6

Elastic Constants Based on Global Coordinate System

6.1 Basic Equations

The engineering properties or elastic constants were introduced in Chap. 2 with respect to the lamina 1-2-3 coordinate system. Their evaluation was presented in Chap. 3 based also on the 1-2-3 coordinate system. We can also define elastic constants with respect to the x-y-z global coordinate system. The elastic constants in the x-y-z coordinate system can be derived directly from their definitions, just as they were derived in Chap. 3 for the 1-2-3 coordinate system.

The elastic constants based on the x-y-z global coordinate system are given as follows [1]:

$$E_x = \frac{E_1}{m^4 + \left(\frac{E_1}{G_{12}} - 2\nu_{12}\right) n^2 m^2 + \frac{E_1}{E_2} n^4} \tag{6.1}$$

$$\nu_{xy} = \frac{\nu_{12}\left(n^4 + m^4\right) - \left(1 + \frac{E_1}{E_2} - \frac{E_1}{G_{12}}\right) n^2 m^2}{m^4 + \left(\frac{E_1}{G_{12}} - 2\nu_{12}\right) n^2 m^2 + \frac{E_1}{E_2} n^2} \tag{6.2}$$

$$E_y = \frac{E_2}{m^4 + \left(\frac{E_2}{G_{12}} - 2\nu_{21}\right) n^2 m^2 + \frac{E_2}{E_1} n^4} \tag{6.3}$$

$$\nu_{yx} = \frac{\nu_{21}\left(n^4 + m^4\right) - \left(1 + \frac{E_2}{E_1} - \frac{E_2}{G_{12}}\right) n^2 m^2}{m^4 + \left(\frac{E_2}{G_{12}} - 2\nu_{21}\right) n^2 m^2 + \frac{E_2}{E_1} n^2} \tag{6.4}$$

$$G_{xy} = \frac{G_{12}}{n^4 + m^4 + 2\left(\frac{2G_{12}}{E_1}\left(1 + 2\nu_{12}\right) + \frac{2G_{12}}{E_2} - 1\right) n^2 m^2} \tag{6.5}$$

It is useful to define several other material properties for fiber-reinforced composite materials that can be used to categorize response [1]. These properties have as their basis the fact that an element of fiber-reinforced composite material with its fiber oriented at some arbitrary angle exhibits a shear strain when subjected to a normal stress, and it also exhibits an extensional strain when subjected to a shear stress.

Poisson's ratio is defined as the ratio of extensional strains, given that the element is subjected only to a simple normal stress. By analogy, the *coefficient of mutual influence of the second kind* is defined as the ratio of a shear strain to an extensional strain, given that the element is subjected to only a single normal stress. The *coefficient of mutual influence of the first kind* is defined as the ratio of an extensional strain to a shear strain, given that the element is subjected to only a single shear stress (see [1]).

One coefficient of mutual influence of the second kind is defines as follows:

$$\eta_{xy,x} = \frac{\gamma_{xy}}{\varepsilon_x} \tag{6.6}$$

where $\sigma_x \neq 0$ and all other stresses are zero. Another coefficient of mutual influence of the second kind is defined as follows:

$$\eta_{xy,y} = \frac{\gamma_{xy}}{\varepsilon_y} \tag{6.7}$$

where $\sigma_y \neq 0$ and all other stresses are zero. It can be shown that the coefficients of mutual influence of the second kind can be written as follows:

$$\eta_{xy,x} = \frac{\bar{S}_{16}}{\bar{S}_{11}} \tag{6.8}$$

$$\eta_{xy,y} = \frac{\bar{S}_{26}}{\bar{S}_{22}} \tag{6.9}$$

The coefficients of mutual influence of the first kind are defined as follows:

$$\eta_{x,xy} = \frac{\varepsilon_x}{\gamma_{xy}} \tag{6.10}$$

$$\eta_{y,xy} = \frac{\varepsilon_y}{\gamma_{xy}} \tag{6.11}$$

where $\tau_{xy} \neq 0$ and all other stresses are zero. It can be shown that the coefficients of mutual influence of the first kind can be written as follows:

$$\eta_{x,xy} = \frac{\bar{S}_{16}}{\bar{S}_{66}} \tag{6.12}$$

$$\eta_{y,xy} = \frac{\bar{S}_{26}}{\bar{S}_{66}} \tag{6.13}$$

6.2 MATLAB Functions Used

The nine MATLAB functions used in this chapter to calculate the constants based on the global coordinate system are :

Ex(E1, E2, NU12, G12, theta) – This function calculates the elastic modulus E_x along the x-direction in the global coordinate system. Its input consists of five arguments representing the four elastic constants E_1, E_2, ν_{12}, G_{12}, and the fiber orientation angle θ.

NUxy(E1, E2, NU12, G12, theta) – This function calculates Poisson's ratio ν_{xy} in the global coordinate system. Its input consists of five arguments representing the four elastic constants E_1, E_2, ν_{12}, G_{12}, and the fiber orientation angle θ.

Ey(E1, E2, NU21, G12, theta) – This function calculates the elastic modulus E_y along the y-direction in the global coordinate system. Its input consists of five arguments representing the four elastic constants E_1, E_2, ν_{21}, G_{12}, and the fiber orientation angle θ.

NUyx(E1, E2, NU21, G12, theta) – This function calculates Poisson's ratio ν_{yx} in the global coordinate system. Its input consists of five arguments representing the four elastic constants E_1, E_2, ν_{21}, G_{12}, and the fiber orientation angle θ.

Gxy(E1, E2, NU12, G12, theta) – This function calculates the shear modulus G_{xy} in the global coordinate system. Its input consists of five arguments representing the four elastic constants E_1, E_2, ν_{12}, G_{12}, and the fiber orientation angle θ.

Etaxyx(Sbar) – This function calculates the coefficient of mutual influence of the second kind $\eta_{xy,x}$. It has one argument – the transformed reduced compliance matrix $[\bar{S}]$.

*Etaxy*y(Sbar) – This function calculates the coefficient of mutual influence of the second kind $\eta_{xy,y}$. It has one argument – the transformed reduced compliance matrix $[\bar{S}]$.

*Etax*xy(Sbar) – This function calculates the coefficient of mutual influence of the first kind $\eta_{x,xy}$. It has one argument – the transformed reduced compliance matrix $[\bar{S}]$.

*Etay*xy(Sbar) – This function calculates the coefficient of mutual influence of the first kind $\eta_{y,xy}$. It has one argument – the transformed reduced compliance matrix $[\bar{S}]$.

The following is a listing of the MATLAB source code for each function:

```
function y = Ex(E1,E2,NU12,G12,theta)
%Ex      This function returns the elastic modulus
%        along the x-direction in the global
%        coordinate system. It has five arguments:
%        E1    - longitudinal elastic modulus
%        E2    - transverse elastic modulus
%        NU12  - Poisson's ratio
%        G12   - shear modulus
%        theta - fiber orientation angle
%        The angle "theta" must be given in degrees.
%        Ex is returned as a scalar
m = cos(theta*pi/180);
n = sin(theta*pi/180);
denom = m^4 + (E1/G12 - 2*NU12)*n*n*m*m + (E1/E2)*n^4;
y = E1/denom;
```

```
function y = NUxy(E1,E2,NU12,G12,theta)
%NUxy   This function returns Poisson's ratio
%       NUxy in the global
%       coordinate system. It has five arguments:
%       E1    - longitudinal elastic modulus
%       E2    - transverse elastic modulus
%       NU12  - Poisson's ratio
%       G12   - shear modulus
%       theta - fiber orientation angle
%       The angle "theta" must be given in degrees.
%       NUxy is returned as a scalar
m = cos(theta*pi/180);
n = sin(theta*pi/180);
denom = m^4 + (E1/G12 - 2*NU12)*n*n*m*m + (E1/E2)*n*n;
numer = NU12*(n^4 + m^4) - (1 + E1/E2 - E1/G12)*n*n*m*m;
y = numer/denom;
```

```
function y = Ey(E1,E2,NU21,G12,theta)
%Ey   This function returns the elastic modulus
%     along the y-direction in the global
%     coordinate system. It has five arguments:
%     E1    - longitudinal elastic modulus
%     E2    - transverse elastic modulus
%     NU21  - Poisson's ratio
%     G12   - shear modulus
%     theta - fiber orientation angle
%     The angle "theta" must be given in degrees.
%     Ey is returned as a scalar
m = cos(theta*pi/180);
n = sin(theta*pi/180);
denom = m^4 + (E2/G12 - 2*NU21)*n*n*m*m + (E2/E1)*n^4;
y = E2/denom;
```

```
function y = NUyx(E1,E2,NU21,G12,theta)
%NUyx   This function returns Poisson's ratio
%       NUyx in the global
%       coordinate system. It has five arguments:
%       E1    - longitudinal elastic modulus
%       E2    - transverse elastic modulus
%       NU21  - Poisson's ratio
%       G12   - shear modulus
%       theta - fiber orientation angle
%       The angle "theta" must be given in degrees.
%       NUyx is returned as a scalar
m = cos(theta*pi/180);
n = sin(theta*pi/180);
denom = m^4 + (E2/G12 - 2*NU21)*n*n*m*m + (E2/E1)*n*n;
numer = NU21*(n^4 + m^4) - (1 + E2/E1 - E2/G12)*n*n*m*m;
y = numer/denom;
```

```
function y = Gxy(E1,E2,NU12,G12,theta)
%Gxy This function returns the shear modulus
%    Gxy in the global
%    coordinate system. It has five arguments:
%    E1    - longitudinal elastic modulus
%    E2    - transverse elastic modulus
%    NU12  - Poisson's ratio
%    G12   - shear modulus
%    theta - fiber orientation angle
%    The angle "theta" must be given in degrees.
%    Gxy is returned as a scalar
m = cos(theta*pi/180);
n = sin(theta*pi/180);
denom = n^4 + m^4 + 2*(2*G12*(1 + 2*NU12)/E1 + 2*G12/E2 - 1)
*n*n*m*m;
 y = G12/denom;
```

```
function y = Etaxyx(Sbar)
%Etaxyx   This function returns the coefficient of
%         mutual influence of the second kind
%         ETAxy,x in the global coordinate system.
%         It has one argument - the reduced
%         transformed compliance matrix Sbar.
%         Etaxyx is returned as a scalar
y = Sbar(1,3)/Sbar(1,1);
```

```
function y = Etaxyy(Sbar)
%Etaxyy   This function returns the coefficient of
%         mutual influence of the second kind
%         ETAxy,y in the global coordinate system.
%         It has one argument - the reduced
%         transformed compliance matrix Sbar.
%         Etaxyy is returned as a scalar
y = Sbar(2,3)/Sbar(2,2);
```

```
function y = Etaxxy(Sbar)
%Etaxxy   This function returns the coefficient of
%         mutual influence of the first kind
%         ETAx,xy in the global coordinate system.
%         It has one argument - the reduced
%         transformed compliance matrix Sbar.
%         Etaxxy is returned as a scalar
y = Sbar(1,3)/Sbar(3,3);
```

```
function y = Etayxy(Sbar)
%Etayxy   This function returns the coefficient of
%         mutual influence of the first kind
%         ETAy,xy in the global coordinate system.
%         It has one argument - the reduced
%         transformed compliance matrix Sbar.
%         Etayxy is returned as a scalar
y = Sbar(2,3)/Sbar(3,3);
```

Example 6.1

Derive the expression for E_x given in (6.1).

Solution

From an elementary course on mechanics of materials, we have the following relation (assuming uniaxial tension with $\sigma_x \neq 0$ and all other stresses zeros):

$$\varepsilon_x = \frac{\sigma_x}{E_x} \tag{6.14}$$

However, from (5.10), we also have the following relation:

$$\varepsilon_x = \bar{S}_{11}\sigma_x \tag{6.15}$$

Comparing (6.14) and (6.15), we conclude the following:

$$\frac{1}{E_x} = \bar{S}_{11} \tag{6.16}$$

Substituting for $\bar{S}_{11}$ from (5.16a) and taking the inverse of (6.16), we obtain the desired result as follows:

$$E_x = \frac{1}{\bar{S}_{11}} = \frac{E_1}{m^4 + \left(\frac{E_1}{G_{12}} - 2\nu_{12}\right) n^2 m^2 + \frac{E_1}{E_2} n^4} \tag{6.17}$$

In the above equation, we have substituted for the elements of the reduced compliance matrix with the appropriate elastic constants.

MATLAB Example 6.2

Consider a graphite-reinforced polymer composite lamina with the elastic constants as given in Example 2.2. Use MATLAB to plot the values of the five elastic constants E_x, ν_{xy}, E_y, ν_{yx}, and G_{xy} as a function of the orientation angle θ in the range $-\pi/2 \leq \theta \leq \pi/2$.

Solution

This example is solved using MATLAB. The elastic modulus E_x is calculated at each value of θ between -90° and 90° in increments of 10° using the MATLAB function Ex.

```
>> Ex1 = Ex(155.0, 12.10, 0.248, 4.40, -90)

Ex1 =

   12.1000

>> Ex2 = Ex(155.0, 12.10, 0.248, 4.40, -80)

Ex2 =

   11.8632

>> Ex3 = Ex(155.0, 12.10, 0.248, 4.40, -70)

Ex3 =

   11.4059

>> Ex4 = Ex(155.0, 12.10, 0.248, 4.40, -60)

Ex4 =

   11.2480

>> Ex5 = Ex(155.0, 12.10, 0.248, 4.40, -50)

Ex5 =

   11.9204

>> Ex6 = Ex(155.0, 12.10, 0.248, 4.40, -40)

Ex6 =

   14.1524

>> Ex7 = Ex(155.0, 12.10, 0.248, 4.40, -30)

Ex7 =

   19.6820

>> Ex8 = Ex(155.0, 12.10, 0.248, 4.40, -20)
```

```
Ex8 =

   34.1218

>> Ex9 = Ex(155.0, 12.10, 0.248, 4.40, -10)

Ex9 =

   78.7623

>> Ex10 = Ex(155.0, 12.10, 0.248, 4.40, 0)

Ex10 =

   155

>> Ex11 = Ex(155.0, 12.10, 0.248, 4.40, 10)

Ex11 =

   78.7623

>> Ex12 = Ex(155.0, 12.10, 0.248, 4.40, 20)

Ex12 =

   34.1218

>> Ex13 = Ex(155.0, 12.10, 0.248, 4.40, 30)

Ex13 =

   19.6820

>> Ex14 = Ex(155.0, 12.10, 0.248, 4.40, 40)

Ex14 =

   14.1524

>> Ex15 = Ex(155.0, 12.10, 0.248, 4.40, 50)

Ex15 =

   11.9204

>> Ex16 = Ex(155.0, 12.10, 0.248, 4.40, 60)
```

```
Ex16 =

   11.2480

>> Ex17 = Ex(155.0, 12.10, 0.248, 4.40, 70)

Ex17 =

   11.4059

>> Ex18 = Ex(155.0, 12.10, 0.248, 4.40, 80)

Ex18 =

   11.8632

>> Ex19 = Ex(155.0, 12.10, 0.248, 4.40, 90)

Ex19 =

   12.1000
```

The x-axis is now setup for the plots as follows:

```
>> x = [-90 -80 -70 -60 -50 -40 -30 -20 -10 0 10 20 30 40 50 60
        70 80 90]

x =

   -90   -80   -70   -60   -50   -40   -30   -20   -10     0    10   20     30
   40    50    60    70    80    90
```

The values of E_x are now calculated for each value of θ between -90° and 90° in increments of 10°.

```
>> y1 = [Ex1 Ex2 Ex3 Ex4 Ex5 Ex6 Ex7 Ex8 Ex9 Ex10 Ex11 Ex12 Ex13 Ex14
         Ex15 Ex16 Ex17 Ex18 Ex19]

y1 =

   Columns 1 through 14

    12.1000    11.8632    11.4059    11.2480     11.9204    14.1524   19.6820
    34.1218    78.7623   155.0000    78.7623    34.1218    19.6820    14.1524

   Columns 15 through 19

    11.9204    11.2480    11.4059    11.8632    12.1000
```

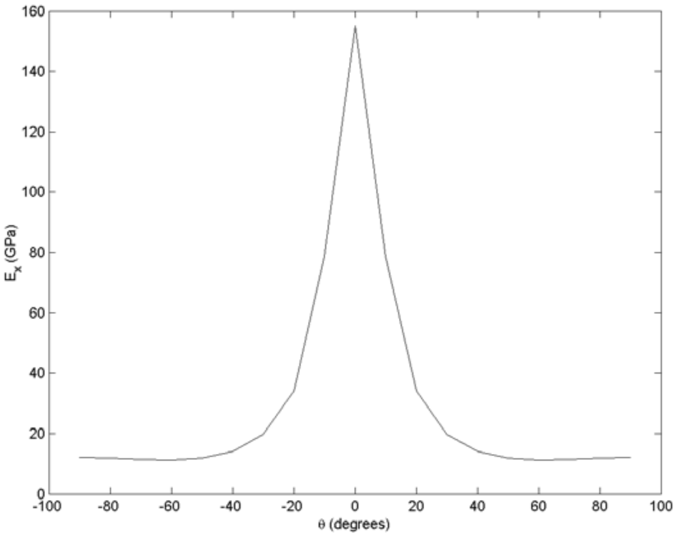

Fig. 6.1. Variation of E_x versus θ for Example 6.2

The plot of the values of E_x versus θ is now generated using the following commands and is shown in Fig. 6.1. Notice that this modulus is an even function of θ. Notice also the rapid variation of the modulus as θ increases or decreases from 0°.

```
>> plot(x,y1)
>> xlabel('\theta (degrees)');
>> ylabel('E_x (GPa)');
```

Next, Poisson's ratio ν_{xy} is calculated at each value of θ between -90° and 90° in increments of 10° using the MATLAB function *NUxy*.

```
>> NUxy1 = NUxy(155.0, 12.10, 0.248, 4.40, -90)

NUxy1 =

    0.0194


>> NUxy2 = NUxy(155.0, 12.10, 0.248, 4.40, -80)


NUxy2 =

    0.0640

>> NUxy3 = NUxy(155.0, 12.10, 0.248, 4.40, -70)
```

```
NUxy3 =

    0.1615

>> NUxy4 = NUxy(155.0, 12.10, 0.248, 4.40, -60)

NUxy4 =

    0.2577

>> NUxy5 = NUxy(155.0, 12.10, 0.248, 4.40, -50)

NUxy5 =

    0.3303

>> NUxy6 = NUxy(155.0, 12.10, 0.248, 4.40, -40)

NUxy6 =

    0.3785

>> NUxy7 = NUxy(155.0, 12.10, 0.248, 4.40, -30)

NUxy7 =

    0.4058

>> NUxy8 = NUxy(155.0, 12.10, 0.248, 4.40, -20)

NUxy8 =

    0.4107

>> NUxy9 = NUxy(155.0, 12.10, 0.248, 4.40, -10)

NUxy9 =

    0.3670

>> NUxy10 = NUxy(155.0, 12.10, 0.248, 4.40, 0)

NUxy10 =

    0.2480

>> NUxy11 = NUxy(155.0, 12.10, 0.248, 4.40, 10)
```

```
NUxy11 =

    0.3670

>> NUxy12 = NUxy(155.0, 12.10, 0.248, 4.40, 20)

NUxy12 =

    0.4107

>> NUxy13 = NUxy(155.0, 12.10, 0.248, 4.40, 30)

NUxy13 =

    0.4058

>> NUxy14 = NUxy(155.0, 12.10, 0.248, 4.40, 40)

NUxy14 =

    0.3785

>> NUxy15 = NUxy(155.0, 12.10, 0.248, 4.40, 50)

NUxy15 =

    0.3303

>> NUxy16 = NUxy(155.0, 12.10, 0.248, 4.40, 60)

NUxy16 =

    0.2577

>> NUxy17 = NUxy(155.0, 12.10, 0.248, 4.40, 70)

NUxy17 =

    0.1615

>> NUxy18 = NUxy(155.0, 12.10, 0.248, 4.40, 80)

NUxy18 =

    0.0640

>> NUxy19 = NUxy(155.0, 12.10, 0.248, 4.40, 90)
```

```
NUxy19 =

    0.0194
```

The values of ν_{xy} are now calculated for each value of θ between -90° and 90° in increments of 10°.

```
>> y2 = [NUxy1 NUxy2 NUxy3 NUxy4 NUxy5 NUxy6 NUxy7 NUxy8 NUxy9 NUxy10
        NUxy11 NUxy12 NUxy13 NUxy14 NUxy15 NUxy16 NUxy17 NUxy18 NUxy19]

y2 =

  Columns 1 through 14

    0.0194    0.0640    0.1615    0.2577    0.3303    0.3785
    0.4058    0.4107   0.3670  0.2480  0.3670    0.4107    0.4058    0.3785

  Columns 15 through 19

    0.3303     0.2577     0.1615     0.0640     0.0194
```

The plot of the values of ν_{xy} versus θ is now generated using the following commands and is shown in Fig. 6.2. Notice that this ratio is an even function of θ. Notice also the rapid variation of the ratio as θ increases or decreases from 0°.

```
>> plot(x,y2)
>> xlabel('\theta (degrees)');
>> ylabel('\nu_{xy}');
```

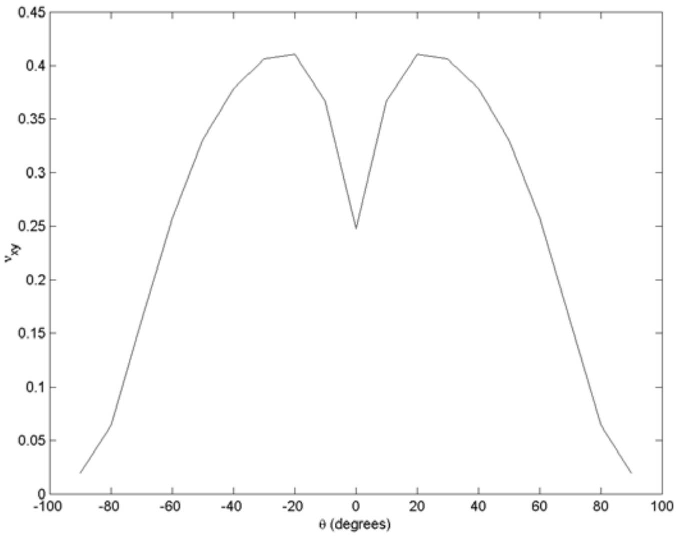

Fig. 6.2. Variation of ν_{xy} versus θ for Example 6.2

Next, the elastic modulus E_y is calculated at each value of θ between -90° and 90° in increments of 10° using the MATLAB function *Ey*.

```
>> Ey1 = Ey(155.0, 12.10, 0.248, 4.40, -90)

Ey1 =

   155

>> Ey2 = Ey(155.0, 12.10, 0.248, 4.40, -80)

Ey2 =

   86.2721

>> Ey3 = Ey(155.0, 12.10, 0.248, 4.40, -70)

Ey3 =

   39.3653

>> Ey4 = Ey(155.0, 12.10, 0.248, 4.40, -60)

Ey4 =

   22.8718

>> Ey5 = Ey(155.0, 12.10, 0.248, 4.40, -50)

Ey5 =

   16.2611

>> Ey6 = Ey(155.0, 12.10, 0.248, 4.40, -40)

Ey6 =

   13.3820

>> Ey7 = Ey(155.0, 12.10, 0.248, 4.40, -30)

Ey7 =

   12.2222

>> Ey8 = Ey(155.0, 12.10, 0.248, 4.40, -20)
```

```
Ey8 =

   11.9374

>> Ey9 = Ey(155.0, 12.10, 0.248, 4.40, -10)

Ey9 =

   12.0208

>> Ey10 = Ey(155.0, 12.10, 0.248, 4.40, 0)

Ey10 =

   12.1000

>> Ey11 = Ey(155.0, 12.10, 0.248, 4.40, 10)

Ey11 =

   12.0208

>> Ey12 = Ey(155.0, 12.10, 0.248, 4.40, 20)

Ey12 =

   11.9374

>> Ey13 = Ey(155.0, 12.10, 0.248, 4.40, 30)

Ey13 =

   12.2222

>> Ey14 = Ey(155.0, 12.10, 0.248, 4.40, 40)

Ey14 =

   13.3820

>> Ey15 = Ey(155.0, 12.10, 0.248, 4.40, 50)

Ey15 =

   16.2611

>> Ey16 = Ey(155.0, 12.10, 0.248, 4.40, 60)
```

```
Ey16 =

   22.8718

>> Ey17 = Ey(155.0, 12.10, 0.248, 4.40, 70)

Ey17 =

   39.3653

>> Ey18 = Ey(155.0, 12.10, 0.248, 4.40, 80)

Ey18 =

   86.2721

>> Ey19 = Ey(155.0, 12.10, 0.248, 4.40, 90)

Ey19 =

   155
```

The values of E_y are now calculated for each value of θ between -90° and 90° in increments of 10°.

```
>> y3 = [Ey1 Ey2 Ey3 Ey4 Ey5 Ey6 Ey7 Ey8 Ey9 Ey10 Ey11 Ey12 Ey13 Ey14
        Ey15 Ey16 Ey17 Ey18 Ey19]

y3 =

  Columns 1 through 14

   155.0000   86.2721   39.3653   22.8718   16.2611   13.3820 12.2222
   11.9374  12.0208  12.1000  12.0208  11.9374  12.2222  13.3820

  Columns 15 through 19

   16.2611   22.8718   39.3653   86.2721  155.0000
```

The plot of the values of E_y versus θ is now generated using the following commands and is shown in Fig. 6.3. Notice that this modulus is an even function of θ. Notice also the rapid variation of the modulus as θ increases or decreases from 0°.

```
>> plot(x,y3)
>> xlabel('\theta (degrees)');
>> ylabel('E_y (GPa)');
```

Next, Poisson's ratio ν_{yx} is calculated at each value of θ between -90° and 90° in increments of 10° using the MATLAB function *NUyx*

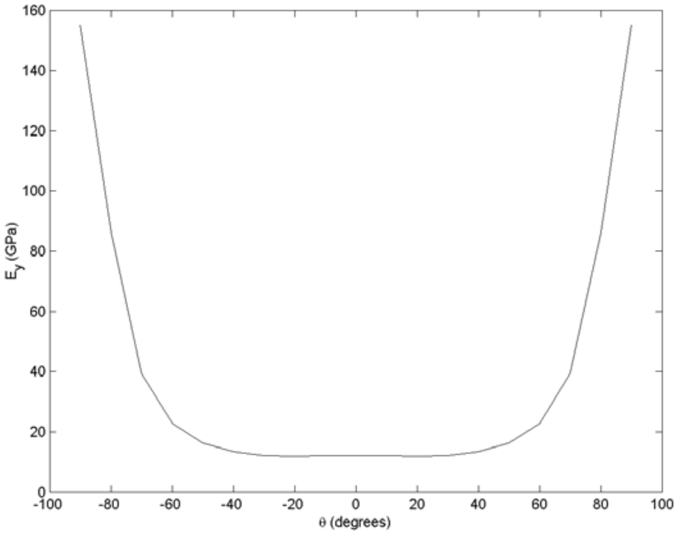

Fig. 6.3. Variation of E_y versus θ for Example 6.2

```
>> NUyx1 = NUyx(155.0, 12.10, 0.248, 4.40, -90)

NUyx1 =

    3.1769

>> NUyx2 = NUyx(155.0, 12.10, 0.248, 4.40, -80)

NUyx2 =

    1.9812

>> NUyx3 = NUyx(155.0, 12.10, 0.248, 4.40, -70)

NUyx3 =

    1.1713

>> NUyx4 = NUyx(155.0, 12.10, 0.248, 4.40, -60)

NUyx4 =

    0.8617

>> NUyx5 = NUyx(155.0, 12.10, 0.248, 4.40, -50)
```

```
NUyx5 =

    0.6987

>> NUyx6 = NUyx(155.0, 12.10, 0.248, 4.40, -40)

NUyx6 =

    0.5775

>> NUyx7 = NUyx(155.0, 12.10, 0.248, 4.40, -30)

NUyx7 =

    0.4663

>> NUyx8 = NUyx(155.0, 12.10, 0.248, 4.40, -20)

NUyx8 =

    0.3616

>> NUyx9 = NUyx(155.0, 12.10, 0.248, 4.40, -10)

NUyx9 =

    0.2799

>> NUyx10 = NUyx(155.0, 12.10, 0.248, 4.40, 0)

NUyx10 =

    0.2480

>> NUyx11 = NUyx(155.0, 12.10, 0.248, 4.40, 10)

NUyx11 =

    0.2799

>> NUyx12 = NUyx(155.0, 12.10, 0.248, 4.40, 20)

NUyx12 =

    0.3616

>> NUyx13 = NUyx(155.0, 12.10, 0.248, 4.40, 30)
```

```
NUyx13 =

    0.4663

>> NUyx14 = NUyx(155.0, 12.10, 0.248, 4.40, 40)

NUyx14 =

    0.5775

>> NUyx15 = NUyx(155.0, 12.10, 0.248, 4.40, 50)

NUyx15 =

    0.6987

>> NUyx16 = NUyx(155.0, 12.10, 0.248, 4.40, 60)

NUyx16 =

    0.8617

>> NUyx17 = NUyx(155.0, 12.10, 0.248, 4.40, 70)

NUyx17 =

    1.1713

>> NUyx18 = NUyx(155.0, 12.10, 0.248, 4.40, 80)

NUyx18 =

    1.9812

>> NUyx19 = NUyx(155.0, 12.10, 0.248, 4.40, 90)

NUyx19 =

    3.1769
```

The values of ν_{yx} are now calculated for each value of θ between -90° and 90° in increments of 10°.

```
>> y4 = [NUyx1 NUyx2 NUyx3 NUyx4 NUyx5 NUyx6 NUyx7 NUyx8 NUyx9 NUyx10
        NUyx11 NUyx12 NUyx13 NUyx14 NUyx15 NUyx16 NUyx17 NUyx18 NUyx19]

y4 =

   Columns 1 through 14
```

```
3.1769  1.9812  1.1713  0.8617  0.6987  0.5775   0.4663
0.3616  0.2799  0.2480  0.2799  0.3616  0.4663  0.5775

Columns 15 through 19

 0.6987    0.8617    1.1713    1.9812    3.1769
```

The plot of the values of ν_{yx} versus θ is now generated using the following commands and is shown in Fig. 6.4. Notice that this ratio is an even function of θ. Notice also the rapid variation of the ratio as θ increases or decreases from 0°.

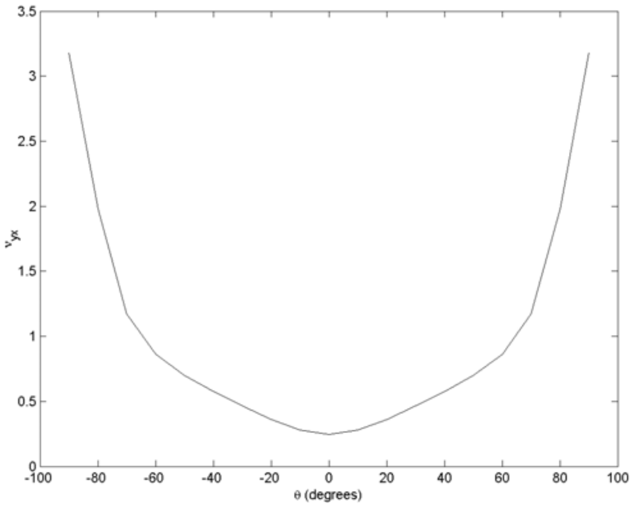

Fig. 6.4. Variation of ν_{yx} versus θ for Example 6.2

```
>> plot(x,y4)
>> xlabel('\theta (degrees)');
>> ylabel('\nu_{yx}');
```

Next, the shear modulus G_{xy} is calculated at each value of θ between -90° and 90° in increments of 10° using the MATLAB function *Gxy*.

```
>> Gxy1 = Gxy(155.0, 12.10, 0.248, 4.40, -90)

Gxy1 =

    4.4000

>> Gxy2 = Gxy(155.0, 12.10, 0.248, 4.40, -80)

Gxy2 =

    4.7285
```

```
>> Gxy3 = Gxy(155.0, 12.10, 0.248, 4.40, -70)

Gxy3 =

    5.8308

>> Gxy4 = Gxy(155.0, 12.10, 0.248, 4.40, -60)

Gxy4 =

    7.9340

>> Gxy5 = Gxy(155.0, 12.10, 0.248, 4.40, -50)

Gxy5 =

   10.3771

>> Gxy6 = Gxy(155.0, 12.10, 0.248, 4.40, -40)

Gxy6 =

   10.3771

>> Gxy7 = Gxy(155.0, 12.10, 0.248, 4.40, -30)

Gxy7 =

    7.9340

>> Gxy8 = Gxy(155.0, 12.10, 0.248, 4.40, -20)

Gxy8 =

    5.8308

>> Gxy9 = Gxy(155.0, 12.10, 0.248, 4.40, -10)

Gxy9 =

    4.7285

>> Gxy10 = Gxy(155.0, 12.10, 0.248, 4.40, 0)

Gxy10 =

    4.4000
```

```
>> Gxy11 = Gxy(155.0, 12.10, 0.248, 4.40, 10)

Gxy11 =

    4.7285

>> Gxy12 = Gxy(155.0, 12.10, 0.248, 4.40, 20)

Gxy12 =

    5.8308

>> Gxy13 = Gxy(155.0, 12.10, 0.248, 4.40, 30)

Gxy13 =

    7.9340

>> Gxy14 = Gxy(155.0, 12.10, 0.248, 4.40, 40)

Gxy14 =

   10.3771

>> Gxy15 = Gxy(155.0, 12.10, 0.248, 4.40, 50)

Gxy15 =

   10.3771

>> Gxy16 = Gxy(155.0, 12.10, 0.248, 4.40, 60)

Gxy16 =

    7.9340

>> Gxy17 = Gxy(155.0, 12.10, 0.248, 4.40, 70)

Gxy17 =

    5.8308

>> Gxy18 = Gxy(155.0, 12.10, 0.248, 4.40, 80)

Gxy18 =

    4.7285
```

```
>> Gxy19 = Gxy(155.0, 12.10, 0.248, 4.40, 90)

Gxy19 =

    4.4000
```

The values of G_{xy} are now calculated for each value of θ between -90° and 90° in increments of 10°.

```
>> y5 = [Gxy1 Gxy2 Gxy3 Gxy4 Gxy5 Gxy6 Gxy7 Gxy8 Gxy9 Gxy10 Gxy11
        Gxy12  Gxy13 Gxy14 Gxy15 Gxy16 Gxy17 Gxy18 Gxy19]

y5 =

  Columns 1 through 14

   4.4000   4.7285   5.8308   7.9340  10.3771  10.3771   7.9340
   5.8308    4.7285    4.4000    4.7285    5.8308    7.9340   10.3771

  Columns 15 through 19

   10.3771    7.9340    5.8308    4.7285    4.4000
```

The plot of the values of G_{xy} versus θ is now generated using the following commands and is shown in Fig. 6.5. Notice that this modulus is an even function of θ. Notice also the rapid variation of the modulus as θ increases or decreases from 0°.

```
>> plot(x,y5)
>> xlabel('\theta (degrees)');
>> ylabel('G_{xy} (GPa)');
```

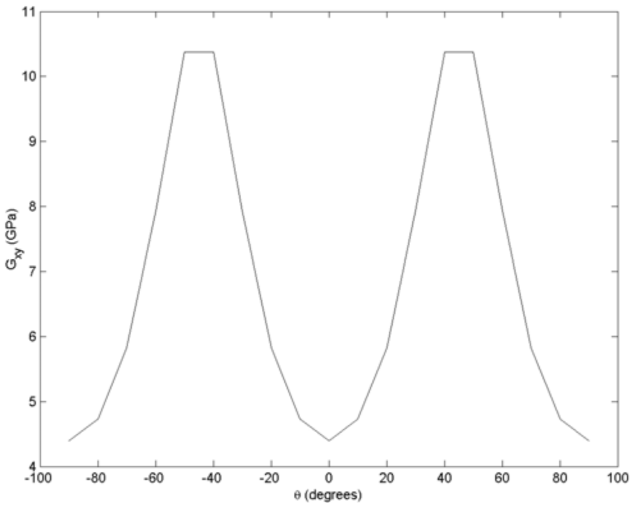

Fig. 6.5. Variation of G_{xy} versus θ for Example 6.2

MATLAB Example 6.3

Consider a graphite-reinforced polymer composite lamina with the elastic constants as given in Example 2.2. Use MATLAB to plot the values of the two coefficients of mutual influence of the second kind $\eta_{xy,x}$ and $\eta_{xy,y}$ as a function of the orientation angle θ in the range $-\pi/2 \leq \theta \leq \pi/2$.

Solution

This example is solved using MATLAB. First, the reduced 3×3 compliance matrix is obtained as follows using the MATLAB function *ReducedCompliance* of Chap. 4.

```
>> S = ReducedCompliance(155.0, 12.10, 0.248, 4.40)

S =

    0.0065   -0.0016         0
   -0.0016    0.0826         0
         0         0    0.2273
```

Next, the transformed reduced compliance matrix $[\bar{S}]$ is calculated at each value of θ between $-90°$ and $90°$ in increments of $10°$ using the MATLAB function *Sbar*.

```
>> S1 = Sbar(S, -90)

S1 =

    0.0826   -0.0016   -0.0000
   -0.0016    0.0065    0.0000
   -0.0000    0.0000    0.2273

>> S2 = Sbar(S, -80)

S2 =

    0.0909   -0.0122   -0.0452
   -0.0122    0.0193    0.0712
   -0.0226    0.0356    0.2061

>> S3 = Sbar(S, -70)

S3 =

    0.1111   -0.0390   -0.0647
   -0.0390    0.0528    0.1137
   -0.0323    0.0568    0.1524

>> S4 = Sbar(S, -60)
```

```
S4 =

    0.1315   -0.0695   -0.0454
   -0.0695    0.0934    0.1114
   -0.0227    0.0557    0.0914

>> S5 = Sbar(S, -50)

S5 =

    0.1390   -0.0894    0.0065
   -0.0894    0.1258    0.0685
    0.0033    0.0342    0.0516

>> S6 = Sbar(S, -40)

S6 =

    0.1258   -0.0894    0.0685
   -0.0894    0.1390    0.0065
    0.0342    0.0033    0.0516

>> S7 = Sbar(S, -30)

S7 =

    0.0934   -0.0695    0.1114
   -0.0695    0.1315   -0.0454
    0.0557   -0.0227    0.0914


>> S8 = Sbar(S, -20)

S8 =

    0.0528   -0.0390    0.1137
   -0.0390    0.1111   -0.0647
    0.0568   -0.0323    0.1524

>> S9 = Sbar(S, -10)

S9 =

    0.0193   -0.0122    0.0712
   -0.0122    0.0909   -0.0452
    0.0356   -0.0226    0.2061

>> S10 = Sbar(S, 0)
```

```
S10 =

    0.0065   -0.0016         0
   -0.0016    0.0826         0
         0         0    0.2273

>> S11 = Sbar(S, 10)

S11 =

    0.0193   -0.0122   -0.0712
   -0.0122    0.0909    0.0452
   -0.0356    0.0226    0.2061

>> S12 = Sbar(S, 20)

S12 =

    0.0528   -0.0390   -0.1137
   -0.0390    0.1111    0.0647
   -0.0568    0.0323    0.1524

>> S13 = Sbar(S, 30)

S13 =

    0.0934   -0.0695   -0.1114
   -0.0695    0.1315    0.0454
   -0.0557    0.0227    0.0914

>> S14 = Sbar(S, 40)

S14 =

    0.1258   -0.0894   -0.0685
   -0.0894    0.1390   -0.0065
   -0.0342   -0.0033    0.0516

>> S15 = Sbar(S, 50)

S15 =

    0.1390   -0.0894   -0.0065
   -0.0894    0.1258   -0.0685
   -0.0033   -0.0342    0.0516

>> S16 = Sbar(S, 60)
```

```
S16 =

    0.1315   -0.0695    0.0454
   -0.0695    0.0934   -0.1114
    0.0227   -0.0557    0.0914

>> S17 = Sbar(S, 70)

S17 =

    0.1111   -0.0390    0.0647
   -0.0390    0.0528   -0.1137
    0.0323   -0.0568    0.1524

>> S18 = Sbar(S, 80)

S18 =

    0.0909   -0.0122    0.0452
   -0.0122    0.0193   -0.0712
    0.0226   -0.0356    0.2061

>> S19 = Sbar(S, 90)

S19 =

    0.0826   -0.0016    0.0000
   -0.0016    0.0065   -0.0000
    0.0000   -0.0000    0.2273
```

The x-axis is now setup for the plots as follows:

```
>> x = [-90  -80  -70  -60  -50  -40  -30  -20  -10  0  10  20  30
       40  50  60  70  80  90]

x =

  -90   -80   -70   -60   -50   -40   -30   -20   -10     0    10
  20    30    40    50    60    70    80    90
```

The values of the coefficient of mutual influence of the second kind $\eta_{xy,x}$ is calculated next for each value of θ in increments of 10° using the MATLAB function *Etaxyx*.

```
>> Etaxyx1 = Etaxyx(S1)

Etaxyx1 =

   -2.1194e-016

>> Etaxyx2 = Etaxyx(S2)
```

```
Etaxyx2 =

   -0.4968

>> Etaxyx3 = Etaxyx(S3)

Etaxyx3 =

   -0.5821

>> Etaxyx4 = Etaxyx(S4)

Etaxyx4 =

   -0.3455

>> Etaxyx5 = Etaxyx(S5)

Etaxyx5 =

    0.0471

>> Etaxyx6 = Etaxyx(S6)

Etaxyx6 =

    0.5446

>> Etaxyx7 = Etaxyx(S7)

Etaxyx7 =

    1.1927

>> Etaxyx8 = Etaxyx(S8)

Etaxyx8 =

    2.1536

>> Etaxyx9 = Etaxyx(S9)

Etaxyx9 =

    3.6831

>> Etaxyx10 = Etaxyx(S10)
```

```
Etaxyx10 =

     0

>> Etaxyx11 = Etaxyx(S11)

Etaxyx11 =

   -3.6831

>> Etaxyx12 = Etaxyx(S12)

Etaxyx12 =

   -2.1536

>> Etaxyx13 = Etaxyx(S13)

Etaxyx13 =

   -1.1927

>> Etaxyx14 = Etaxyx(S14)

Etaxyx14 =

   -0.5446

>> Etaxyx15 = Etaxyx(S15)

Etaxyx15 =

   -0.0471

>> Etaxyx16 = Etaxyx(S16)

Etaxyx16 =

    0.3455

>> Etaxyx17 = Etaxyx(S17)

Etaxyx17 =

    0.5821

>> Etaxyx18 = Etaxyx(S18)
```

```
Etaxyx18 =

    0.4968

>> Etaxyx19 = Etaxyx(S19)

Etaxyx19 =

    2.1194e-016
```

The values of $\eta_{xy,x}$ are now calculated for each value of θ between -90° and 90° in increments of 10°.

```
>> y6 = [Etaxyx1 Etaxyx2 Etaxyx3 Etaxyx4 Etaxyx5 Etaxyx6 Etaxyx7
        Etaxyx8 Etaxyx9 Etaxyx10 Etaxyx11 Etaxyx12 Etaxyx13 Etaxyx14
        Etaxyx15 Etaxyx16 Etaxyx17 Etaxyx18 Etaxyx19]

y6 =

  Columns 1 through 14

    -0.0000   -0.4968   -0.5821   -0.3455    0.0471    0.5446
    1.1927    2.1536    3.6831         0   -3.6831   -2.1536   -1.1927
    -0.5446

  Columns 15 through 19

    -0.0471    0.3455    0.5821    0.4968    0.0000
```

The plot of the values of $\eta_{xy,x}$ versus θ is now generated using the following commands and is shown in Fig. 6.6. Notice that this coefficient is an odd function of θ. Notice also the rapid variation of the coefficient as θ increases or decreases from 0°.

```
>> plot(x,y6)
>> xlabel('\theta (degrees)');
>> ylabel('\eta_{xy,x}');
```

The values of the coefficient of mutual influence of the second kind $\eta_{xy,y}$ is calculated next for each value of θ in increments of 10° using the MATLAB function *Etaxyy*.

```
>> Etaxyy1 = Etaxyy(S1)

Etaxyy1 =

  4.1613e-015

>> Etaxyy2 = Etaxyy(S2)
```

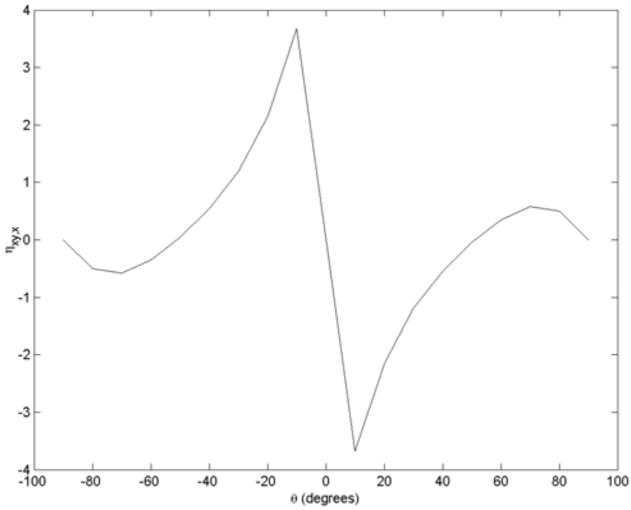

Fig. 6.6. Variation of $\eta_{xy,x}$ versus θ for Example 6.3

```
Etaxyy2 =

    3.6831

>> Etaxyy3 = Etaxyy(S3)

Etaxyy3 =

    2.1536

>> Etaxyy4 = Etaxyy(S4)

Etaxyy4 =

    1.1927

>> Etaxyy5 = Etaxyy(S5)

Etaxyy5 =

    0.5446

>> Etaxyy6 = Etaxyy(S6)

Etaxyy6 =

    0.0471
```

```
>> Etaxyy7 = Etaxyy(S7)

Etaxyy7 =

   -0.3455

>> Etaxyy8 = Etaxyy(S8)

Etaxyy8 =

   -0.5821

>> Etaxyy9 = Etaxyy(S9)

Etaxyy9 =

   -0.4968

>> Etaxyy10 = Etaxyy(S10)

Etaxyy10 =

     0

>> Etaxyy11 = Etaxyy(S11)

Etaxyy11 =

    0.4968

>> Etaxyy12 = Etaxyy(S12)

Etaxyy12 =

    0.5821

>> Etaxyy13 = Etaxyy(S13)

Etaxyy13 =

    0.3455

>> Etaxyy14 = Etaxyy(S14)

Etaxyy14 =

   -0.0471
```

```
>> Etaxyy15 = Etaxyy(S15)

Etaxyy15 =

   -0.5446

>> Etaxyy16 = Etaxyy(S16)

Etaxyy16 =

   -1.1927

>> Etaxyy17 = Etaxyy(S17)

Etaxyy17 =

   -2.1536

>> Etaxyy18 = Etaxyy(S18)

Etaxyy18 =

   -3.6831

>> Etaxyy19 = Etaxyy(S19)

Etaxyy19 =

 -4.1613e-015
```

The values of $\eta_{xy,y}$ are now calculated for each value of θ between -90° and 90° in increments of 10°.

```
>> y7 = [Etaxyy1 Etaxyy2 Etaxyy3 Etaxyy4 Etaxyy5 Etaxyy6 Etaxyy7
        Etaxyy8 Etaxyy9 Etaxyy10 Etaxyy11 Etaxyy12 Etaxyy13 Etaxyy14
        Etaxyy15 Etaxyy16 Etaxyy17 Etaxyy18 Etaxyy19]

y7 =

  Columns 1 through 14

    0.0000    3.6831    2.1536    1.1927    0.5446    0.0471 -0.3455
    -0.5821   -0.4968         0    0.4968    0.5821    0.3455   -0.0471

  Columns 15 through 19

    -0.5446   -1.1927   -2.1536   -3.6831   -0.0000
```

The plot of the values of $\eta_{xy,y}$ versus θ is now generated using the following commands and is shown in Fig. 6.7. Notice that this coefficient is an odd function

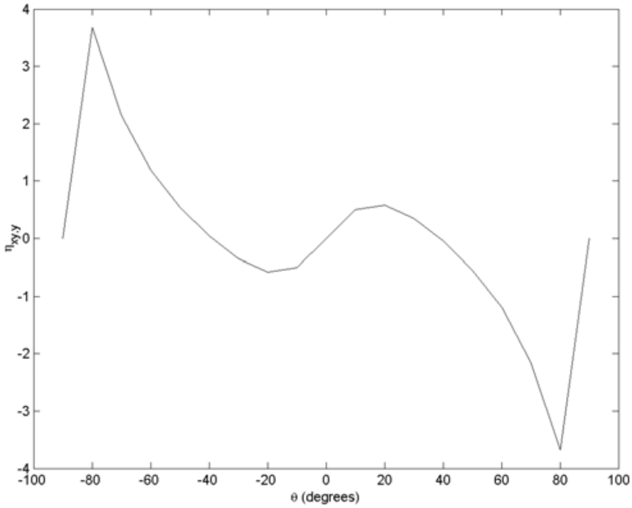

Fig. 6.7. Variation of $\eta_{xy,y}$ versus θ for Example 6.3

of θ. Notice also the rapid variation of the coefficient as θ increases or decreases from $0°$.

```
>> plot(x,y7)
>> xlabel('\theta (degrees)');
>> ylabel('\eta_{xy,y}');
```

Problems

Problem 6.1

Derive the expression for ν_{xy} given in (6.2).

Problem 6.2

Derive the expression for E_y given in (6.3).

Problem 6.3

Derive the expression for ν_{yx} given in (6.4).

Problem 6.4

Derive the expression for G_{xy} given in (6.5).

MATLAB Problem 6.5

Consider a glass-reinforced polymer composite lamina with the elastic constants as given in Problem 2.7. Use MATLAB to plot the values of the five elastic constants E_x, ν_{xy}, E_y, ν_{yx}, and G_{xy} as a function of the orientation angle θ in the range $-\pi/2 \leq \theta \leq \pi/2$.

Problem 6.6

Derive the expressions for the coefficients of mutual influence of the second kind $\eta_{xy,x}$ and $\eta_{xy,y}$ given in (6.8) and (6.9).

Problem 6.7

Derive the expressions for the coefficients of mutual influence of the first kind $\eta_{x,xy}$ and $\eta_{y,xy}$ given in (6.12) and (6.13).

MATLAB Problem 6.8

Consider a graphite-reinforced polymer composite lamina with the elastic constants as given in Example 2.2. Use MATLAB to plot the values of the two coefficients of mutual influence of the first kind $\eta_{x,xy}$ and $\eta_{y,xy}$ as a function of the orientation angle θ in the range $-\pi/2 \leq \theta \leq \pi/2$.

MATLAB Problem 6.9

Consider a glass-reinforced polymer composite lamina with the elastic constants as given in Problem 2.7. Use MATLAB to plot the values of the two coefficients of mutual influence of the second kind $\eta_{xy,x}$ and $\eta_{xy,y}$ as a function of the orientation angle θ in the range $-\pi/2 \leq \theta \leq \pi/2$.

MATLAB Problem 6.10

Consider a glass-reinforced polymer composite lamina with the elastic constants as given in Problem 2.7. Use MATLAB to plot the values of the two coefficients of mutual influence of the first kind $\eta_{x,xy}$ and $\eta_{y,xy}$ as a function of the orientation angle θ in the range $-\pi/2 \leq \theta \leq \pi/2$.