CHAPTER 5

Filament Winding and Fiber Placement

1. FILAMENT WINDING

1.1. Introduction

Filament winding is a process used to make composite structures such as pressure vessels, storage tanks or pipes. Composite pressure vessels offer light weight and high strength. Applications include oxygen tanks used in aircraft and by mountain climbers, compressed natural gas cylinders for vehicles, drive shafts for automobiles, and pipes for conducting corrosive liquids.

Filament winding is a comparatively simple operation in which continuous reinforcements in the form of rovings or monofilaments are wound over a rotating mandrel. Specially designed machines, traversing at speeds synchronized with the mandrel rotation, control the winding angles and the placement of the reinforcements. Structures may be plain cylinders or pipes or tubing, varying from a few centimeters to one or two meters in diameter. Spherical, conical, and geodesic shapes are within winding capability. End closures can be incorporated into the winding to produce pressure vessels and storage tanks.

A schematic of a simple filament winding setup is shown in Figure 1.2(b) and repeated here as Figure 5.1. Figure 5.2 shows a photo of a filament winding machine [repeat of Figure 1.2(c)].

The basic mechanism consists of pulling a roving (number of strands) of fibers from the creels. These are spread out using a bank of combs. The fibers then go through a bath of resin (for the case of wet winding). On exit from the bath of resin, the fibers are collimated into a band. The band

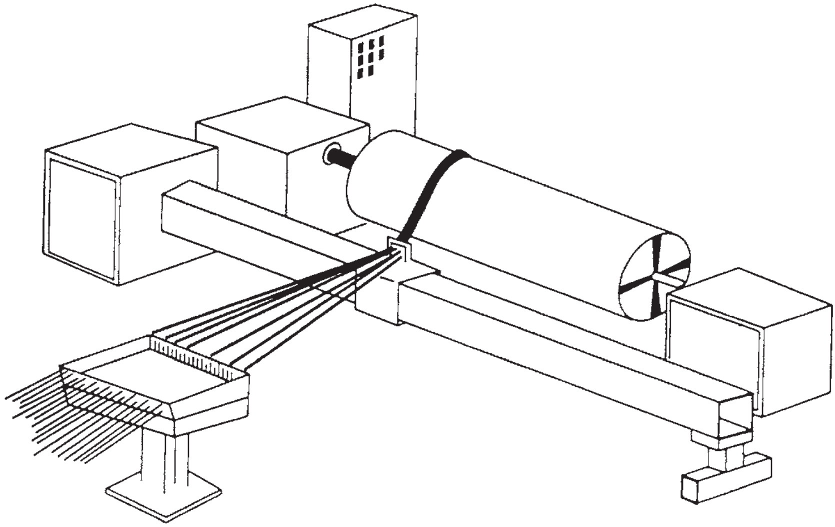

FIGURE 5.1 Schematic of the filament winding process (courtesy of Wiley Interscience).

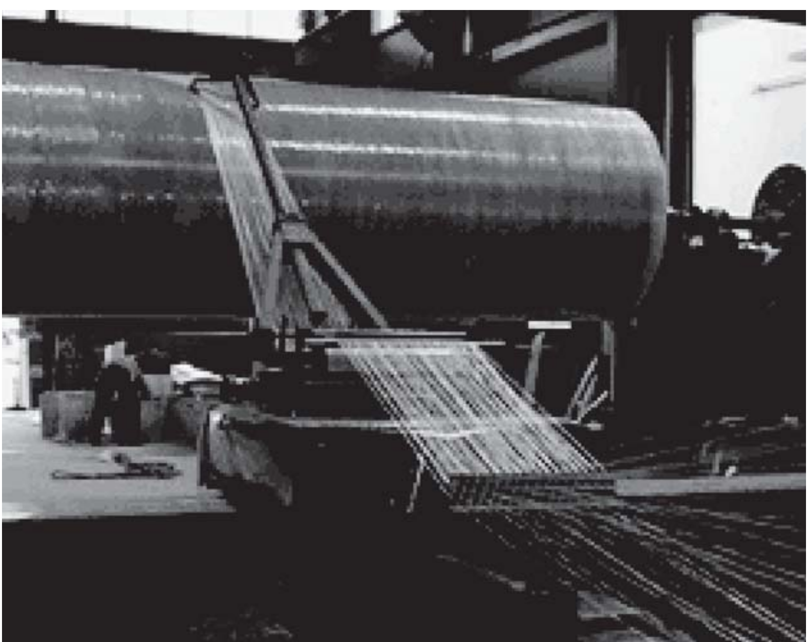

FIGURE 5.2 The placement of fiber band on the mandrel. (www.gilgwang.com/english/frp/grp01.html).

goes through a fiber feed and is then placed on the surface of a mandrel. The fiber feed traverses back and forth along the length of the mandrel. The mandrel is attached to a motor, which gives it rotational motion. The combined motion of the fiber feed and the rotation of the mandrel make the fiber bands spread over the surface of the mandrel. By covering the surface of the mandrel with many layers, one can build up the thickness of the part. The fiber orientation can be controlled by varying the speed of traverse of the fiber feed and the rotational speed of the mandrel. Filament winding is usually used to make a composite structure in the form of surfaces of revolution, such as cylinders or spheres. The surfaces of these structures are usually convex due to the need to apply tension on the tows while these tows are placed on the surface of the mandrel. If the surface is concave, bridging of the fibers over the concave surface may occur. As can be seen from these figures, the basic components of a filament winding system consist of a mandrel and devices to place the fiber tows on the surface of the mandrel to build up the thickness for the part.

1.2. The Winding Process

The operation of filament winding is the reverse of the conventional machining process of milling on a lathe. In milling, one starts with a cylindrical surface and one removes the material from the surface one strip at a time. In filament winding, one deposits the material on the surface of the mandrel one strip at a time. The most basic form of filament winding is a two-degrees-of-freedom operation. This consists of the rotation of the mandrel and the linear movement of the feed along the axis of the mandrel. Two-axis filament winding machines can be used to wind pipes. Filament winding machines with more degrees of freedom exist. The availability of the additional degrees of freedom can be useful in winding at the end of the part, such as heads of pressure vessels, or the winding of shapes more complex than straight cylinders such as those with variation in cross section (i.e. cones) or spheres. For example, for the case of a four-axis winding machine, the basic movements are mandrel rotation and feed traverse. To these are added a cross-slide perpendicular to the mandrel axis and a fourth axis of motion, rotation of the feed eye. These latter permit more accurate fiber placement at the ends. Winding machines with more degrees of freedom up to the level of multi-degrees-of-freedom robots are available. To illustrate the concept of filament winding, only the simple operation of machines with two degrees of freedom will be described in this chapter. Depending on the coordination between the rotational motion and the axial motion, different types of winding can be obtained. These are: polar, helical, circuit and pattern, layer, hoop, longitudinal, and combination.

1.2.1. Polar Winding

This is also called planar winding. In this process, the mandrel remains stationary while a fiber feed arm rotates about the longitudinal axis, inclined at the prescribed angle of the wind. The mandrel is indexed to advance one fiber bandwidth for each rotation of the feed arm. This pattern is described as a single circuit polar wrap (Figure 5.3) .The fiber bands lie adjacent to each other; a completed layer consists of two plies oriented at plus and minus the winding angle.

1.2.2. Helical Winding

In this process, the mandrel rotates continuously while the fiber feed carriage traverses back and forth. The carriage speed and mandrel rotation are regulated to generate the desired winding angle. The normal pattern is multi-circuit helical. After the first traverse, the fiber bands are not adjacent. Several circuits are required before the pattern repeats. A typical 10-circuit pattern is shown in Figure 5.4.

In the above configuration one needs to distinguish between the straight cylindrical part and the head (or dome). In the straight cylindrical part, the relation between the rotational displacement and axial displacement can be established. Refer to Figure 5.5. This figure shows the developed surface of the straight part of the cylinder. The dimension of the base is πD where D is the diameter of the mandrel. Let α be the winding angle (angle between fiber path and the axis of the cylinder), b be the band width of the fiber band, and L be the axial distance traveled by the

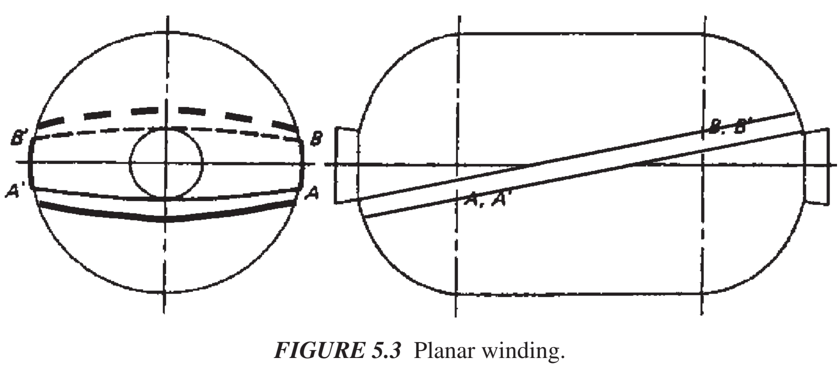

FIGURE 5.3 Planar winding.

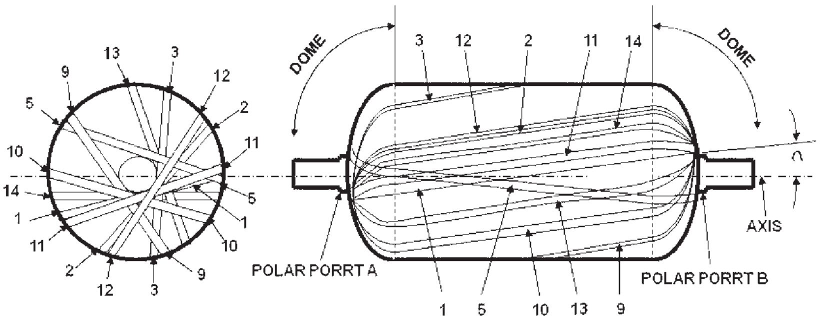

FIGURE 5.4 An example of a helical winding pattern.

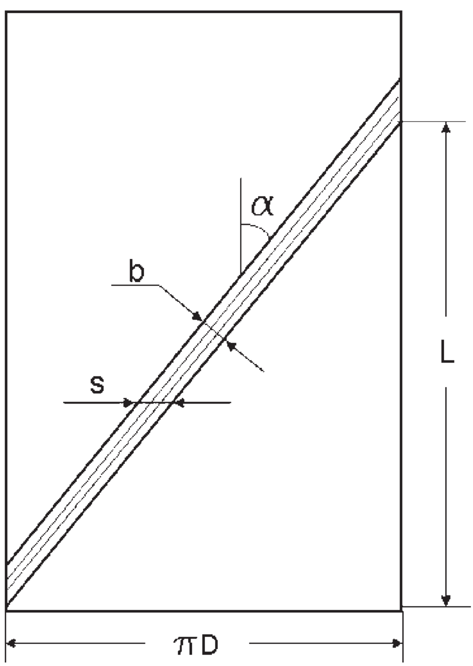

FIGURE 5.5 Developed envelope with fiber path.

fiber feed corresponding to one rotational revolution. The relation between the rotational distance and axial distance can be written as:

$$L = \frac{\pi D}{\tan\alpha} \tag{5.1}$$

If h represents the length of the straight part of the cylinder to be built, the number of revolutions required for the fiber feed to travel this distance is given as:

$$n = \frac{h}{L} = \frac{h\tan\alpha}{\pi D} \tag{5.2}$$

Equation (5.2) gives the number of revolutions. This can be a whole number or a decimal number. One needs to convert this into the number of degrees (by multiplying n by 360) in order to determine the number of degrees of revolution.

Since filament winding is a continuous process, the fiber feed has to reverse its motion to go back to the other end. Also it is essential that tension be maintained in the fibers to ensure good properties of the final product. One also needs to identify the location of the fiber feed (point A in Figure 5.6) and the point of separation between the fiber band and the surface of the mandrel (point B).

When the point B reaches the end of the straight part of the cylinder, this point will move over the surface of the head of the component to be built (i.e. a vessel). The fiber feed (point A) starts to go into reverse. It takes some time before point B touches the end of the straight part of the cylinder again (point B′). The number of degrees of rotation of the man-

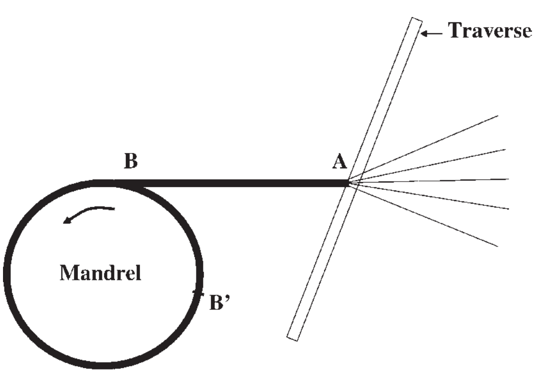

FIGURE 5.6 Relative position of the fiber feed (point A) and point of separation (point B) between fiber band and mandrel surface.

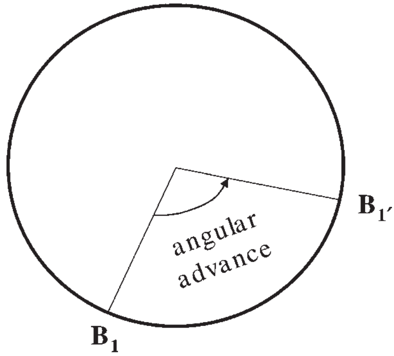

FIGURE 5.7 Relative angular position of the point of separation at the end and at the beginning of a circuit.

drel during this time is termed the *dwell angle*. A dwell angle exists at both ends of the cylinder.

1.2.3. Circuit and Pattern

When the point B has gone one complete cycle and returns to the same axial position along the length of the cylinder and goes in the same direction, a circuit has been completed. Due to the complexity of the motion, there is no guarantee that after one circuit, the point $B_{1'}$ at the end of one circuit will have the same angular position as its position at the beginning of the circuit (B_1). Figure 5.7 illustrates this point.

It takes a number of circuits before the point B can return to its position at the beginning. When this happens, one has a pattern. This can be illustrated in the following example.

Example 5.1

It is desirable to wind a 30 cm diameter by 100 cm long cylinder at a 30° wind angle. The fiber band width is assumed to be 0.6 cm and the dwell angle is 180°. Determine the number of circuits required to make a pattern.

Solution

First, define the *reference circle* as the circle of the cross-sectional area of the cylinder at one end, say, the left end. Assume that winding starts from a point B on that circle. In one circuit, the feed moves twice the length of the mandrel. This means forward once and backward once along the length of the mandrel. When a pattern is complete, a set of circuits has been made and the fiber path returns to the initial position.

Equation (5.2) gives the number of revolutions required for the fiber feed to move a distance h which is equal to the length of the cylinder. For a circuit, two cylinder lengths need to be traveled. The corresponding number of revolutions will therefore be:

$$2n = \frac{2h\tan\alpha}{\pi D} = \frac{(2)(100\text{ cm})\tan 30}{\pi(30\text{ cm})} = 1.23 \qquad \text{(a)}$$

The corresponding number of degrees is:

$$(1.23)(360) = 441° \qquad \text{(b)}$$

In addition to the number of degrees in Equation (b), one has to add two times the dwell angle in order to obtain the total number of degrees required to make a circuit. This gives:

$$\theta = 441 + 2(180) = 801° \qquad \text{(c)}$$

If one subtracts the above number by a whole multiple of 360°, one would obtain the angular advance of the starting point (new point $B_{1'}$) as compared to starting point B_1 on the reference circle. This angular advance is 801 – (360)(2) = 81°. This is shown in Figure 5.7.

In order to make a pattern, one needs to have a multiple of the advance angles such that this multiple will be equal to a multiple of 360°. This can be expressed as:

$$(m)(81) = (n)(360) \qquad \text{(d)}$$

where m and n are integers and should be as small as possible.

Equation (d) shows that m and n can be quite large before the equation is satisfied. This may not be practical. In order to reduce the numbers m and n, one needs to adjust the operation to make the advance angle a good whole number. One good whole number close to 81 is 90. This can be done by adjusting the dwell angle to be 180 + 9/2 = 184.5°.

(This can be done by adjusting the machine setting.) If this is done, Equation (d) becomes:

$$(m)(90) = (n)(360) \quad \text{or} \quad \frac{m}{n} = 4 \qquad \text{(e)}$$

One can select $m = 4$ and $n = 1$. What this means is that it takes 4 times the advance angle (or 4 circuits) to make a pattern.

Note: In the pattern calculated above, the fiber band will go back exactly to the same position on the reference circle as at the beginning of the winding process. This may not be desirable because if one continues this process, the fiber will follow the same path as before and one may not be able to cover the whole surface of the mandrel. It is desirable to advance the position $B_{1'}$ one bandwidth distance along the circumferential direction after one pattern. This distance in angular value can be calculated to be (note that the circumferential coverage of a bandwidth b is $b/\cos\alpha$):

$$\Delta\theta = \frac{b}{\pi D \cos\alpha}(360) = \frac{0.6}{\pi(30)\cos(30)}(360) = 2.65^{\circ} \qquad \text{(f)}$$

This advanced angular value is accumulated over 4 circuits. The value for each circuit is 2.65/4 = 0.66°. This angle is then divided by two dwell angles. The dwell angle is then adjusted to be: 184.5 + 0.66/2 = 184.8°.

1.2.4. Layer

A pattern may consist of fiber intersections (fiber crossovers—see Figure 5.4) at certain sections. Crossovers may occur at more than one section, depending on the wind angle. A layer is defined as a set of patterns that completely cover the surface of the mandrel with fibers.

From Figure 5.5, it can be seen that the relation between the circumferential coverage S and the bandwidth b can be written as:

$$S = \frac{b}{\cos\alpha} \qquad (5.3)$$

In order to make a layer, the whole circumferential distance πD has to be covered. The number of circuits per layer C can be calculated as:

$$C = \frac{\pi D}{S} = \frac{\pi D \cos\alpha}{b} \qquad (5.4)$$

Example 5.2

Continue with Example 5.1 and determine the number of circuits required to make a layer.

Solution

For $\alpha = 30°$, one has (from Equation 5.4):

$$C = \frac{\pi(30)\cos 30}{0.60} = 136$$

There are 136 circuits to make up a layer. Recall from Example 5.1 that it takes 4 circuits to make a pattern. The number of patterns per layer is then 136/4 = 34.

1.2.5. Hoop Winding

Hoop or circumferential layers are wound close to 90°. The feed advances one bandwidth per revolution. The layer is considered a single ply. Hoop layers may also be applied as doublers or localized stiffeners at strategic points along the cylinder.

1.2.6. Longitudinal Winding

Longitudinal winding applies to low angle wrap which is either planar or helical. For closed pressure vessels, the minimum angle is determined by the polar openings at each end.

1.2.7. Combination Winding

Longitudinals are reinforced with hoop layers. The customary practice with pressure vessels is to place the bulk of the hoop wraps in the outer layer. A balance of hoop and longitudinal reinforcement can also be achieved by winding at two or more helical angles.

1.2.8. Wet/dry Winding

In addition to the classification of different winding patterns, one also distinguishes the type of winding depending on whether fibers are wetted with liquid resin in-situ or prepregs are used. These are referred to as wet winding or dry winding. In wet winding, the resin is applied during the winding stage (Figure 5.1). The alternate dry winding method utilizes the pre-impregnated B-staged rovings. Wet winding tends to be messy due to the possible dripping of the wet resin. Dry winding is cleaner but the raw materials (prepregs) are more expensive than the tows.

1.3. End Closures

End closures for pressure vessels are either mechanically fastened to the cylindrical portion or are integrally wound. If end closures are fastened to the cylindrical portion, both the end closures and cylindrical portion need to have flanges. Integrally wound end closures can provide better pressure containment than mechanically fastened heads. The fiber path yields a balance of meridional and circumferential forces and is consistent with winding conditions so that no slippage occurs. The head contours and related polar bosses are critical in vessel design.

One common contour follows the geodesic isotensoid. This contour is normally adapted to helical winding. The fiber path is taken as tangential to the polar boss (Figure 5.8). The geodesic path is the path of shortest distance between two points on a curved surface. The reason to select the

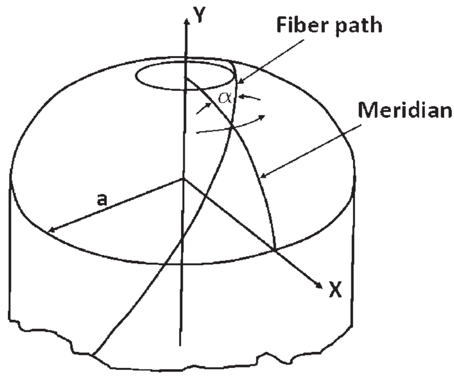

FIGURE 5.8 Geodesic path.

geodesic path is because if winding is made along this path, no slippage of the fiber relative to the surface of the mandrel will occur. For winding on frictionless surface, geodesic paths need to be followed. If one wants to wind a long non-geodesic path, some friction needs to be created between the fibers and the surface, either by rough surface or some form of adhesive applied to the fibers.

For the path to be geodesic (connecting the two points with the shortest distance on the curved surface), Equation (5.5) holds:

$$X \sin \alpha = const \tag{5.5}$$

where X is the coordinate of the point and α is the angle between the fiber direction and the meridian at the point X.

At the tangency point, $\alpha = 90°$ and:

$$X \sin \alpha = X_o \tag{5.6}$$

where X_o is the boss radius.

1.4. Materials

1.4.1. Reinforcements

Nearly all filament winding for making pipes or low-pressure vessels is conducted with continuous E glass roving as reinforcement. A stronger but higher priced S glass roving is used less frequently, principally in the aerospace industry. Pressure vessels are not normally subjected to bending loads. As such the low modulus of glass fibers is not of concern.

However for pipes that are supported by saddles over long spans, deflection and ovalization of the cross section may be a concern. For these cases, thickness higher than what is required from internal pressure may be required. Graphite fibers and Kevlar fibers have a higher modulus than glass fibers but are more expensive. It is possible to have hybrid windings where different layers in the thickness of the structures can be wound using different type of fibers. Combinations of different types of fibers and different winding angles can provide the wound structures with unique capability to withstand internal pressure and also bending loads.

1.4.2. Resins

The major matrix systems for filament windings are based on epoxy, polyester or vinyl ester resins. The viscosity of the resin system should be sufficiently low so that wetting of the fibers can be done quickly and easily as the fibers are run through the bath of resin. However the viscosity should thicken rapidly after the fiber bands are deposited onto the surface of the mandrel to avoid resin dripping and to avoid resin from being squeezed from the pressure of subsequent layers. Normal curing is conducted either at room temperature or at elevated temperature without pressurization. Figure 5.9 shows the viscosity of a few resin systems used for filament winding.

Epoxy resins for filament winding are essentially the same as the laminating resins. The diglycidyl ether of bisphenol-A (DGEBA) is the most important resin type. Epoxy novolacs and cycloaliphatics are utilized to a lesser extent. Other available systems are based on brominated epoxy for improved ignition resistance, resorcinol diglycidyl ether for extended processability, and flexible epoxies for impact resistance and greater elongation.

Due to their lower cost and a balance of physical and chemical properties, polyestser and vinyl ester resins find extensive use in commercial practice. Their handling characteristics are readily adapted to filament winding. Processing viscosity is comparatively easy to control. As with the epoxies, no fundamental distinctions can be made between filament winding and laminating systems.

1.5. Mandrels

Invariably all pressure vessels or pipes made of composites have a liner. The function of the liner is to seal the liquid or gas inside the vessel or pipe. Normally the fibers provide the strength and stiffness for the

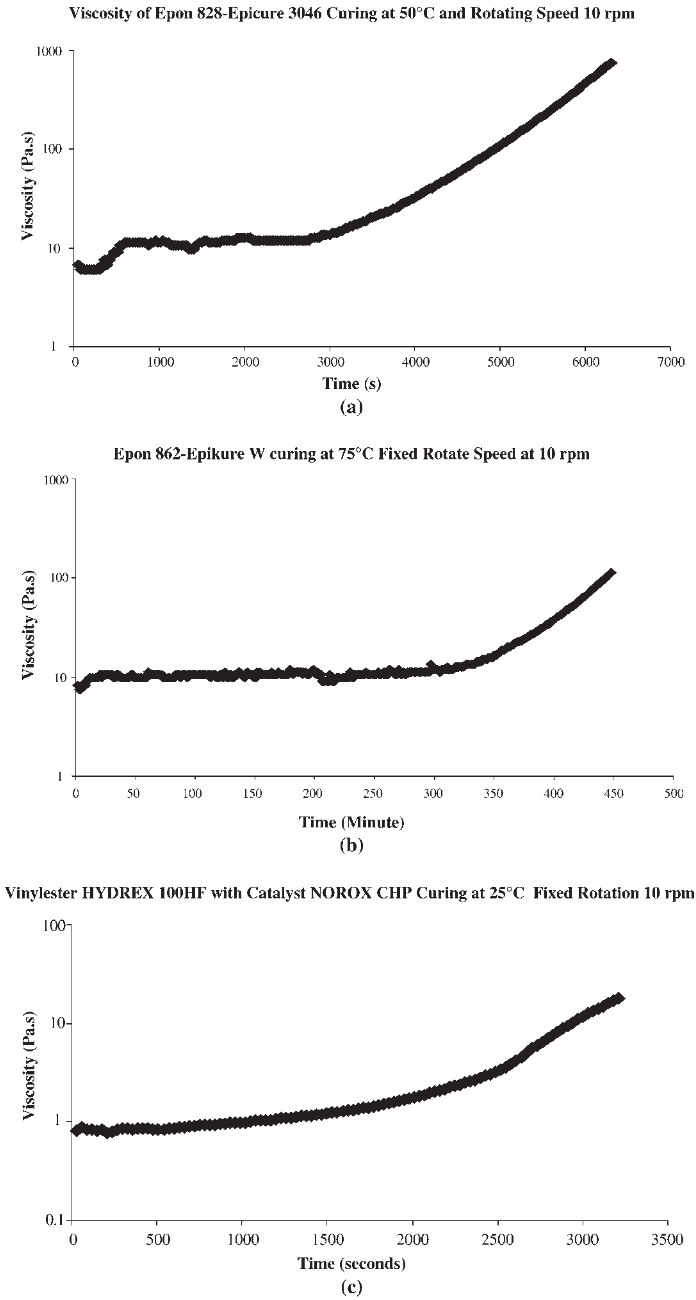

FIGURE 5.9 Viscosity of (a) Epon 828/Epicure 3046 at 50°C, 10 rpm rotating speed. 100 phr Epon: 47 phr 3046, (b) Epon 82-Epikure W at 75°C, 10 rpm rotating speed, (c) Vinylester at 25°C.

structure. In cases where there are cracks in the matrix, these cracks may not cause rupture in the vessel or pipe, but the fluid inside may leak or weep out of the container. In case of flammable fluid, this can be dangerous, even though the fiber network is sufficient to contain the pressure. In order to seal the fluid, a flexible liner is usually used. A liner can be a rubber bladder, a soft layer of thermoplastic such as PVC, or a thin layer of aluminum. When the liner is stiff enough, the liner may be used as a mandrel or over-winding of the fiber and resin.

In a situation where the liner is not stiff enough to withstand the compression due to winding force, or in cases where it is essential to take the mandrel out, strategies for mandrels should be developed. There are many requirements for mandrels. The important considerations for mandrels are:

- Mandrel should be sufficiently stiff to withstand the compression imposed by the winding force.
- The resin should not stick to the surface of the mandrel. Release agents need to be applied.
- The mandrel should be extractable from the part after curing.

Mandrels can be classified according the following categories:

1.5.1. Extractable Mandrels

For winding fiberglass pipes, steel tubes can be used as mandrels. The steel tube is made longer than the length of the pipe to be made, and a pin can be inserted into the steel tubes end. After the composite has cured, a winch can be used to extract the mandrel from the composite pipe. If machining can be done, the mandrel may have a slight taper along the length to facilitate extraction. A release agent should be placed on the outside of the mandrel to facilitate extraction. Figure 5.10 shows the mandrel for a pipe.

Inflatable mandrels made of a rubber bladder can also be used. The rubber bladder is inflated with air pressure to press against the wall of a form. The pressure provides stiffness for the mandrel. This may be used to supplement the stiffness of a thin liner.

1.5.2. Collapsible Mandrels

These mandrels consist of a segmented surface consisting of many pieces. These pieces are expanded to take the shape of the final form by collapsible linkages, similar to the operation of a collapsible umbrella. After the part is cured, the mandrel is collapsed to be removed.

1.5.3. Breakable Mandrels

Mandrels can be made out of plaster, which can be molded to take the form of the final part. After the part is cured, the plaster is broken to be removed. For some class projects where small tubes of composites are made, glass tubes such as the tube used for neon light can be used as a mandrel. The tube is broken to be removed after the part is cured.

1.5.4. Dissolvable Mandrels

Mandrels can be made of materials that can be dissolved in solution. One example is low melting alloy. These are high in density and tend to creep under moderate winding tension. They are limited to small vessels in the order of 0.3 m in diameter to 0.3 m in length. Another example is eutectic salts. These can be melted by moderately high temperature. These are better suited than the alloys and are applicable up to 0.6 m in diameter. With care, they can be flush molded, and they are easy to remove. Another example is soluble plaster. These have a long plastic stage and can be wiped to contour. They are easily washed out. Another example is a mixture of sand-polyvinyl alcohol (PVA). This material is an excellent choice for diameter up to 1.5 m and for limited quantities. It dissolves readily in hot water, but requires careful molding control. Low compressive strength is a limitation.

1.6. Material Handling and Process Controls

In filament winding, there are many machine components that are required for the handling of the fibers. Referring back to Figures 5.1 and 5.2, the process begins with the creels. Many creels are used to provide sufficient number of tows or yarns to make a fiber band. After the fibers leave the creels, they need to be tensioned so that the fibers are kept straight and taut. Tension is provided by guide eyes, drum eye brakes, scissor bars, and the drag through the resin tank. Typically, tension ranges from 1.1–4.4 N per end (an end is like a tow or yarn). Normal pro-

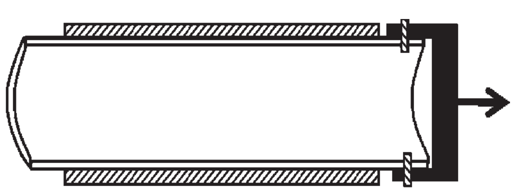

FIGURE 5.10 A mandrel to manufacture composite pipes.

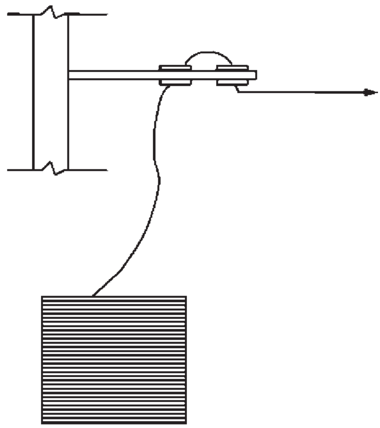

FIGURE 5.11 Example of a tensioning device.

cedure is to keep tension on the dry fibers to a minimum to prevent excessive abrasion and snarling. Figure 5.11 shows one example of the tensioner device. The friction between the fibers and the plates provides the tension force.

After the tensioner, the fiber tows or yarns are fed though a bath of resin for impregnation. Figure 5.12 shows an example of the bath of resin and the impregnation mechanism.

Before the fibers are deposited onto the mandrel, a band of fibers is formed. A device similar to that shown in Figure 5.13 is used to form the band. A uniform flat band will result in improved strength as well as a more uniform thickness. The thickness of a single layer can be calculated for a specific band density (ends/cm) and glass content. Wider band width can cover the surface of the mandrel more quickly. However with a constant amount of fibers, wider band means smaller thickness of the band. The band width has an influence on the number of circuits required to cover the mandrel surface as discussed earlier.

1.7. Netting Analysis for Pressure Vessels under Internal Pressure

The behavior of filament wound composites is analogous to that of other angle-plied laminates so that the analytical methods developed for laminates can be applied to filament wound structures. The netting analysis is a simplified procedure (as compared to laminate theory) used mainly to estimate fiber stresses in a cylindrical vessel subjected to internal pressure. This method is based on the assumption that only the rein-

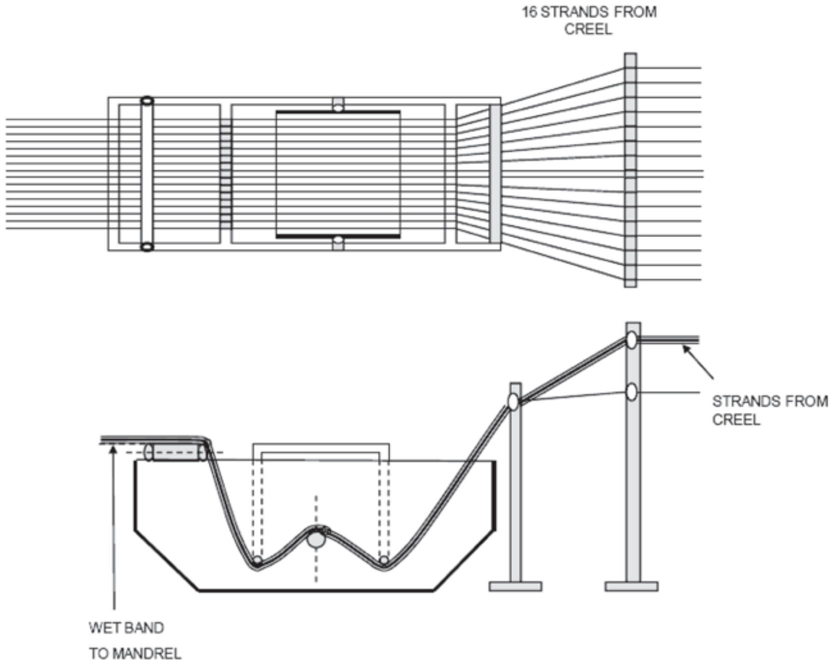

FIGURE 5.12 Impregnation tank with feed from double level.

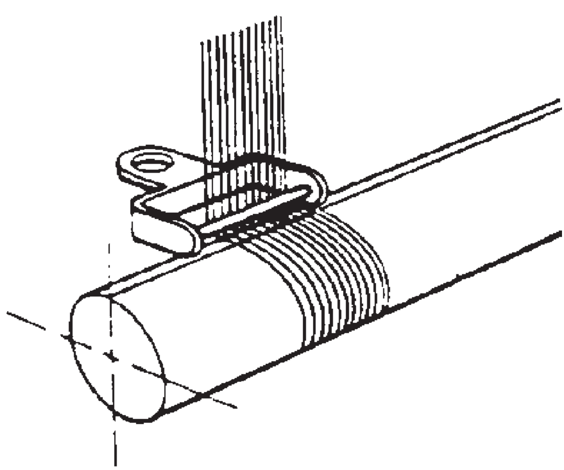

FIGURE 5.13 A band forming mechanism.

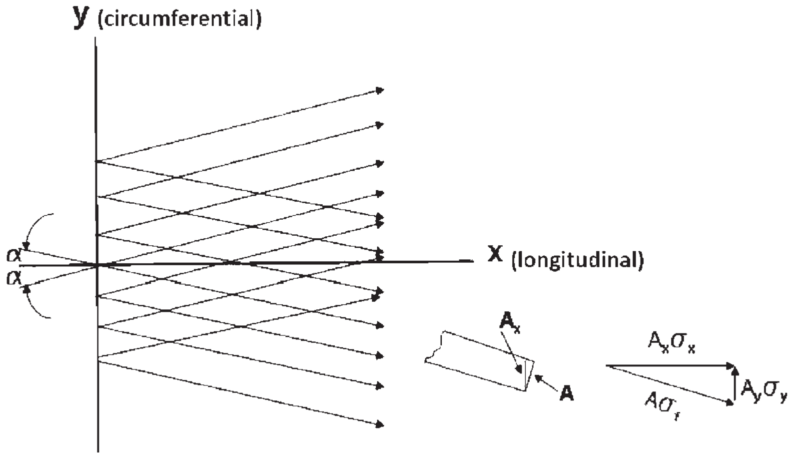

FIGURE 5.14 Netting analysis.

forcing fibers have a load carrying capability and that all fibers are uniformly stressed in tension. Figure 5.14 represents a two-layered system of parallel fibers from which the netting equations can be derived. In this figure, the x axis is along the length of the vessel whereas the y axis represents the circumferential direction.

The stress in each fiber is σ_f, acting on the cross-sectional area A of the fiber band. The force on the fiber is $A\sigma_f$. The component of this force along the x direction (along the length of the cylinder) is $A\sigma_f \cos\alpha$. This force is acting over an area $A_x = A/\cos\alpha$. Dividing the force along the x direction over the area normal to the x direction yields the stress σ_x as:

$$\sigma_x = \frac{F_x}{A_x} = \frac{\sigma_f A \cos\alpha}{A / \cos\alpha} = \sigma_f \cos^2\alpha \tag{5.7}$$

Similarly, for the stress along the y direction (or hoop direction):

$$\sigma_y = \sigma_f \sin^2\alpha \tag{5.8}$$

where α is the winding angle. It can easily be seen that:

$$\frac{\sigma_y}{\sigma_x} = \tan^2\alpha \tag{5.9}$$

Winding with only one angle: In the case where filament winding is only with one angle, there is an optimal angle such that the structure may fail along the longitudinal and hoop directions simultaneously. From equilibrium conditions, it can be shown that for a thin-walled pressure vessel under internal pressure, the longitudinal and hoop stresses are given by:

$$\sigma_x = \frac{pr}{2t} \tag{5.10}$$

and

$$\sigma_y = \frac{pr}{t} \tag{5.11}$$

where

σ_x, σ_y = longitudinal and hoop stresses respectively
p = internal pressure
t = thickness
r = radius of the cylinder (for thin cylinder either inside or outside radius can be used)

From equations (5.10) and (5.11) one has:

$$\frac{\sigma_y}{\sigma_x} = 2 \tag{5.12}$$

Equating equations (5.10) and (5.11) yields:

$$\tan^2 \alpha = 2 \quad \text{or} \quad \alpha = 54.7^\circ \tag{5.13}$$

What the above indicates is that if the winding is done at 54.7°, the vessel will fail both along the longitudinal and hoop directions at the same time.

Winding at layers at two different angles: When winding consists of two layers at different angles, the following equations can be derived. Using equilibrium along the longitudinal direction:

$$\sigma_x(t_1 + t_2) = \sigma_{f1} t_1 \cos^2 \alpha_1 + \sigma_{f2} t_2 \cos^2 \alpha_2 \tag{5.14}$$

Using equilibrium along the hoop direction:

$$\sigma_y(t_1 + t_2) = \sigma_{f1} t_1 \sin^2 \alpha_1 + \sigma_{f2} t_2 \sin^2 \alpha_2 \tag{5.15}$$

The above two equations are general for winding with two layers at two different angles α_1 and α_2. These equations also allow for different materials to be used at the two layers, each with different thicknesses. For example, one layer can be made of carbon/epoxy (with fiber strength σ_{f1} of a certain thickness t_1 while the other layer can be made of glass/epoxy (with fiber strength σ_{f2} of another thickness t_2) etc.

When the outer layer is composed of a hoop layer only (i.e. $\alpha_2 = 90°$) (this is a common case where the inner layers are helical while the outer layer is hoop), the above equations reduce to:

$$\sigma_x (t_1 + t_2) = \sigma_{f1} t_1 \cos^2 \alpha_1 \tag{5.16}$$

and

$$\sigma_y (t_1 + t_2) = \sigma_{f1} t_1 \sin^2 \alpha_1 + \alpha_{f2} t_2 \tag{5.17}$$

If the fibers of the two different layers are made of the same material, i.e., $\sigma_{f1} = \sigma_{f2} = \sigma_F$, and utilizing also the relation in Equation (5.13), it can be shown that:

$$\frac{t_{\text{hoop}}}{t_{\text{longitudinal}}} = 2\cos^2 \alpha_1 - \sin^2 \alpha_1 = 3\cos^2 \alpha_1 - 1 \tag{5.18}$$

Winding with many layers: The above analysis can be extended to the case of winding with many layers. For this general case, equations (5.14) and (5.15) can be generalized as:

$$\sigma_x (t_1 + t_2 + t_3 + \ldots + t_n) =$$
$$\sigma_{f1} t_1 \cos^2 \alpha_1 + \sigma_{f2} t_2 \cos^2 \alpha_2 + \sigma_{f3} t_3 \cos^2 \alpha_3 + \ldots + \sigma_{fn} t_n \cos^2 \alpha_n \tag{5.19}$$

and

$$\sigma_y (t_1 + t_2 + t_3 + \ldots + t_n) =$$
$$\sigma_{f1} t_1 \sin^2 \alpha_1 + \sigma_{f2} t_2 \sin^2 \alpha_2 + \sigma_{f3} t_3 \sin^2 \alpha_3 + \ldots + \sigma_{fn} t_n \sin^2 \alpha_n \tag{5.20}$$

Example 5.3

A pressure vessel with internal diameter of 40 cm is subjected to an internal pressure of 7 MPa. It is to be wound using fibers at 90° and at +/–45°. Fiberglass with a strength of 1 GPa is used. If the thickness of the hoop (90°) layer is 2 mm, what would be the required thickness of the +/–45° layer?

Solution

From Equations (5.19) and (5.20) we have:

$$\sigma_x(t_1 + t_2) = \sigma_{f1} t_1 \cos^2\alpha_1 + \sigma_{f2} t_2 \cos^2\alpha_2$$

$$\sigma_y(t_1 + t_2) = \sigma_{f1} t_1 \sin^2\alpha_1 + \sigma_{f2} t_2 \sin^2\alpha_2$$

Let layer 1 be the 90° layer and layer 2 be the +/–45° layer.

$$\sigma_x(0.002 + t_2) = (10^9 \text{ Pa}) t_2 (0.5)$$

$$\sigma_y(0.002 + t_2) = (10^9 \text{ Pa})(0.002 \text{ m}) + (10^9) t_2 (0.5)$$

From equilibrium:

$$\sigma_x = \frac{pr}{2t} \qquad \sigma_y = \frac{pr}{t} \quad \text{yielding} \quad \frac{\sigma_y}{\sigma_x} = 2$$

Substituting this relation to the above expression yields:

$$(10^9 \text{ Pa})(0.002 \text{ m}) + (10^9 \text{ Pa})(0.5 t_2) = (10^9 \text{ Pa}) t_2$$

This gives:

$$t_2 = 4 \text{ mm}$$

1.8. Fiber Motion [1,2]

During processing, the fibers may move causing a change in fiber tension and in fiber position. Springer et al. [1,2] developed a model for the determination of the stress and the position of the fiber. Assume that the fiber angle remains constant during processing. A fiber layer consists of a fiber sheet of thickness $\Delta\xi$ surrounded by the resin. The cross-sectional area of the fiber sheet is:

$$A_f = V_f A \tag{5.21}$$

where A is the cross-sectional area of the entire layer (resin + fiber sheet, $A = b\Delta h$), V_f is the fiber volume fraction, b is the width of the fiber band, and Δh is the band thickness.

Motions of the fiber sheet in the axial and hoop directions are not considered because the axial and hoop components of the fiber tension are in equilibrium. However, the radial position of the fiber sheet, r_f, may change for the following reasons:

1. Fiber tension in the curved fibers causes the fibers to move through the resin while the resin viscosity is low. The radial displacement of a fiber sheet relative to the resin is denoted as u_f.

2. Temperature changes in the mandrel and the composite and the chemical changes (shrinkage) of the resin may cause the mandrel and the composite to expand or to contract, causing radial displacement of the fiber, denoted by

$$u_{mc} = u_m + u_c$$

where

u_{mc} = the radial displacement due to changes in mandrel dimensions and chemical shrinkage of the resin.
u_m = the radial displacement due to changes in mandrel dimensions (thermal expansion or contraction).
u_c = the radial displacement due to chemical shrinkage of resin.

Thus, the instantaneous fiber position relative to the axis of the cylinder is:

$$r_f = R_f^o + u_f + u_{mc} \tag{5.22}$$

where R_f^o is the radial position of the fiber sheet at time t_o. u_f is obtained by assuming that the entire layer is deposited instantaneously at time t_o, so that the radial position of the entire sheet is the same at every point in a given layer. Furthermore, end effects are neglected. Thus, the stress in the direction of the fibers is:

$$\sigma_f = \frac{F}{A_f} \tag{5.23}$$

where F is the instantaneous fiber tension and A_f is the cross section of the fiber sheet. The circumferential component of the fiber tension is (from Equation 5.8):

$$\sigma_\theta \sigma_f \sin^2 \alpha \tag{5.24}$$

There is a pressure difference Δp across a fiber sheet (Figure 5.15). For a fiber sheet at the position r_f, force equilibrium gives:

$$\frac{\Delta p}{\Delta \xi} = \frac{dp}{dr} = \frac{\sigma_\theta}{r_f} \tag{5.25}$$

The relative velocity between the fibers and the resin is described by Darcy's law:

$$u_f = \frac{S}{\mu}\frac{dp}{dr} \tag{5.26}$$

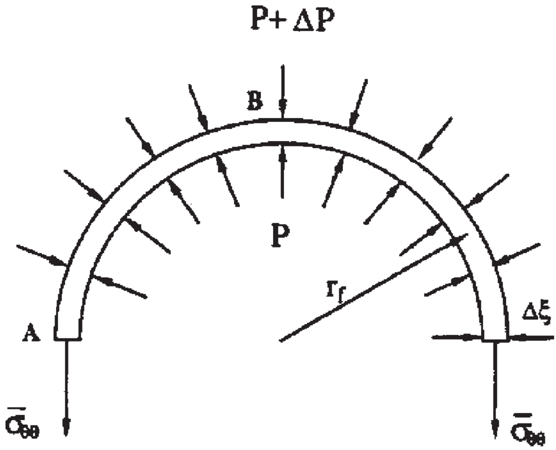

FIGURE 5.15 Pressures on the inside and outside of a fiber sheet.

where S is apparent permeability of the fiber sheet and μ is the resin viscosity. Then the change in fiber position Δu_f during a small time step Δt is:

$$\Delta u_f = -\frac{S\Delta t}{\mu}\frac{\sigma_f}{r_f}\sin^2\alpha \tag{5.27}$$

From geometric considerations, the change in length of the fiber band can be obtained by referring to Figure 5.16.

The change in length of the fiber can be expressed as:

$$\Delta L_f = AB - L_f$$

$$\begin{aligned} AB^2 &= L_f^2 + 2\pi(\Delta u_f)^2 + 2L_f[2\pi(\Delta u_f)]\sin\phi_o \\ &\approx L_f^2 + 2L_f[2\pi(\Delta u_f)]\sin\phi_o \end{aligned}$$

$$\Delta L_f = L_f\sqrt{1+\frac{4\pi\Delta u_f\sin\phi_o}{L_f}} - L_f \approx 2\pi\Delta u_f\sin\phi_o$$

From the lower part of Figure 5.16:

$$2\pi r_f = L_f\sin\phi_o$$

Substituting this into the above equation yields:

$$\frac{\Delta u_f}{r_f} \sin^2 \alpha = \frac{\Delta L_f}{L_f} = \Delta \varepsilon_f \tag{5.28}$$

where ΔL_f and L_f are the elongation and original length of the fiber sheet, respectively. The change in fiber stress corresponding to the change in strain $\Delta\varepsilon_f$ is:

$$\Delta \sigma_f = \sigma_f^{t+\Delta t} - \sigma_f^t = E_f \Delta \varepsilon_f \tag{5.29}$$

where E_f is the longitudinal modulus of the fiber. These equations give the fiber stress at time $t + \Delta t$:

$$\sigma_t^{t+\Delta t} = \sigma_f^t \left[1 - \frac{E_f S \Delta t}{\mu} \frac{\sin^4 \alpha}{r_f^2} \right] \tag{5.30}$$

Solutions of Equations (5.27) to (5.30) provide the fiber position and

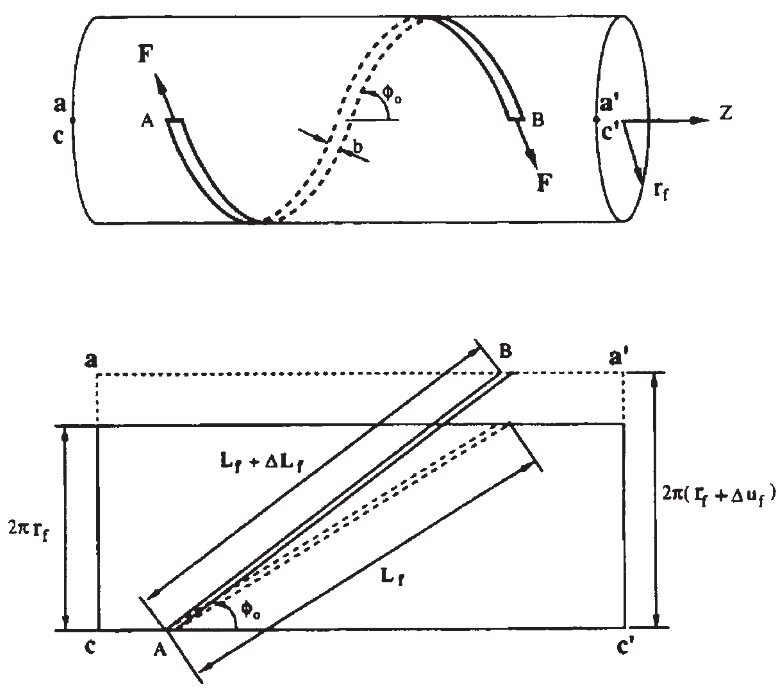

FIGURE 5.16 Fiber length configuration.

fiber stress (fiber tension) at time $t + \Delta t$. The corresponding initial conditions are:

At $t = t_o$,

$$u_f = 0$$

$$\sigma_f = \sigma_f^o = \frac{F_o}{A_f}$$

Example 5.4

It is desired to filament wind a cylinder using glass/epoxy at room temperature using a resin system with a viscosity of 600 cps. The diameter of the mandrel is 30 cm. The width of the fiber band is 10 mm and its height is 1 mm. The initial tension in the fiber band is 10 kPa. The fiber modulus is 70 GPa. The fiber volume fraction is 0.65. The fiber winding angle is +/–45°. Assuming that the permeability of the fiber network is 10^{-12} m^2, determine the position of the fiber and tension in the fiber as a function of time.

Solution

The solution utilizes Equations (5.27) and (5.30). Initial fiber tension is 10 kPa. For the first 4.5 hours, the viscosity is constant at 600 cps. The fiber tension as a function of time during this period is:

$$\sigma_f^t = \sigma_f^o\left[1 - \frac{E_f S\Delta t}{\mu}\frac{\sin^4\alpha}{r_f^2}\right]$$

$$= (10\text{ kPa})\left[1 - \frac{(70\text{ GPa})(10^{-12}\text{ m}^2)t}{600\times 10^{-3}\text{ Pa - sec}}\frac{0.25}{(0.15\text{ m})^2}\right] = 10\text{ kPa }(1 - 1.3t)$$

$$\sigma_f^t = 10 - 13t\text{ kPa}$$

The fiber position at any time is given by:

$$\Delta u_f = -\frac{S\Delta t}{\mu}\frac{\sigma_f}{r_f}\sin^2\alpha$$

$$= -\frac{(10^{-12}\text{ m}^2)t}{600\times 10^{-3}\text{ Pa - sec}}\frac{(10 - 13t)10^3\text{ Pa}}{0.15\text{ m}}(0.5) = 5.55\times 10^{-9}(10 - 13t)\text{ m}$$

It can be seen from the above results that the fiber tension decreases quickly due to the fiber motion even though the position of the fiber does not change very much. The fiber tension is reduced to nil in less than a second. It is essential that a tensioning device be used to take up this slack by maintaining tension in the fiber and avoiding fiber waviness.

2. FIBER PLACEMENT PROCESS

Fiber placement is a process similar to filament winding in which the fibers are placed onto the surface of the mandrel one strip at a time. Figure 5.17 shows a schematic for the fiber placement process. The differences between fiber placement and filament winding are as follows.

- In filament winding, the fiber tows are subject to tension while they are being placed onto the surface of the mandrel. In fiber placement, the fibers are pushed toward the surface of the mandrel. As such, flexible tows cannot be used for fiber placement. Instead, tapes with a certain degree of rigidity are used. Filament winding can be done using both wet winding and dry winding, whereas for fiber placement, only prepregs or tapes can be used (i.e. no wet fiber placement process).
- In fiber placement process, there is a pressure applicator that presses the fibers as they are being placed on the surface of the mandrel. This pressure applicator consolidates the fibers as they are being wrapped around the mandrel. With the pressure applicator, surfaces other than convex (as required for filament winding) can used.
- In the fiber placement process, usually heat is applied at the nip point (point where the fiber bands meet the surface of the mandrel). The application of heat allows the liquefication of the resin. Combination of heat and pressure provides the drive of flow and consolidation. Due to the presence of heat and pressure,

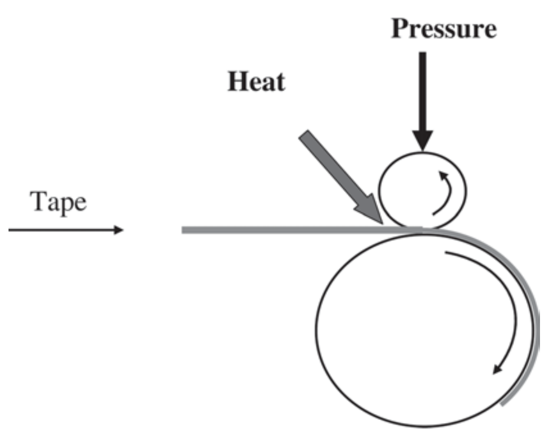

FIGURE 5.17 Schematic of the fiber placement process.

the resin systems used for the fiber placement process can be different from those used for filament winding.

- The fiber placement process can be applied to both thermoset and thermoplastic composites without significant change in the machine setup (except for the laying head).

3. REFERENCES

1. Calius E. P. and Springer G. S. A model of filament wound thin cylinders, *Internat. J. Solids Structures,* 1990, 26, pp. 271–297.
2. Lee S. Y. and Springer G. S. Filament winding cylinders: 1. Process model, *J. Compos. Mater.,* 24, pp. 1270–1298, 1990.

4. HOMEWORK

1. It is desired to filament wind a 300 mm diameter pipe with a winding angle of +/−54°. The length of the pipe is 1300 mm. The bandwidth of the fibers is 6 mm. Assume a dwell angle of 180° at each end. Assume also that there is no overlap or gap between adjacent bandwidths. Determine:

   - The number of circuits per pattern
   - The number of patterns per layer
   - The number of circuits per layer

   You are free to adjust the dwell angle and the bandwidth slightly to obtain your results.

2. A pipe of 300 mm in diameter is filament wound with fiberglass. Two types of wind angle will be used. One is 30° and the other is 45°. Assume that there is 1/3 of the thickness of the 30° type and 2/3 of the thickness of the 45° type. Assume also that the strength of the fiberglass used in this case is 1360 MPa. Determine the thickness of each type of winding needed to contain an internal pressure of 41 MPa.