danger of it being ejected from between the platens when loaded, and thus need not be constrained. The fixture has recesses machined into it to permit the use of strain gages, but as for the Boeing fixture, strain gages are not normally used. Compressive strength is calculated using Equation (13.16).

13.5.3 Comparison of the Boeing and Northrop Open-Hole Compression Test Methods

The performance of the Boeing and Northrop open-hole compression test methods and corresponding fixtures are directly compared in Reference [21]. In addition to 25-mm-wide specimens tested in the standard Northrop fixture, 38-mm-wide specimens were tested in a special fixture fabricated with this increased width capability. Although there were some minor differences in the results, there were no distinct trends to report. This included whether the Boeing specimen was shear loaded or end loaded, and whether the Northrop specimen was 25 mm or 38 mm wide.

As previously noted in Section 13.4, the open-hole tests, whether tension or compression, are usually used as comparative tests. Thus, until additional studies such as that of Reference [21] are conducted, it is best to select one open-hole compression test method and use it consistently, knowing that the results obtained will not be significantly different from those obtained using one of the other open-hole compression test methods.

13.5.4 Filled-Hole Tension and Compression Test Methods

The discussion in this chapter addresses the influence of empty (unfilled) holes in composite laminates. An unfilled hole will deform under loading. However, often a hole is created in a laminate to accommodate a fastener of some type, e.g., a bolt, pin, or rivet. The presence of a close-fitting fastener will restrict the deformation of the hole, thus changing the state of stress in the laminate, and possibly the failure stress.

ASTM is currently preparing a standard [23] to govern the tensile and compressive testing of laminates with filled holes. The test fixtures and procedures are similar to those outlined in the previous sections for unfilled holes. However, the laminate failure modes may change because of the fastener interference. The differences in failure mode are discussed in Reference [24].

References

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