Solutions to Problems

Problem 2.1

In this case, $[S]$ is symmetric given as follows:

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{13} & S_{23} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix}$$

$$\begin{aligned} |S| &= [S_{11}(S_{22}S_{33} - S_{23}S_{23}) - S_{12}(S_{12}S_{33} - S_{13}S_{23}) \\ &\quad + S_{13}(S_{12}S_{23} - S_{13}S_{22})]\, S_{44}S_{55}S_{66} \\ &= (S_{11}S_{22}S_{33} - S_{11}S_{23}S_{23} - S_{33}S_{12}S_{12} \\ &\quad - S_{22}S_{13}S_{13} + 2S_{12}S_{23}S_{13})\, S_{44}S_{55}S_{66} \end{aligned}$$

Next, use the following formula to calculate the inverse of $[S]$:

$$[C] = [S]^{-1} = \frac{adj[S]}{|S|}$$

Only C_{11} will be calculated in detail as follows:

$$C_{11} = \frac{(adj[S])_{11}}{|S|} = \frac{(S_{22}S_{33} - S_{23}S_{23})\, S_{44}S_{55}S_{66}}{|S|} = \frac{1}{S}(S_{22}S_{33} - S_{23}S_{23})$$

where S is given in the book in (2.5). The same procedure can be followed to derive the other elements of $[C]$ given in (2.5).

Problem 2.2

The reciprocity relations of (2.6) are valid for linear elastic analysis. They can be derived by applying the Maxwell-Betti Reciprocal Theorem. For more details, see [1].

Problem 2.3

$$[S] = \begin{bmatrix} 1/E_1 & -\nu_{12}/E_1 & -\nu_{13}/E_1 & 0 & 0 & 0 \\ -\nu_{12}/E_1 & 1/E_2 & -\nu_{23}/E_2 & 0 & 0 & 0 \\ -\nu_{13}/E_1 & -\nu_{23}/E_2 & 1/E_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{2(1+\nu_{23})}{E_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{12} \end{bmatrix}$$

Problem 2.4

$$\begin{aligned} S &= S_{11}S_{22}S_{33} - S_{11}S_{23}S_{23} - S_{22}S_{13}S_{13} - S_{33}S_{12}S_{12} + 2S_{12}S_{23}S_{13} \\ &= \frac{1}{E_1}\frac{1}{E_2}\frac{1}{E_2} - \frac{1}{E_1}\left(\frac{-\nu_{23}}{E_2}\right)\left(\frac{-\nu_{23}}{E_2}\right) - \frac{1}{E_2}\left(\frac{-\nu_{12}}{E_1}\right)\left(\frac{-\nu_{21}}{E_2}\right) \\ &\quad - \frac{1}{E_2}\left(\frac{-\nu_{12}}{E_1}\right)\left(\frac{-\nu_{21}}{E_2}\right) + 2\left(\frac{-\nu_{12}}{E_1}\right)\left(\frac{-\nu_{23}}{E_2}\right)\left(\frac{-\nu_{21}}{E_2}\right) \\ &= \frac{1-\nu_{23}^2 - 2\nu_{12}\nu_{21} - 2\nu_{12}\nu_{23}\nu_{21}}{E_1E_2^2} \\ &= \frac{1-\nu'}{E_1E_2^2} \end{aligned}$$

where ν' is given by:

$$\nu' = \nu_{23}^2 + 2\nu_{12}\nu_{21} + 2\nu_{12}\nu_{23}\nu_{21}$$

Next, C_{11} is calculated in detail as follows:

$$\begin{aligned} C_{11} &= \frac{1}{S}(S_{22}S_{33} - S_{23}S_{23}) \\ &= \frac{E_1E_2^2}{1-\nu'}\left[\frac{1}{E_2}\frac{1}{E_2} - \left(\frac{-\nu_{23}}{E_2}\right)\left(\frac{-\nu_{23}}{E_2}\right)\right] \\ &= \frac{\left(1-\nu_{23}^2\right)E_1}{1-\nu'} \end{aligned}$$

Similarly, the other elements of $[C]$ are obtained as follows:

$$C_{12} = \frac{(1+\nu_{23})\nu_{12}E_2}{1-\nu'}$$
$$C_{13} = \frac{(1+\nu_{23})\nu_{12}E_2}{1-\nu'} = C_{12}$$
$$C_{22} = \frac{(1-\nu_{12}\nu_{21})E_2}{1-\nu'}$$
$$C_{23} = \frac{(\nu_{23}+\nu_{12}\nu_{21})E_2}{1-\nu'}$$
$$C_{33} = \frac{(1-\nu_{12}\nu_{21})E_2}{1-\nu'} = C_{22}$$
$$C_{44} = \frac{E_2}{2(1+\nu_{23})}$$
$$C_{55} = G_{12}$$
$$C_{66} = G_{12} = C_{55}$$

Problem 2.5

$$[S] = \begin{bmatrix} \frac{1}{E} & \frac{-\nu}{E} & \frac{-\nu}{E} & 0 & 0 & 0 \\ \frac{-\nu}{E} & \frac{1}{E} & \frac{-\nu}{E} & 0 & 0 & 0 \\ \frac{-\nu}{E} & \frac{-\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2(1+\nu)}{E} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2(1+\nu)}{E} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2(1+\nu)}{E} \end{bmatrix}$$

$$[S] = \frac{1}{E}\begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+\nu) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(1+\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+\nu) \end{bmatrix}$$

Problem 2.6

$$[C] = \frac{E}{(1+\nu)(1+2\nu)} \begin{bmatrix} 1 & 1-\nu & 1-\nu & 0 & 0 & 0 \\ 1-\nu & 1 & 1-\nu & 0 & 0 & 0 \\ 1-\nu & 1-\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1+2\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1+2\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1+2\nu}{2} \end{bmatrix}$$

Problem 2.7

```
>> sigma3 = 150/(40*40)

sigma3 =

   0.0938

>> sigma = [0 ; 0 ; sigma3 ; 0 ; 0 ; 0]

sigma =

        0
        0
   0.0938
        0
        0
        0

>> [S] =
OrthotropicCompliance(50.0,15.2,15.2,0.254,0.428,0.254,4.70,
   3.28,4.70)

S =
    0.0200   -0.0051   -0.0051         0         0         0
   -0.0051    0.0658   -0.0282         0         0         0
   -0.0051   -0.0282    0.0658         0         0         0
         0         0         0    0.3049         0         0
         0         0         0         0    0.2128         0
         0         0         0         0         0    0.2128

>> epsilon = S*sigma
```

```
epsilon =

   -0.0005
   -0.0026
    0.0062
         0
         0
         0

>> format short e
>> epsilon

epsilon =

   -4.7625e-004
   -2.6398e-003
    6.1678e-003
              0
              0
              0

>> d1 = epsilon(1)*40

d1 =

   -1.9050e-002

>> d2 = epsilon(2)*40

d2 =

   -1.0559e-001

>> d3 = epsilon(3)*40

d3 =

   2.4671e-001
```

Problem 2.8

```
>> sigma3 = 150/(40*40)

sigma3 =

    0.0938
```

```
>> sigma = [0 ; 0 ; sigma3 ; 0 ; 0 ; 0]

sigma =

         0
         0
    0.0938
         0
         0
         0

>> [S] = IsotropicCompliance(72.4,0.3)

S =

    0.0138   -0.0041   -0.0041         0         0         0
   -0.0041    0.0138   -0.0041         0         0         0
   -0.0041   -0.0041    0.0138         0         0         0
         0         0         0    0.0359         0         0
         0         0         0         0    0.0359         0
         0         0         0         0         0    0.0359

>> epsilon = S*sigma

epsilon =
   -0.0004
   -0.0004
    0.0013
         0
         0
         0

>> format short e
>> epsilon

epsilon =

   -3.8847e-004
   -3.8847e-004
    1.2949e-003
              0
              0
              0

>> d1 = epsilon(1)*40

d1 =

   -1.5539e-002
```

```
>> d2 = epsilon(2)*40

d2 =

   -1.5539e-002

>> d3 = epsilon(3)*40

d3 =

    5.1796e-002
```

Problem 2.9

```
>> sigma2 = 100/(60*60)

sigma2 =

    0.0278

>> sigma = [0 ; sigma2 ; 0 ; 0 ; 0 ; 0]

sigma =

         0
    0.0278
         0
         0
         0
         0

>> [S] =

OrthotropicCompliance(155.0,12.10,12.10,0.248,0.458,0.248,
   4.40,3.20,4.40)

S =

     0.0065   -0.0016   -0.0016         0         0         0
    -0.0016    0.0826   -0.0379         0         0         0
    -0.0016   -0.0379    0.0826         0         0         0
          0         0         0    0.3125         0         0
          0         0         0         0    0.2273         0
          0         0         0         0         0    0.2273
```

```
>> EpsilonMechanical = S*sigma

EpsilonMechanical =

   -0.0000
    0.0023
   -0.0011
         0
         0
         0

>> format short e
>> EpsilonMechanical

EpsilonMechanical =

   -4.4444e-005
    2.2957e-003
   -1.0514e-003
              0
              0
              0

>> EpsilonThermal(1) = -0.01800e-6*30

EpsilonThermal =

   -5.4000e-007

>> EpsilonThermal(2) = 24.3e-6*30

EpsilonThermal =

   -5.4000e-007  7.2900e-004

>> EpsilonThermal(3) = 24.3e-6*30

EpsilonThermal =

   -5.4000e-007  7.2900e-004  7.2900e-004

>> EpsilonThermal(4) = 0

EpsilonThermal =

   -5.4000e-007  7.2900e-004  7.2900e-004            0

>> EpsilonThermal(5) = 0
```

```
EpsilonThermal =

   -5.4000e-007  7.2900e-004  7.2900e-004          0          0

>> EpsilonThermal(6) = 0

EpsilonThermal =

   -5.4000e-007  7.2900e-004  7.2900e-004          0          0          0

>> EpsilonThermal = EpsilonThermal'

EpsilonThermal =

   -5.4000e-007
    7.2900e-004
    7.2900e-004
              0
              0
              0

>> Epsilon = EpsilonMechanical + EpsilonThermal

Epsilon =

   -4.4984e-005
    3.0247e-003
   -3.2242e-004
              0
              0
              0

>> d1 = Epsilon(1)*60

d1 =

  -2.6991e-003

>> d2 = Epsilon(2)*60

d2 =

  1.8148e-001

>> d3 = Epsilon(3)*60
```

```
d3 =

   -1.9345e-002

>>
```

Problem 2.10

$$\begin{Bmatrix} \varepsilon_1 - \alpha_1 \Delta T - \beta_1 \Delta T \\ \varepsilon_2 - \alpha_2 \Delta T - \beta_2 \Delta T \\ \varepsilon_3 - \alpha_3 \Delta T - \beta_3 \Delta T \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{13} & S_{23} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 - \alpha_1 \Delta T - \beta_1 \Delta T \\ \varepsilon_2 - \alpha_2 \Delta T - \beta_2 \Delta T \\ \varepsilon_3 - \alpha_3 \Delta T - \beta_3 \Delta T \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix}$$

Problem 3.1

Let A be the total cross-sectional area of the unit cell and let A^f and A^m be the cross-sectional areas of the fiber and matrix, respectively. Then, we have the following relations based on the geometry of the problem:

$$A^f + A^m = A$$

Divide both sides of the above equation by A to obtain:

$$\frac{A^f}{A} + \frac{A^m}{A} = 1$$

Substituting $A^f/A = V^f$ and $A^m/A = V^m$, we obtain (3.1) as follows:

$$V^f + V^m = 1$$

Problem 3.2

Let W be the width of the cross-section in Fig. 3.3 (see book). Also, let W^f and W^m be the widths of the fiber and matrix, respectively.

$$\begin{aligned}
\nu_{12}^f &= -\frac{\Delta W^f/W^f}{\Delta L/L} \\
\nu^m &= -\frac{\Delta W^m/W^m}{\Delta L/L} \\
\Delta W^f &= -\nu_{12}^f W^f \frac{\Delta L}{L} \\
\Delta W^m &= -\nu^m W^m \frac{\Delta L}{L} \\
\Delta W &= \Delta W^f + \Delta W^m \\
&= -\left(\nu_{12}^f W^f + \nu^m W^m\right) \frac{\Delta L}{L} \\
\frac{\Delta W}{W} &= -\left(\nu_{12}^f \frac{W^f}{W} + \nu^m \frac{W^m}{W}\right) \frac{\Delta L}{L} \\
-\frac{\Delta W/W}{\Delta L/L} &= \nu_{12}^f V^f + \nu^m V^m
\end{aligned}$$

where $W^f/W = V^f$ and $W^m/W = V^m$. Then, we obtain:

$$\nu_{12} = \nu_{12}^f V^f + \nu^m V^m$$

Problem 3.3

Let W be the width of the cross-section in Fig. 3.3 (see book). Also, let W^f and W^m be the widths of the fiber and matrix, respectively. Also, from equilibrium, we have $\sigma_2^f = \sigma_2^m = \sigma_2$.

$$\begin{aligned}
\sigma_2^f &= \sigma_2 = E_2^f \varepsilon_2^f = E_2^f \frac{\Delta W^f}{W^f} \\
\sigma_2^m &= \sigma_2 = E^m \varepsilon_2^m = E^m \frac{\Delta W^m}{W^m} \\
\Delta W^f &= \frac{W^f}{E_2^f} \sigma_2 \\
\Delta W^m &= \frac{W^m}{E^m} \sigma_2 \\
\varepsilon_2 &= \frac{\Delta W}{W} = \frac{\Delta W^f + \Delta W^m}{W}
\end{aligned}$$

$$= \frac{\left(\frac{W^f}{E_2^f} + \frac{W^m}{E^m}\right)\sigma_2}{W}$$

$$\varepsilon_2 = \left(\frac{W^f/W}{E_2^f} + \frac{W^m/W}{E^m}\right)\sigma_2$$

$$\varepsilon_2 = \frac{1}{E_2}\sigma_2$$

$$\frac{1}{E_2} = \frac{V^f}{E_2^f} + \frac{V^m}{E^m}$$

where $W^f/W = V^f$ and $W^m/W = V^m$.

Problem 3.4

The following is a listing of the modified MATLAB function $E2$ called *E2Modified*. Note that this modified function is available with the M-files for the book on the CD-ROM that accompanies the book.

```
function y = E2Modified(Vf,E2f,Em,Eta,NU12f,NU21f,NUm,E1f,p)
%E2Modified  This function returns Young's modulus in the
%            transverse direction. Its input are nine values:
%            Vf    - fiber volume fraction
%            E2f   - transverse Young's modulus of the fiber
%            Em    - Young's modulus of the matrix
%            Eta   - stress-partitioning factor
%            NU12f - Poisson's ratio NU12 of the fiber
%            NU21f - Poisson's ratio NU21 of the fiber
%            NUm   - Poisson's ratio of the matrix
%            E1f   - longitudinal Young's modulus of the fiber
%            p     - parameter used to determine which equation to use:
%                  p = 1 - use equation (3.4)
%                  p = 2 - use equation (3.9)
%                  p = 3 - use equation (3.10)
%                  p = 4 - use the modified formula using (3.23)
%            Use the value zero for any argument not needed
%            in the calculations.
Vm = 1 - Vf;
if p == 1
     y = 1/(Vf/E2f + Vm/Em);
elseif p == 2
     y = 1/((Vf/E2f + Eta*Vm/Em)/(Vf + Eta*Vm));
elseif p == 3
     deno = E1f*Vf + Em*Vm;
     etaf = (E1f*Vf + ((1-NU12f*NU21f)*Em + NUm*NU21f
            *E1f)*Vm)/deno;
```

```
     etam = (((1-NUm*NUm)*E1f - (1-NUm*NU12f)*Em)*Vf
            + Em*Vm)/deno;
     y = 1/(etaf*Vf/E2f + etam*Vm/Em);
elseif p == 4
     EmPrime = Em/(1 - NUm*NUm);
     y = 1/(Vf/E2f + Vm/EmPrime);
end
```

Problem 3.5

The transverse modulus E_2 is calculated in GPa using the three different formulas with the MATLAB function $E2$ as follows. Note that the three values obtained are comparable and very close to each other.

```
>> E2(0.65, 14.8, 3.45, 0, 0, 0, 0, 0, 1)

ans =

   6.8791

>> E2(0.65, 14.8, 3.45, 0.5, 0, 0, 0, 0, 2)

ans =

   8.7169

>> E2(0.65, 14.8, 3.45, 0, 0.3, 0.3, 0.36, 85.6, 3)

ans =

   7.6135
```

Problem 3.6

```
>> y(1) = E2(0, 14.8, 3.45, 0, 0, 0, 0, 0, 1)

y =

  3.4500

>> y(2) = E2(0.1, 14.8, 3.45, 0, 0, 0, 0, 0, 1)

y =

  3.4500   3.7366
```

```
>> y(3) = E2(0.2, 14.8, 3.45, 0, 0, 0, 0, 0, 1)

y =

  3.4500  3.7366  4.0750

>> y(4) = E2(0.3, 14.8, 3.45, 0, 0, 0, 0, 0, 1)

y =

  3.4500  3.7366  4.0750  4.4809

>> y(5) = E2(0.4, 14.8, 3.45, 0, 0, 0, 0, 0, 1)

y =

  3.4500  3.7366  4.0750  4.4809  4.9766

>> y(6) = E2(0.5, 14.8, 3.45, 0, 0, 0, 0, 0, 1)

y =

  3.4500  3.7366  4.0750  4.4809  4.9766  5.5956

>> y(7) = E2(0.6, 14.8, 3.45, 0, 0, 0, 0, 0, 1)

y =

  3.4500  3.7366  4.0750  4.4809  4.9766  5.5956  6.3905

>> y(8) = E2(0.7, 14.8, 3.45, 0, 0, 0, 0, 0, 1)

y =

  3.4500  3.7366  4.0750  4.4809  4.9766  5.5956  6.3905
  7.4486

>> y(9) = E2(0.8, 14.8, 3.45, 0, 0, 0, 0, 0, 1)

y =

  3.4500  3.7366  4.0750  4.4809  4.9766  5.5956  6.3905
  7.4486  8.9266
```

```
>> y(10) = E2(0.9, 14.8, 3.45, 0, 0, 0, 0, 0, 1)

y =

  3.4500  3.7366  4.0750  4.4809  4.9766  5.5956  6.3905
  7.4486  8.9266  11.1363

>> y(11) = E2(1, 14.8, 3.45, 0, 0, 0, 0, 0, 1)

y =

  3.4500  3.7366  4.0750  4.4809  4.9766  5.5956  6.3905
  7.4486  8.9266  11.1363  14.8000

>> z(1) = E2(0, 14.8, 3.45, 0.4, 0, 0, 0, 0, 2)

z =

  3.4500

>> z(2) = E2(0.1, 14.8, 3.45, 0.4, 0, 0, 0, 0, 2)

z =

  3.4500  4.1402

>> z(3) = E2(0.2, 14.8, 3.45, 0.4, 0, 0, 0, 0, 2)

z =

  3.4500  4.1402  4.8933

>> z(4) = E2(0.3, 14.8, 3.45, 0.4, 0, 0, 0, 0, 2)

z =

  3.4500  4.1402  4.8933  5.7182

>> z(5) = E2(0.4, 14.8, 3.45, 0.4, 0, 0, 0, 0, 2)

z =

  3.4500  4.1402  4.8933  5.7182  6.6258

>> z(6) = E2(0.5, 14.8, 3.45, 0.4, 0, 0, 0, 0, 2)

z =

  3.4500  4.1402  4.8933  5.7182  6.6258  7.6290
```

```
>> z(7) = E2(0.6, 14.8, 3.45, 0.4, 0, 0, 0, 0, 2)

z =

  3.4500  4.1402  4.8933  5.7182  6.6258  7.6290  8.7439


>> z(8) = E2(0.7, 14.8, 3.45, 0.4, 0, 0, 0, 0, 2)

z =

  3.4500  4.1402  4.8933  5.7182  6.6258  7.6290  8.7439
  9.9903

>> z(9) = E2(0.8, 14.8, 3.45, 0.4, 0, 0, 0, 0, 2)

z =

  3.4500  4.1402  4.8933  5.7182  6.6258  7.6290  8.7439
  9.9903  11.3927

>> z(10) = E2(0.9, 14.8, 3.45, 0.4, 0, 0, 0, 0, 2)

z =

  3.4500  4.1402  4.8933  5.7182  6.6258  7.6290  8.7439
  9.9903  11.3927  12.9825

>> z(11) = E2(1, 14.8, 3.45, 0.4, 0, 0, 0, 0, 2)

z =

  3.4500  4.1402  4.8933  5.7182  6.6258  7.6290  8.7439
  9.9903  11.3927  12.9825  14.8000
>> w(1) = E2(0, 14.8, 3.45, 0.5, 0, 0, 0, 0, 2)

w =

  3.4500

>> w(2) = E2(0.1, 14.8, 3.45, 0.5, 0, 0, 0, 0, 2)

w =

  3.4500  4.0090

>> w(3) = E2(0.2, 14.8, 3.45, 0.5, 0, 0, 0, 0, 2)
```

```
w =

  3.4500  4.0090  4.6348

>> w(4) = E2(0.3, 14.8, 3.45, 0.5, 0, 0, 0, 0, 2)



w =

  3.4500  4.0090  4.6348  5.3401

>> w(5) = E2(0.4, 14.8, 3.45, 0.5, 0, 0, 0, 0, 2)

w =

  3.4500  4.0090  4.6348  5.3401  6.1412

>> w(6) = E2(0.5, 14.8, 3.45, 0.5, 0, 0, 0, 0, 2)

w =

  3.4500  4.0090  4.6348  5.3401  6.1412  7.0590

>> w(7) = E2(0.6, 14.8, 3.45, 0.5, 0, 0, 0, 0, 2)

w =

  3.4500  4.0090  4.6348  5.3401  6.1412  7.0590  8.1209

>> w(8) = E2(0.7, 14.8, 3.45, 0.5, 0, 0, 0, 0, 2)

w =

  3.4500  4.0090  4.6348  5.3401  6.1412  7.0590  8.1209
  9.3638

>> w(9) = E2(0.8, 14.8, 3.45, 0.5, 0, 0, 0, 0, 2)

w =

  3.4500  4.0090  4.6348  5.3401  6.1412  7.0590  8.1209
  9.3638  10.8382

>> w(10) = E2(0.9, 14.8, 3.45, 0.5, 0, 0, 0, 0, 2)
```

```
w =

  3.4500  4.0090  4.6348  5.3401  6.1412  7.0590  8.1209
  9.3638  10.8382  12.6156

>> w(11) = E2(1, 14.8, 3.45, 0.5, 0, 0, 0, 0, 2)

w =

  3.4500  4.0090  4.6348  5.3401  6.1412  7.0590  8.1209
  9.3638  10.8382  12.6156  14.8000

>> u(1) = E2(0, 14.8, 3.45, 0.6, 0, 0, 0, 0, 2)

u =

  3.4500

>> u(2) = E2(0.1, 14.8, 3.45, 0.6, 0, 0, 0, 0, 2)

u =

  3.4500  3.9197

>> u(3) = E2(0.2, 14.8, 3.45, 0.6, 0, 0, 0, 0, 2)

u =

  3.4500  3.9197  4.4548

>> u(4) = E2(0.3, 14.8, 3.45, 0.6, 0, 0, 0, 0, 2)

u =

  3.4500  3.9197  4.4548  5.0701

>> u(5) = E2(0.4, 14.8, 3.45, 0.6, 0, 0, 0, 0, 2)

u =

  3.4500  3.9197  4.4548  5.0701  5.7850

>> u(6) = E2(0.5, 14.8, 3.45, 0.6, 0, 0, 0, 0, 2)

u =

  3.4500  3.9197  4.4548  5.0701  5.7850  6.6258
```

```
>> u(7) = E2(0.6, 14.8, 3.45, 0.6, 0, 0, 0, 0, 2)

u =

  3.4500  3.9197  4.4548  5.0701  5.7850  6.6258  7.6290

>> u(8) = E2(0.7, 14.8, 3.45, 0.6, 0, 0, 0, 0, 2)

u =

  3.4500  3.9197  4.4548  5.0701  5.7850  6.6258  7.6290
  8.8468

>> u(9) = E2(0.8, 14.8, 3.45, 0.6, 0, 0, 0, 0, 2)

u =

  3.4500  3.9197  4.4548  5.0701  5.7850  6.6258  7.6290
  8.8468  10.3561

>> u(10) = E2(0.9, 14.8, 3.45, 0.6, 0, 0, 0, 0, 2)

u =

  3.4500  3.9197  4.4548  5.0701  5.7850  6.6258  7.6290
  8.8468  10.3561  12.2759

>> u(11) = E2(1, 14.8, 3.45, 0.6, 0, 0, 0, 0, 2)

u =

  3.4500  3.9197  4.4548  5.0701  5.7850  6.6258  7.6290
  8.8468  10.3561  12.2759  14.8000

>> v(1) = E2(0, 14.8, 3.45, 0, 0.3, 0.3, 0.36, 85.6, 3)

v =

  3.4500

>> v(2) = E2(0.1, 14.8, 3.45, 0, 0.3, 0.3, 0.36, 85.6, 3)

v =

  3.4500  4.1564

>> v(3) = E2(0.2, 14.8, 3.45, 0, 0.3, 0.3, 0.36, 85.6, 3)
```

```
v =

  3.4500  4.1564  4.6041

>> v(4) = E2(0.3, 14.8, 3.45, 0, 0.3, 0.3, 0.36, 85.6, 3)

v =

  3.4500  4.1564  4.6041  5.0767

>> v(5) = E2(0.4, 14.8, 3.45, 0, 0.3, 0.3, 0.36, 85.6, 3)

v =

  3.4500  4.1564  4.6041  5.0767  5.6249

>> v(6) = E2(0.5, 14.8, 3.45, 0, 0.3, 0.3, 0.36, 85.6, 3)

v =

  3.4500  4.1564  4.6041  5.0767  5.6249  6.2878

>> v(7) = E2(0.6, 14.8, 3.45, 0, 0.3, 0.3, 0.36, 85.6, 3)

v =

  3.4500  4.1564  4.6041  5.0767  5.6249  6.2878  7.1155

>> v(8) = E2(0.7, 14.8, 3.45, 0, 0.3, 0.3, 0.36, 85.6, 3)

v =

  3.4500  4.1564  4.6041  5.0767  5.6249  6.2878  7.1155
  8.1845

>> v(9) = E2(0.8, 14.8, 3.45, 0, 0.3, 0.3, 0.36, 85.6, 3)

v =

  3.4500  4.1564  4.6041  5.0767  5.6249  6.2878  7.1155
  8.1845  9.6228

>> v(10) = E2(0.9, 14.8, 3.45, 0, 0.3, 0.3, 0.36, 85.6, 3)

v =

  3.4500  4.1564  4.6041  5.0767  5.6249  6.2878  7.1155
  8.1845  9.6228  11.6657
```

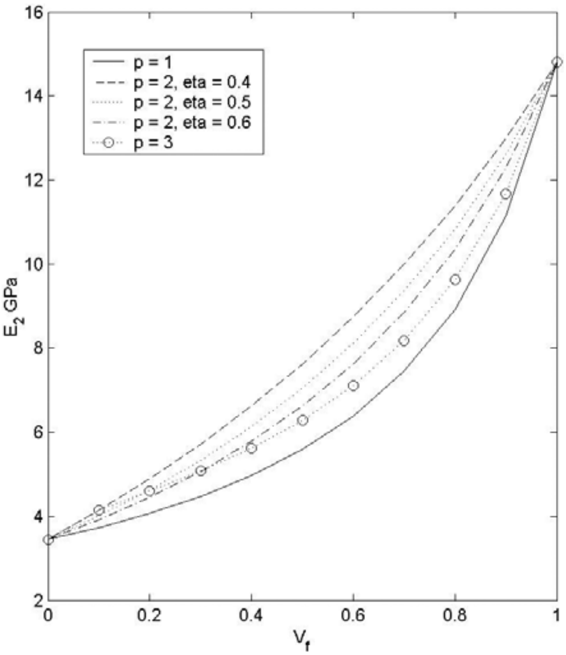

Fig. Variation of E_2 versus V^f for Problem 3.6

```
>> v(11) = E2(1, 14.8, 3.45, 0, 0.3, 0.3, 0.36, 85.6, 3)

v =

  3.4500  4.1564  4.6041  5.0767  5.6249  6.2878  7.1155
  8.1845  9.6228  11.6657  14.8000

>> x = [0 ; 0.1 ; 0.2 ; 0.3 ; 0.4 ; 0.5 ; 0.6 ; 0.7 ; 0.8 ;
       0.9 ; 1]

x =

        0
   0.1000
   0.2000
   0.3000
   0.4000
   0.5000
   0.6000
   0.7000
   0.8000
   0.9000
   1.0000
```

```
>> plot(x,y,'k-',x,z,'k--',x,w,'k:',x,u,'k-.',x,v,'ko:')
>> xlabel('V_f');
>> ylabel('E_2 GPa');
>> legend('p = 1', 'p = 2, eta = 0.4', 'p = 2, eta = 0.5',
   'p = 2, eta = 0.6', 'p = 3', 5);
```

Problem 3.7

The shear modulus G_{12} is calculated in GPa using three different formulas using the MATLAB function $G12$ as follows. Notice that the second and third values obtained are very close.

```
>> G12(0.55, 28.3, 1.27, 0, 1)

ans =

   2.6755

>> G12(0.55, 28.3, 1.27, 0.6, 2)

ans =

   3.5340

>> G12(0.55, 28.3, 1.27, 0, 3)

ans =

   3.8382
```

Problem 3.8

```
>> y(1) = G12(0, 28.3, 1.27, 0, 1)

y =

  1.2700

>> y(2) = G12(0.1, 28.3, 1.27, 0, 1)

y =

  1.2700  1.4041

>> y(3) = G12(0.2, 28.3, 1.27, 0, 1)
```

```
y =

  1.2700  1.4041  1.5699

>> y(4) = G12(0.3, 28.3, 1.27, 0, 1)

y =

  1.2700  1.4041  1.5699  1.7801

>> y(5) = G12(0.4, 28.3, 1.27, 0, 1)

y =

  1.2700  1.4041  1.5699  1.7801  2.0552

>> y(6) = G12(0.5, 28.3, 1.27, 0, 1)

y =

  1.2700  1.4041  1.5699  1.7801  2.0552  2.4309

>> y(7) = G12(0.6, 28.3, 1.27, 0, 1)

y =

  1.2700  1.4041  1.5699  1.7801  2.0552  2.4309  2.9748

>> y(8) = G12(0.7, 28.3, 1.27, 0, 1)

y =

  1.2700  1.4041  1.5699  1.7801  2.0552  2.4309  2.9748
  3.8321

>> y(9) = G12(0.8, 28.3, 1.27, 0, 1)

y =

  1.2700  1.4041  1.5699  1.7801  2.0552  2.4309  2.9748
  3.8321  5.3836

>> y(10) = G12(0.9, 28.3, 1.27, 0, 1)

y =

  1.2700  1.4041  1.5699  1.7801  2.0552  2.4309  2.9748
  3.8321  5.3836  9.0463
```

```
>> y(11) = G12(1, 28.3, 1.27, 0, 1)

y =

  Columns 1 through 10

    1.2700  1.4041  1.5699  1.7801  2.0552  2.4309  2.9748
    3.8321  5.3836  9.0463

  Column 11

    28.3000

>> z(1) = G12(0, 28.3, 1.27, 0.6, 2)

z =

  1.2700

>> z(2) = G12(0.1, 28.3, 1.27, 0.6, 2)

z =

  1.2700  1.4928

>> z(3) = G12(0.2, 28.3, 1.27, 0.6, 2)

z =

  1.2700  1.4928  1.7661

>> z(4) = G12(0.3, 28.3, 1.27, 0.6, 2)

z =

  1.2700  1.4928  1.7661  2.1095

>> z(5) = G12(0.4, 28.3, 1.27, 0.6, 2)

z =

  1.2700  1.4928  1.7661  2.1095  2.5538

>> z(6) = G12(0.5, 28.3, 1.27, 0.6, 2)

z =

  1.2700  1.4928  1.7661  2.1095  2.5538  3.1510
```

```
>> z(7) = G12(0.6, 28.3, 1.27, 0.6, 2)

z =

  1.2700  1.4928  1.7661  2.1095  2.5538  3.1510  3.9966

>> z(8) = G12(0.7, 28.3, 1.27, 0.6, 2)

z =

  1.2700  1.4928  1.7661  2.1095  2.5538  3.1510  3.9966
  5.2863

>> z(9) = G12(0.8, 28.3, 1.27, 0.6, 2)

z =

  1.2700  1.4928  1.7661  2.1095  2.5538  3.1510  3.9966
  5.2863  7.4945

>> z(10) = G12(0.9, 28.3, 1.27, 0.6, 2)

z =

  1.2700  1.4928  1.7661  2.1095  2.5538  3.1510  3.9966
  5.2863  7.4945  12.1448

>> z(11) = G12(1, 28.3, 1.27, 0.6, 2)

z =

  Columns 1 through 10

    1.2700  1.4928  1.7661  2.1095  2.5538  3.1510  3.9966
    5.2863  7.4945  12.1448

  Column 11

    28.3000

>> w(1) = G12(0, 28.3, 1.27, 0, 3)

w =

   1.2700
```

```
>> w(2) = G12(0.1, 28.3, 1.27, 0, 3)

w =

   1.2700  1.5255

>> w(3) = G12(0.2, 28.3, 1.27, 0, 3)

w =

   1.2700  1.5255  1.8383

>> w(4) = G12(0.3, 28.3, 1.27, 0, 3)

w =

   1.2700  1.5255  1.8383  2.2297

>> w(5) = G12(0.4, 28.3, 1.27, 0, 3)

w =

   1.2700  1.5255  1.8383  2.2297  2.7340

>> w(6) = G12(0.5, 28.3, 1.27, 0, 3)

w =

   1.2700  1.5255  1.8383  2.2297  2.7340  3.4082

>> w(7) = G12(0.6, 28.3, 1.27, 0, 3)

w =

   1.2700  1.5255  1.8383  2.2297  2.7340  3.4082  4.3552

>> w(8) = G12(0.7, 28.3, 1.27, 0, 3)

w =

   1.2700  1.5255  1.8383  2.2297  2.7340  3.4082  4.3552
   5.7830

>> w(9) = G12(0.8, 28.3, 1.27, 0, 3)
```

```
w =

   1.2700  1.5255  1.8383  2.2297  2.7340  3.4082  4.3552
   5.7830  8.1823

>> w(10) = G12(0.9, 28.3, 1.27, 0, 3)

w =

   1.2700  1.5255  1.8383  2.2297  2.7340  3.4082  4.3552
   5.7830  8.1823  13.0553

>> w(11) = G12(1, 28.3, 1.27, 0, 3)

w =

  Columns 1 through 10

    1.2700  1.5255  1.8383  2.2297  2.7340  3.4082  4.3552
    5.7830  8.1823  13.0553

  Column 11

    28.3000

>> x = [0 ; 0.1 ; 0.2 ; 0.3 ; 0.4 ; 0.5 ; 0.6 ; 0.7 ; 0.8 ;
      0.9 ; 1]

x =

        0
   0.1000
   0.2000
   0.3000
   0.4000
   0.5000
   0.6000
   0.7000
   0.8000
   0.9000
   1.0000

>> plot(x,y,'k-',x,z,'k--',x,w,'k-.')
>> xlabel('V ^ f');
>> ylabel('G_{}12{} GPa');
>> legend('p = 1', 'p = 2', 'p = 3', 3);
```

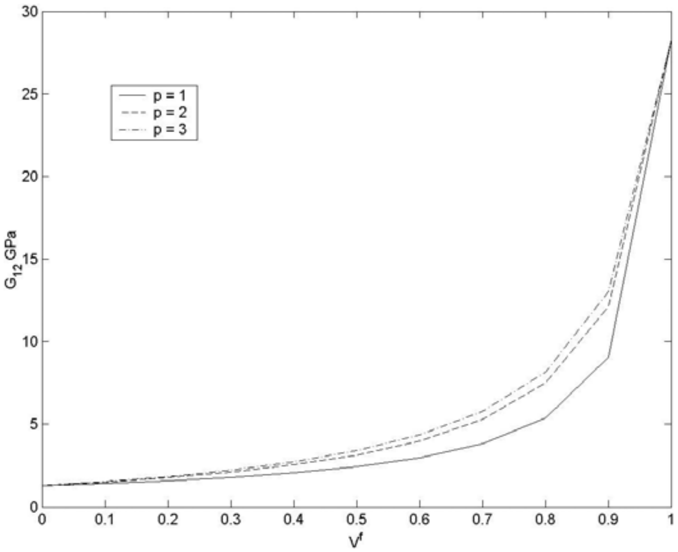

Fig. Variation of G_{12} versus V^f for Problem 3.8

Problem 3.9

First, the longitudinal coefficient of thermal expansion α_1 is calculated in /K as follows:

```
>> Alpha1(0.6, 233, 4.62, -0.540e-6, 41.4e-6)

ans =

  7.1671e-009
```

Next, the transverse coefficient of thermal expansion α_2 is calculated in /K using two different formulas as follows. Notice that in the second formula, we need to calculate also the value of the longitudinal modulus E_1. Note also that the two values obtained are comparable and very close to each other.

```
>> Alpha2(0.6, 10.10e-6, 41.4e-6, 0, 0, 0, 0, 0, 0, 1)

ans =

  2.2620e-005

>> E1 = E1(0.6, 233, 4.62)

E1 =

  141.6480
```

```
>> Alpha2(0.6, 10.10e-6, 41.4e-6, E1, 233, 4.62, 0.200,
        0.360, -0.540e-6, 2)

ans =

  2.8515e-005
```

Problem 3.10

$$E_1 = E^f V^f + E^m V^m + E^i V^i$$

Note that the derivation of the above equation is very similar to the derivation in Example 3.1.

Problem 4.1

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_1} & \frac{-\nu_{12}}{E_1} & 0 \\ \frac{-\nu_{12}}{E_1} & \frac{1}{E_2} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}$$

Problem 4.2

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} \frac{E_1}{1-\nu_{12}\nu_{21}} & \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} & 0 \\ \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} & \frac{E_2}{1-\nu_{12}\nu_{21}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}$$

where $\nu_{12}E_2 = \nu_{21}E_1$.

Problem 4.3

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E} & \frac{-\nu}{E} & 0 \\ \frac{-\nu}{E} & \frac{1}{E} & 0 \\ 0 & 0 & \frac{2(1+\nu)}{E} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}$$

Problem 4.4

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} \dfrac{E}{1-\nu^2} & \dfrac{\nu E}{1-\nu^2} & 0 \\ \dfrac{\nu E}{1-\nu^2} & \dfrac{E}{1-\nu^2} & 0 \\ 0 & 0 & \dfrac{E}{2(1+\nu)} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}$$

Problem 4.5

```
>> S = ReducedCompliance(50.0, 15.2, 0.254, 4.70)

S =

   0.0200   -0.0051         0
  -0.0051    0.0658         0
        0         0    0.2128

>> Q = ReducedStiffness(50.0, 15.2, 0.254, 4.70)

Q =

   51.0003    3.9380         0
    3.9380   15.5041         0
         0         0    4.7000

>> S*Q

ans =

    1.0000         0         0
         0    1.0000         0
         0         0    1.0000
```

Problem 4.6

```
>> S = OrthotropicCompliance(155.0, 12.10, 12.10, 0.248, 0.458,
       0.248, 4.40, 3.20, 4.40)

S =

   0.0065   -0.0016   -0.0016         0         0         0
  -0.0016    0.0826   -0.0379         0         0         0
```

```
  -0.0016   -0.0379    0.0826         0         0         0
        0         0         0    0.3125         0         0
        0         0         0         0    0.2273         0
        0         0         0         0         0    0.2273

>> sigma1 = 0

sigma1 =

     0

>> sigma2 = -2.5/(200*0.200)

sigma2 =

   -0.0625

>> epsilon3 = S(1,3)*sigma1 + S(2,3)*sigma2

epsilon3 =

   0.0024
```

Problem 4.7

```
>> S = ReducedIsotropicCompliance(72.4, 0.3)

S =

   0.0138   -0.0041         0
  -0.0041    0.0138         0
        0         0    0.0359

>> Q = ReducedIsotropicStiffness(72.4, 0.3)

Q =

   79.5604   23.8681         0
   23.8681   79.5604         0
         0         0   27.8462

>> S*Q

ans =

   1.0000         0         0
        0    1.0000         0
        0         0    1.0000
```

Problem 4.8

```
>> S = OrthotropicCompliance(155.0, 12.10, 12.10, 0.248, 0.458,
       0.248, 4.40, 3.20, 4.40)

S =

   0.0065   -0.0016   -0.0016         0         0         0
  -0.0016    0.0826   -0.0379         0         0         0
  -0.0016   -0.0379    0.0826         0         0         0
        0         0         0    0.3125         0         0
        0         0         0         0    0.2273         0
        0         0         0         0         0    0.2273

>> sigma1 = 4/(200*0.200)

sigma1 =

    0.1000

>> sigma2 = 0

sigma2 =

    0

>> epsilon3 = S(1,3)*sigma1 + S(2,3)*sigma2

epsilon3 =

 -1.6000e-004
```

Problem 4.9

```
function y = ReducedStiffness2(E1,E2,NU12,G12)
%ReducedStiffness2  This function returns the reduced
%                   stiffness matrix for fiber-reinforced
%                   materials.
%                   There are four arguments representing
%                   four material constants.
%                   The size of the reduced compliance
%                   matrix is 3 x 3. The reuduced stiffness
%                   matrix is calculated as the inverse of
%                   the reduced compliance matrix.
z = [1/E1 -NU12/E1 0 ; -NU12/E1 1/E2 0 ; 0 0 1/G12];
y = inv(z);
```

```
function y = ReducedIsotropicStiffness2(E,NU)
%ReducedIsotropicStiffness2  This function returns the
%                            reduced isotropic stiffness
%                            matrix for fiber-reinforced
%                            materials.
%                            There are two arguments
%                            representing two material
%                            constants. The size of the
%                            reduced compliance matrix is
%                            3 x 3. The reduced stiffness
%                            matrix is calculated
%                            as the inverse of the reduced
%                            compliance matrix.
z = [1/E -NU/E 0 ; -NU/E 1/E 0 ; 0 0 2*(1+NU)/E];
y = inv(z);
```

Problem 4.10

$$\begin{Bmatrix} \varepsilon_1 - \alpha_1 \Delta T - \beta_1 \Delta M \\ \varepsilon_2 - \alpha_2 \Delta T - \beta_2 \Delta M \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 - \alpha_1 \Delta T - \beta_1 \Delta M \\ \varepsilon_2 - \alpha_2 \Delta T - \beta_2 \Delta M \\ \gamma_{12} \end{Bmatrix}$$

Problem 5.1

From an introductory course on mechanics of materials, we have the following stress transformation equations between the 1-2-3 coordinate system and the x-y-z global coordinate system:

$$\begin{aligned}
\sigma_1 &= \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta \\
\sigma_2 &= \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\tau_{xy}\sin\theta\cos\theta \\
\sigma_3 &= \sigma_z \\
\tau_{23} &= \tau_{yz}\cos\theta - \tau_{xz}\sin\theta \\
\tau_{13} &= \tau_{yz}\sin\theta + \tau_{xz}\cos\theta \\
\tau_{12} &= -\sigma_x \sin\theta\cos\theta + \sigma_y \sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)
\end{aligned}$$

For the case of plane stress, we already have $\sigma_3 = \tau_{23} = \tau_{13} = 0$. Substitute this into the third, fourth, and fifth equations above and rearrange the terms to obtain:

$$\begin{aligned}\sigma_z &= 0\\ \tau_{yz}\cos\theta - \tau_{xz}\sin\theta &= 0\\ \tau_{yz}\sin\theta + \tau_{xz}\cos\theta &= 0\end{aligned}$$

It is clear now that $\sigma_z = 0$. Next, we solve the last two equations above by multiplying the first equation by $\cos\theta$ and the second equation by $\sin\theta$. Then, we add the two equation to obtain:

$$\tau_{yz}(\cos^2\theta + \sin^2\theta) = 0$$

However, we know that $\cos^2\theta + \sin^2\theta = 1$. Therefore, we conclude that $\tau_{yz} = 0$. It also follows immediately that $\tau_{xz} = 0$ also.

Problem 5.2

From an introductory course on mechanics of materials, we have the following stress transformation equations between the 1-2-3 coordinate system and the x-y-z global coordinate system:

$$\begin{aligned}\sigma_1 &= \sigma_x\cos^2\theta + \sigma_y\sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta\\ \sigma_2 &= \sigma_x\sin^2\theta + \sigma_y\cos^2\theta - 2\tau_{xy}\sin\theta\cos\theta\\ \tau_{12} &= -\sigma_x\sin\theta\cos\theta + \sigma_y\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)\end{aligned}$$

Write the above three equations in matrix form as follows:

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix} = \begin{bmatrix}\cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta\\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta\\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta - \sin^2\theta\end{bmatrix}\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}$$

Let $m = \cos\theta$ and $n = \sin\theta$. Therefore, we obtain the desired equation as follows:

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix} = \begin{bmatrix}m^2 & n^2 & 2mn\\ n^2 & m^2 & -2mn\\ -mn & mn & m^2 - n^2\end{bmatrix}\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}$$

Problem 5.3

$$[T] = \begin{bmatrix}m^2 & n^2 & 2mn\\ n^2 & m^2 & -2mn\\ -mn & mn & m^2 - n^2\end{bmatrix}$$

Calculate the determinant of $[T]$ as follows:

$$\begin{aligned}|T| &= m^2\begin{vmatrix} m^2 & -2mn \\ mn & m^2-n^2\end{vmatrix} - n^2\begin{vmatrix} n^2 & -2mn \\ -mn & m^2-n^2\end{vmatrix} + 2mn\begin{vmatrix} n^2 & m^2 \\ -mn & mn\end{vmatrix} \\ &= \left(m^2+n^2\right)^3 \\ &= 1\end{aligned}$$

The above is true since $m^2+n^2=\cos^2\theta+\sin^2\theta=1$. Therefore, we obtain:

$$\begin{aligned}[T]^{-1} &= \frac{adj[T\}}{|T|} = adj[T] \\ &= \begin{bmatrix} m^2 & n^2 & -2mn \\ n^2 & m^2 & 2mn \\ mn & -mn & m^2-n^2\end{bmatrix}\end{aligned}$$

Problem 5.4

$$\begin{aligned}[\bar{S}] &= [T]^{-1}[S][T] \\ [\bar{S}]^{-1} &= \left([T]^{-1}[S][T]\right)^{-1} = [T]^{-1}[S]^{-1}\left([T]^{-1}\right)^{-1} = [T]^{-1}[Q][T] = [\bar{Q}]\end{aligned}$$

Similarly, we also have the other way:

$$\begin{aligned}[\bar{Q}] &= [T]^{-1}[Q][T] \\ [\bar{Q}]^{-1} &= \left([T]^{-1}[Q][T]\right)^{-1} = [T]^{-1}[Q]^{-1}\left([T]^{-1}\right)^{-1} = [T]^{-1}[S][T] = [\bar{S}]\end{aligned}$$

Problem 5.5

Multiply the three matrices in (5.13) in book as follows:

$$\begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}\end{bmatrix} = \begin{bmatrix} m^2 & n^2 & -2mn \\ n^2 & m^2 & 2mn \\ mn & -mn & m^2-n^2\end{bmatrix}\begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66}\end{bmatrix}$$
$$\begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2-n^2\end{bmatrix}$$

The above multiplication can be performed either manually or using a computer algebra system like MAPLE or MATHEMATICA or the MATLAB Symbolic Math Toolbox. Therefore, we obtain the following expression:

$$
\begin{aligned}
\bar{Q}_{11} &= Q_{11}m^4 + 2(Q_{12} + 2Q_{66})n^2m^2 + Q_{22}n^4 \\
\bar{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})n^2m^2 + Q_{12}(n^4 + m^4) \\
\bar{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})nm^3 + (Q_{12} - Q_{22} + 2Q_{66})n^3m \\
\bar{Q}_{22} &= Q_{11}n^4 + 2(Q_{12} + 2Q_{66})n^2m^2 + Q_{22}m^4 \\
\bar{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})n^3m + (Q_{12} - Q_{22} + 2Q_{66})nm^3 \\
\bar{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})n^2m^2 + Q_{66}(n^4 + m^4)
\end{aligned}
$$

Problem 5.6

```
function y = Tinv2(theta)
%Tinv2  This function returns the inverse of the
%       transformation matrix T
%       given the orientation angle "theta".
%       There is only one argument representing "theta"
%       The size of the matrix is 3 x 3.
%       The angle "theta" must be given in degrees.
m = cos(theta*pi/180);
n = sin(theta*pi/180);
x = [m*m n*n 2*m*n ; n*n m*m -2*m*n ; -m*n m*n m*m-n*n];
y = inv(x);
```

Problem 5.7

```
function y = Sbar2(S,T)
%Sbar2  This function returns the transformed reduced
%       compliance matrix "Sbar" given the reduced
%       compliance matrix S and the transformation
%       matrix T.
%       There are two arguments representing S and T
%       The size of the matrix is 3 x 3.
Tinv = inv(T);
y = Tinv*S*T;
```

```
function y = Qbar2(Q,T)
%Qbar2  This function returns the transformed reduced
%       stiffness matrix "Qbar" given the reduced
%       stiffness matrix Q and the transformation
%       matrix T.
%       There are two arguments representing Q and T
```

```
%       The size of the matrix is 3 x 3.
Tinv = inv(T);
y = Tinv*Q*T;
```

Problem 5.8

```
>> S = ReducedCompliance(50.0, 15.20, 0.254, 4.70)

S =

    0.0200   -0.0051         0
   -0.0051    0.0658         0
         0         0    0.2128

>> S1 = Sbar(S, -90)

S1 =

    0.0658   -0.0051   -0.0000
   -0.0051    0.0200    0.0000
   -0.0000    0.0000    0.2128

>> S2 = Sbar(S, -80)

S2 =

    0.0740   -0.0147   -0.0451
   -0.0147    0.0310    0.0608
   -0.0226    0.0304    0.1935

>> S3 = Sbar(S, -70)

S3 =

    0.0945   -0.0391   -0.0664
   -0.0391    0.0594    0.0959
   -0.0332    0.0479    0.1447

>> S4 = Sbar(S, -60)

S4 =

    0.1161   -0.0669   -0.0515
   -0.0669    0.0932    0.0912
   -0.0258    0.0456    0.0892
```

```
>> S5 = Sbar(S, -50)

S5 =

    0.1268   -0.0850   -0.0056
   -0.0850    0.1188    0.0507
   -0.0028    0.0254    0.0529

>> S6 = Sbar(S, -40)

S6 =

    0.1188   -0.0850    0.0507
   -0.0850    0.1268   -0.0056
    0.0254   -0.0028    0.0529

>> S7 = Sbar(S, -30)

S7 =

    0.0932   -0.0669    0.0912
   -0.0669    0.1161   -0.0515
    0.0456   -0.0258    0.0892

>> S8 = Sbar(S, -20)

S8 =

    0.0594   -0.0391    0.0959
   -0.0391    0.0945   -0.0664
    0.0479   -0.0332    0.1447

>> S9 = Sbar(S, -10)

S9 =

    0.0310   -0.0147    0.0608
   -0.0147    0.0740   -0.0451
    0.0304   -0.0226    0.1935

>> S9 = Sbar(S, -10)

S9 =

    0.0310   -0.0147    0.0608
   -0.0147    0.0740   -0.0451
    0.0304   -0.0226    0.1935
```

```
>> S10 = Sbar(S, 0)

S10 =

    0.0200   -0.0051         0
   -0.0051    0.0658         0
         0         0    0.2128

>> S11 = Sbar(S, 10)

S11 =

    0.0310   -0.0147   -0.0608
   -0.0147    0.0740    0.0451
   -0.0304    0.0226    0.1935

>> S12 = Sbar(S, 20)

S12 =

    0.0594   -0.0391   -0.0959
   -0.0391    0.0945    0.0664
   -0.0479    0.0332    0.1447

>> S13 = Sbar(S, 30)

S13 =

    0.0932   -0.0669   -0.0912
   -0.0669    0.1161    0.0515
   -0.0456    0.0258    0.0892

>> S14 = Sbar(S, 40)

S14 =

    0.1188   -0.0850   -0.0507
   -0.0850    0.1268    0.0056
   -0.0254    0.0028    0.0529

>> S15 = Sbar(S, 50)

S15 =

    0.1268   -0.0850    0.0056
   -0.0850    0.1188   -0.0507
    0.0028   -0.0254    0.0529
```

```
>> S16 = Sbar(S, 60)

S16 =

    0.1161   -0.0669    0.0515
   -0.0669    0.0932   -0.0912
    0.0258   -0.0456    0.0892

>> S17 = Sbar(S, 70)

S17 =

    0.0945   -0.0391    0.0664
   -0.0391    0.0594   -0.0959
    0.0332   -0.0479    0.1447

>> S18 = Sbar(S, 80)

S18 =

    0.0740   -0.0147    0.0451
   -0.0147    0.0310   -0.0608
    0.0226   -0.0304    0.1935

>> S19 = Sbar(S, 90)

S19 =

    0.0658   -0.0051    0.0000
   -0.0051    0.0200   -0.0000
    0.0000   -0.0000    0.2128

>> x = [-90 -80 -70 -60 -50 -40 -30 -20 -10 0 10 20 30 40 50
       60 70 80 90]

x =

   -90   -80   -70   -60   -50   -40   -30   -20   -10
    0    10    20    30    40    50    60    70    80    90

>> y1 = [S1(1,1) S2(1,1) S3(1,1) S4(1,1) S5(1,1) S6(1,1)
        S7(1,1) S8(1,1) S9(1,1) S10(1,1) S11(1,1) S12(1,1) S13(1,1)
        S14(1,1) S15(1,1) S16(1,1) S17(1,1) S18(1,1) S19(1,1)]

y1 =

  Columns 1 through 14

     0.0658    0.0740    0.0945    0.1161    0.1268    0.1188
```

```
     0.0932    0.0594    0.0310    0.0200    0.0310    0.0594
     0.0932    0.1188

  Columns 15 through 19

     0.1268    0.1161    0.0945    0.0740    0.0658

>> plot(x,y1)
>> xlabel('\theta (degrees)');
>> ylabel('S^{-}_{11} (GPa)^{-1}');
```

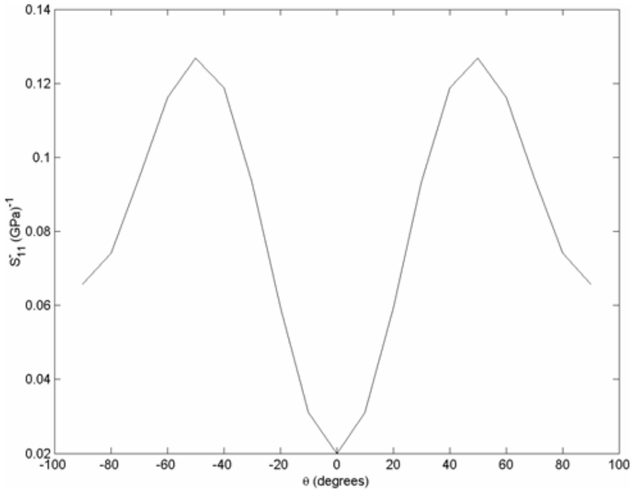

Fig. Variation of $\bar{S}_{11}$ versus θ for Problem 5.8

```
>> y2 = [S1(1,2) S2(1,2) S3(1,2) S4(1,2) S5(1,2) S6(1,2) S7(1,2)
        S8(1,2) S9(1,2) S10(1,2) S11(1,2) S12(1,2) S13(1,2) S14(1,2)
        S15(1,2) S16(1,2) S17(1,2) S18(1,2) S19(1,2)]

y2 =

  Columns 1 through 14

    -0.0051   -0.0147   -0.0391   -0.0669   -0.0850   -0.0850
    -0.0669   -0.0391   -0.0147   -0.0051   -0.0147   -0.0391
    -0.0669   -0.0850

  Columns 15 through 19

    -0.0850   -0.0669   -0.0391   -0.0147   -0.0051
```

```
>> plot(x,y2)
>> xlabel('\theta (degrees)');
>> ylabel('S^{-}_{12} (GPa)^{-1}');
```

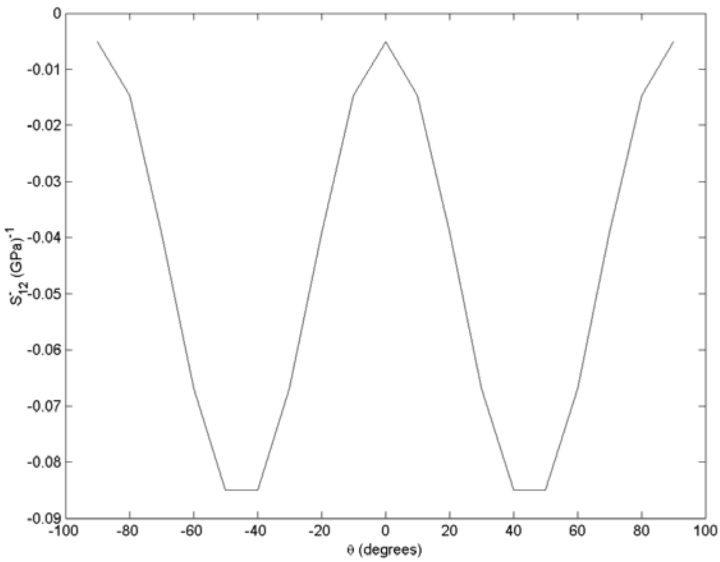

Fig. Variation of $\bar{S}_{12}$ versus θ for Problem 5.8

```
>> y3 = [S1(1,3) S2(1,3) S3(1,3) S4(1,3) S5(1,3) S6(1,3) S7(1,3)
        S8(1,3) S9(1,3) S10(1,3) S11(1,3) S12(1,3) S13(1,3) S14(1,3)
        S15(1,3) S16(1,3) S17(1,3) S18(1,3) S19(1,3)]

y3 =

  Columns 1 through 14

    -0.0000   -0.0451   -0.0664   -0.0515   -0.0056    0.0507
     0.0912    0.0959    0.0608         0   -0.0608   -0.0959
    -0.0912   -0.0507

  Columns 15 through 19

     0.0056    0.0515    0.0664    0.0451    0.0000

>> plot(x,y3)
>> xlabel('\theta (degrees)');
>> ylabel('S^{-}_{16} (GPa)^{-1}');
```

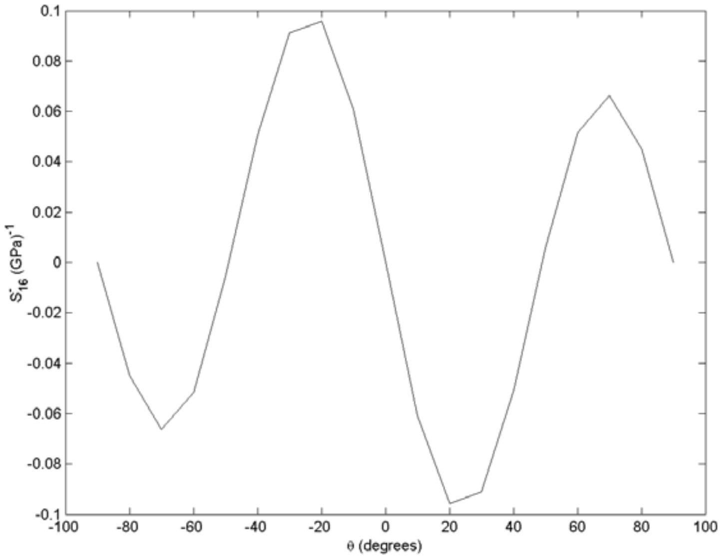

Fig. Variation of $\bar{S}_{16}$ versus θ for Problem 5.8

```
>> y4 = [S1(2,2) S2(2,2) S3(2,2) S4(2,2) S5(2,2) S6(2,2) S7(2,2)
        S8(2,2) S9(2,2) S10(2,2) S11(2,2) S12(2,2) S13(2,2) S14(2,2)
        S15(2,2) S16(2,2) S17(2,2) S18(2,2) S19(2,2)]

y4 =

  Columns 1 through 14

    0.0200    0.0310    0.0594    0.0932    0.1188    0.1268
    0.1161    0.0945    0.0740    0.0658    0.0740    0.0945
    0.1161    0.1268

  Columns 15 through 19

    0.1188    0.0932    0.0594    0.0310    0.0200

>> plot(x,y4)
>> xlabel('\theta (degrees)');
>> ylabel('S^{-}_{22} (GPa)^{-1}');
```

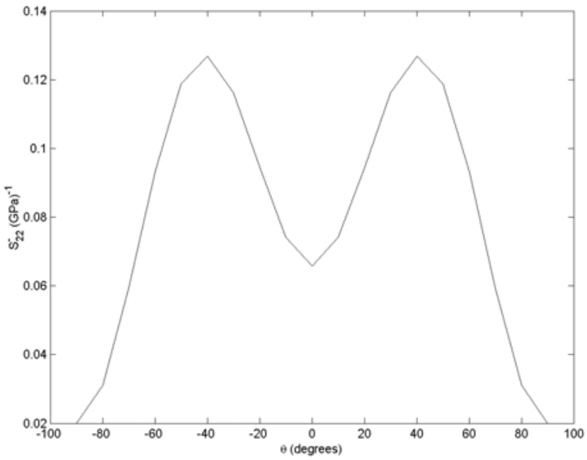

Fig. Variation of $\bar{S}_{22}$ versus θ for Problem 5.8

```
>> y5 = [S1(2,3) S2(2,3) S3(2,3) S4(2,3) S5(2,3) S6(2,3) S7(2,3)
        S8(2,3) S9(2,3) S10(2,3) S11(2,3) S12(2,3) S13(2,3) S14(2,3)
        S15(2,3) S16(2,3) S17(2,3) S18(2,3) S19(2,3)]
y5 =

  Columns 1 through 14

     0.0000    0.0608    0.0959    0.0912    0.0507   -0.0056
    -0.0515   -0.0664   -0.0451         0    0.0451    0.0664
     0.0515    0.0056

  Columns 15 through 19

    -0.0507   -0.0912   -0.0959   -0.0608   -0.0000

>> plot(x,y5)
>> xlabel('\theta (degrees)');
>> ylabel('S^{-}_{26} (GPa)^{-1}');

>> y6 = [S1(3,3) S2(3,3) S3(3,3) S4(3,3) S5(3,3) S6(3,3) S7(3,3)
        S8(3,3) S9(3,3) S10(3,3) S11(3,3) S12(3,3) S13(3,3) S14(3,3)
        S15(3,3) S16(3,3) S17(3,3) S18(3,3) S19(3,3)]

y6 =

  Columns 1 through 14

    0.2128    0.1935    0.1447    0.0892    0.0529    0.0529
    0.0892    0.1447    0.1935    0.2128    0.1935    0.1447
    0.0892    0.0529
```

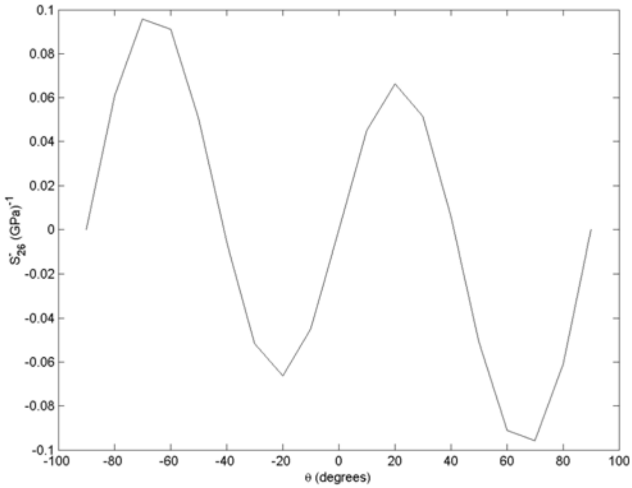

Fig. Variation of $\bar{S}_{26}$ versus θ for Problem 5.8

```
  Columns 15 through 19

    0.0529    0.0892    0.1447    0.1935    0.2128

>> plot(x,y6)
>> xlabel('\theta (degrees)');
>> ylabel('S^{-}_{66} (GPa)^{-1}');
```

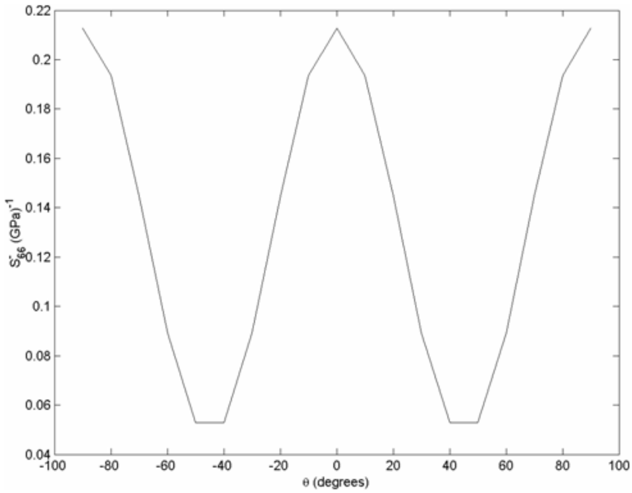

Fig. Variation of $\bar{S}_{66}$ versus θ for Problem 5.8

Problem 5.9

```
>> Q = ReducedStiffness(155.0, 12.10, 0.248, 4.40)

Q =

  155.7478    3.0153         0
    3.0153   12.1584         0
         0         0    4.4000

>> Q1 = Qbar(Q, -90)

Q1 =

   12.1584    3.0153    0.0000
    3.0153  155.7478   -0.0000
    0.0000   -0.0000    4.4000

>> Q2 = Qbar(Q, -80)

Q2 =

   12.0115    7.4919    0.0435
    7.4919  146.9414  -49.1540
    0.0218  -24.5770   13.3532

>> Q3 = Qbar(Q, -70)

Q3 =

   13.1434   18.8271   -8.4612
   18.8271  123.1392  -83.8363
   -4.2306  -41.9181   36.0236

>> Q4 = Qbar(Q, -60)

Q4 =

   19.3541   31.7170  -29.0342
   31.7170   91.1488  -95.3179
  -14.5171  -47.6589   61.8034

>> Q5 = Qbar(Q, -50)

Q5 =

   34.3711   40.1302  -57.6152
   40.1302   59.3051  -83.7927
  -28.8076  -41.8964   78.6299
```

```
>> Q6 = Qbar(Q, -40)

Q6 =

   59.3051   40.1302  -83.7927
   40.1302   34.3711  -57.6152
  -41.8964  -28.8076   78.6299

>> Q7 = Qbar(Q, -30)

Q7 =

   91.1488   31.7170  -95.3179
   31.7170   19.3541  -29.0342
  -47.6589  -14.5171   61.8034

>> Q8 = Qbar(Q, -20)

Q8 =

  123.1392   18.8271  -83.8363
   18.8271   13.1434   -8.4612
  -41.9181   -4.2306   36.0236

>> Q9 = Qbar(Q, -10)

Q9 =

  146.9414    7.4919  -49.1540
    7.4919   12.0115    0.0435
  -24.5770    0.0218   13.3532

>> Q10 = Qbar(Q, 0)

Q10 =

  155.7478    3.0153         0
    3.0153   12.1584         0
         0         0    4.4000

>> Q11 = Qbar(Q, 10)

Q11 =

  146.9414    7.4919   49.1540
    7.4919   12.0115   -0.0435
   24.5770   -0.0218   13.3532
```

```
>> Q12 = Qbar(Q, 20)

Q12 =

  123.1392   18.8271   83.8363
   18.8271   13.1434    8.4612
   41.9181    4.2306   36.0236

>> Q13 = Qbar(Q, 30)

Q13 =

   91.1488   31.7170   95.3179
   31.7170   19.3541   29.0342
   47.6589   14.5171   61.8034

>> Q14 = Qbar(Q, 40)

Q14 =

   59.3051   40.1302   83.7927
   40.1302   34.3711   57.6152
   41.8964   28.8076   78.6299

>> Q15 = Qbar(Q, 50)

Q15 =

   34.3711   40.1302   57.6152
   40.1302   59.3051   83.7927
   28.8076   41.8964   78.6299

>> Q16 = Qbar(Q, 60)

Q16 =

   19.3541   31.7170   29.0342
   31.7170   91.1488   95.3179
   14.5171   47.6589   61.8034


>> Q17 = Qbar(Q, 70)

Q17 =

   13.1434   18.8271    8.4612
   18.8271  123.1392   83.8363
    4.2306   41.9181   36.0236
```

```
>> Q18 = Qbar(Q, 80)

Q18 =

   12.0115    7.4919   -0.0435
    7.4919  146.9414   49.1540
   -0.0218   24.5770   13.3532

>> Q19 = Qbar(Q, 90)

Q19 =

   12.1584    3.0153   -0.0000
    3.0153  155.7478    0.0000
   -0.0000    0.0000    4.4000

>> x = [-90 -80 -70 -60 -50 -40 -30 -20 -10 0 10 20 30 40 50 60
       70 80 90]

x =

   -90   -80   -70   -60   -50   -40   -30   -20   -10     0    10
   20    30    40    50    60    70    80    90

>> y1 = [Q1(1,1) Q2(1,1) Q3(1,1) Q4(1,1) Q5(1,1) Q6(1,1) Q7(1,1)
        Q8(1,1) Q9(1,1) Q10(1,1) Q11(1,1) Q12(1,1) Q13(1,1) Q14(1,1)
        Q15(1,1) Q16(1,1) Q17(1,1) Q18(1,1) Q19(1,1)]

y1 =

  Columns 1 through 14

    12.1584   12.0115   13.1434   19.3541   34.3711   59.3051
    91.1488  123.1392  146.9414  155.7478  146.9414  123.1392
    91.1488   59.3051

  Columns 15 through 19

    34.3711   19.3541   13.1434   12.0115   12.1584


>> plot(x,y1)
>> xlabel('\theta (degrees)');
>> ylabel('Q^{-}_{11} (GPa)');
```

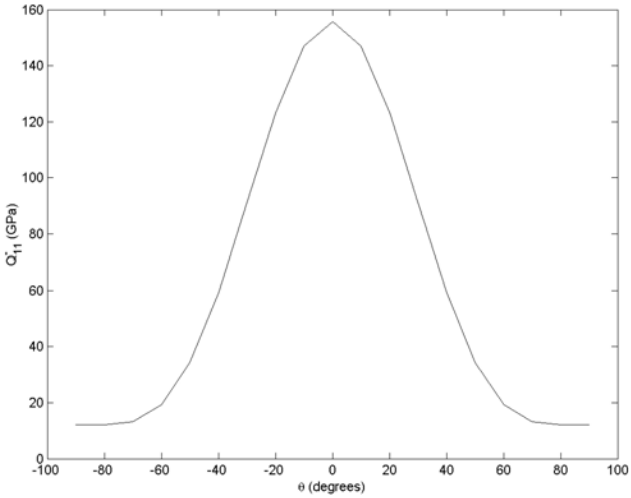

Fig. Variation of $\bar{Q}_{11}$ versus θ for Problem 5.9

```
>> y2 = [Q1(1,2) Q2(1,2) Q3(1,2) Q4(1,2) Q5(1,2) Q6(1,2) Q7(1,2)
        Q8(1,2) Q9(1,2) Q10(1,2) Q11(1,2) Q12(1,2) Q13(1,2) Q14(1,2)
        Q15(1,2) Q16(1,2) Q17(1,2) Q18(1,2) Q19(1,2)]

y2 =

  Columns 1 through 14

    3.0153    7.4919   18.8271   31.7170   40.1302   40.1302
    31.7170   18.8271    7.4919    3.0153    7.4919   18.8271
    31.7170   40.1302

  Columns 15 through 19

    40.1302   31.7170   18.8271    7.4919    3.0153

>> plot(x,y2)
>> xlabel('\theta (degrees)');
>> ylabel('Q^{-}_{12} (GPa)');
```

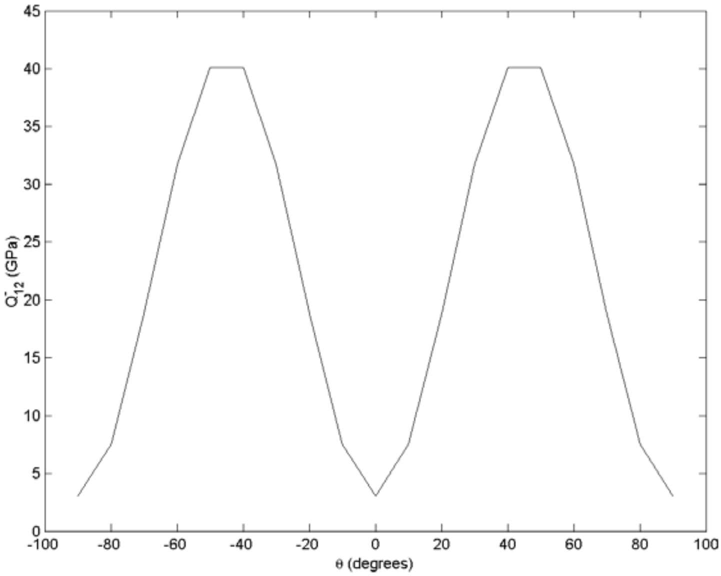

Fig. Variation of $\bar{Q}_{12}$ versus θ for Problem 5.9

```
>> y3 = [Q1(1,3) Q2(1,3) Q3(1,3) Q4(1,3) Q5(1,3) Q6(1,3) Q7(1,3)
        Q8(1,3) Q9(1,3) Q10(1,3) Q11(1,3) Q12(1,3) Q13(1,3) Q14(1,3)
        Q15(1,3) Q16(1,3) Q17(1,3) Q18(1,3) Q19(1,3)]

y3 =

  Columns 1 through 14

     0.0000    0.0435    -8.4612    -29.0342    -57.6152    -83.7927
    -95.3179    -83.8363    -49.1540    0    49.1540    83.8363
     95.3179    83.7927

  Columns 15 through 19

    57.6152    29.0342    8.4612    -0.0435    -0.0000

>> plot(x,y3)
>> xlabel('\theta (degrees)');
>> ylabel('Q^{-}_{16} (GPa)');
```

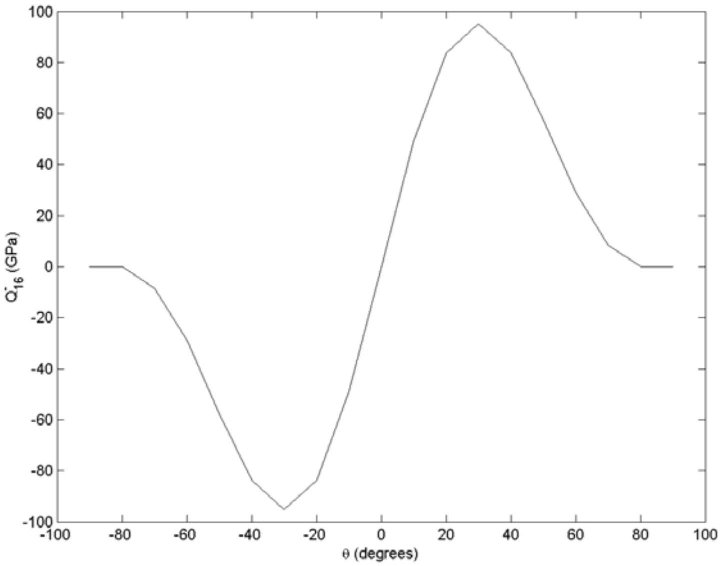

Fig. Variation of $\bar{Q}_{16}$ versus θ for Problem 5.9

```
>> y4 = [Q1(2,2) Q2(2,2) Q3(2,2) Q4(2,2) Q5(2,2) Q6(2,2) Q7(2,2)
        Q8(2,2) Q9(2,2) Q10(2,2) Q11(2,2) Q12(2,2) Q13(2,2) Q14(2,2)
        Q15(2,2) Q16(2,2) Q17(2,2) Q18(2,2) Q19(2,2)]

y4 =

  Columns 1 through 14

    155.7478  146.9414  123.1392   91.1488   59.3051   34.3711
    19.3541   13.1434   12.0115   12.1584   12.0115   13.1434   19.3541
    34.3711

  Columns 15 through 19

    59.3051   91.1488  123.1392  146.9414  155.7478

>> plot(x,y4)
>> xlabel('\theta (degrees)');
>> ylabel('Q^{-}_{22} (GPa)');
```

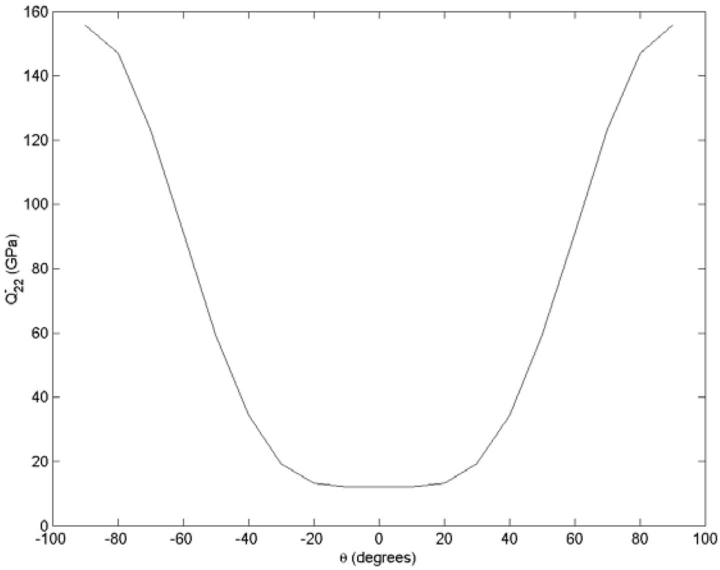

Fig. Variation of $\bar{Q}_{22}$ versus θ for Problem 5.9

```
>> y5 = [Q1(2,3) Q2(2,3) Q3(2,3) Q4(2,3) Q5(2,3) Q6(2,3) Q7(2,3)
        Q8(2,3) Q9(2,3) Q10(2,3) Q11(2,3) Q12(2,3) Q13(2,3) Q14(2,3)
        Q15(2,3) Q16(2,3) Q17(2,3) Q18(2,3) Q19(2,3)]

y5 =

  Columns 1 through 14

    -0.0000   -49.1540   -83.8363   -95.3179   -83.7927   -57.6152
    -29.0342   -8.4612   0.0435   0   -0.0435   8.4612
     29.0342   57.6152

  Columns 15 through 19

    83.7927   95.3179   83.8363   49.1540    0.0000

>> plot(x,y5)
>> xlabel('\theta (degrees)');
>> ylabel('Q^{-}_{26} (GPa)');
```

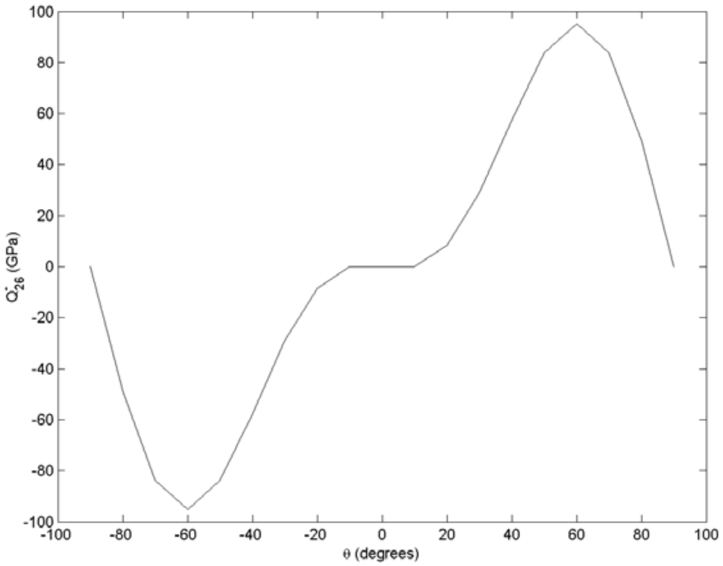

Fig. Variation of $\bar{Q}_{26}$ versus θ for Problem 5.9

```
>> y6 = [Q1(3,3) Q2(3,3) Q3(3,3) Q4(3,3) Q5(3,3) Q6(3,3) Q7(3,3)
        Q8(3,3) Q9(3,3) Q10(3,3) Q11(3,3) Q12(3,3) Q13(3,3) Q14(3,3)
        Q15(3,3) Q16(3,3) Q17(3,3) Q18(3,3) Q19(3,3)]

y6 =

  Columns 1 through 14

    4.4000   13.3532   36.0236   61.8034   78.6299   78.6299
    61.8034   36.0236   13.3532    4.4000   13.3532   36.0236
    61.8034   78.6299

  Columns 15 through 19

    78.6299   61.8034   36.0236   13.3532    4.4000

>> plot(x,y6)
>> xlabel('\theta (degrees)');
>> ylabel('Q^{-}_{66} (GPa)');
```

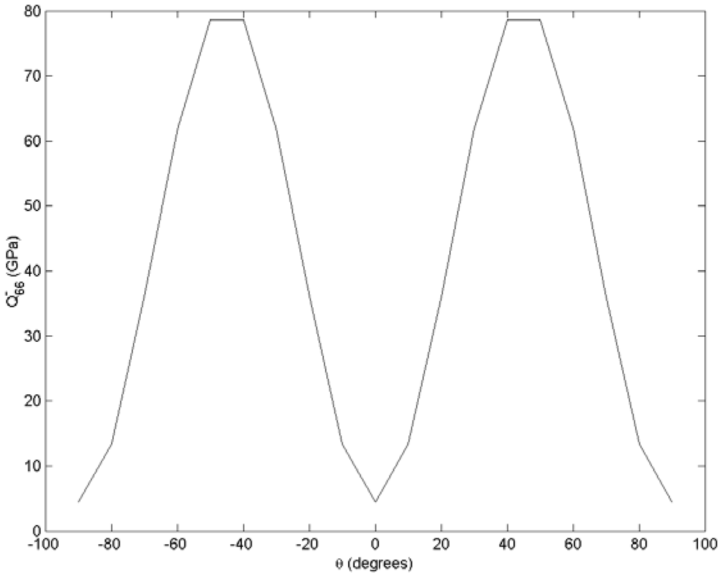

Fig. Variation of $\bar{Q}_{66}$ versus θ for Problem 5.9

Problem 5.10

```
>> Q = ReducedStiffness(50.0, 15.20, 0.254, 4.70)

Q =

   51.0003    3.9380         0
    3.9380   15.5041         0
         0         0    4.7000
>> Q1 = Qbar(Q, -90)

Q1 =

   15.5041    3.9380    0.0000
    3.9380   51.0003   -0.0000
    0.0000   -0.0000    4.7000

>> Q2 = Qbar(Q, -80)

Q2 =

   15.1348    5.3777    1.8406
    5.3777   48.4903  -13.9810
    0.9203   -6.9905    7.5793
```

```
>> Q3 = Qbar(Q, -70)

Q3 =

   14.5714    9.0230    0.7118
    9.0230   41.7630  -23.5283
    0.3559  -11.7642   14.8700

>> Q4 = Qbar(Q, -60)

Q4 =

   15.1478   13.1683   -4.7121
   13.1683   32.8959  -26.0285
   -2.3560  -13.0143   23.1606

>> Q5 = Qbar(Q, -50)

Q5 =

   18.2343   15.8740  -13.2692
   15.8740   24.3981  -21.6877
   -6.6346  -10.8439   28.5719

>> Q6 = Qbar(Q, -40)

Q6 =

   24.3981   15.8740  -21.6877
   15.8740   18.2343  -13.2692
  -10.8439   -6.6346   28.5719


>> Q7 = Qbar(Q, -30)

Q7 =

   32.8959   13.1683  -26.0285
   13.1683   15.1478   -4.7121
  -13.0143   -2.3560   23.1606

>> Q8 = Qbar(Q, -20)

Q8 =

   41.7630    9.0230  -23.5283
    9.0230   14.5714    0.7118
  -11.7642    0.3559   14.8700
```

```
>> Q9 = Qbar(Q, -10)

Q9 =

   48.4903    5.3777  -13.9810
    5.3777   15.1348    1.8406
   -6.9905    0.9203    7.5793

>> Q10 = Qbar(Q, 0)

Q10 =

   51.0003    3.9380         0
    3.9380   15.5041         0
         0         0    4.7000

>> Q11 = Qbar(Q, 10)

Q11 =

   48.4903    5.3777   13.9810
    5.3777   15.1348   -1.8406
    6.9905   -0.9203    7.5793

>> Q12 = Qbar(Q, 20)

Q12 =

   41.7630    9.0230   23.5283
    9.0230   14.5714   -0.7118
   11.7642   -0.3559   14.8700


>> Q13 = Qbar(Q, 30)

Q13 =

   32.8959   13.1683   26.0285
   13.1683   15.1478    4.7121
   13.0143    2.3560   23.1606

>> Q14 = Qbar(Q, 40)

Q14 =

   24.3981   15.8740   21.6877
   15.8740   18.2343   13.2692
   10.8439    6.6346   28.5719
```

```
>> Q15 = Qbar(Q, 50)

Q15 =

   18.2343   15.8740   13.2692
   15.8740   24.3981   21.6877
    6.6346   10.8439   28.5719

>> Q16 = Qbar(Q, 60)

Q16 =

   15.1478   13.1683    4.7121
   13.1683   32.8959   26.0285
    2.3560   13.0143   23.1606

>> Q17 = Qbar(Q, 70)

Q17 =

   14.5714    9.0230   -0.7118
    9.0230   41.7630   23.5283
   -0.3559   11.7642   14.8700

>> Q18 = Qbar(Q, 80)

Q18 =

   15.1348    5.3777   -1.8406
    5.3777   48.4903   13.9810
   -0.9203    6.9905    7.5793


>> Q19 = Qbar(Q, 90)

Q19 =

   15.5041    3.9380   -0.0000
    3.9380   51.0003    0.0000
   -0.0000    0.0000    4.7000

>> x = [-90 -80 -70 -60 -50 -40 -30 -20 -10 0 10 20 30 40 50 60 70
        80 90]

x =

   -90    -80    -70    -60    -50    -40    -30    -20    -10      0     10
   20     30     40     50     60     70     80     90
```

```
>> y1 = [Q1(1,1) Q2(1,1) Q3(1,1) Q4(1,1) Q5(1,1) Q6(1,1) Q7(1,1)
        Q8(1,1) Q9(1,1) Q10(1,1) Q11(1,1) Q12(1,1) Q13(1,1) Q14(1,1)
        Q15(1,1) Q16(1,1) Q17(1,1) Q18(1,1) Q19(1,1)]

y1 =

  Columns 1 through 14

    15.5041   15.1348   14.5714   15.1478   18.2343   24.3981
    32.8959   41.7630   48.4903   51.0003   48.4903   41.7630
    32.8959   24.3981

  Columns 15 through 19

    18.2343   15.1478   14.5714   15.1348   15.5041

>> plot(x,y1)
>> xlabel('\theta (degrees)');
>> ylabel('Q^{-}_{11} (GPa)');
```

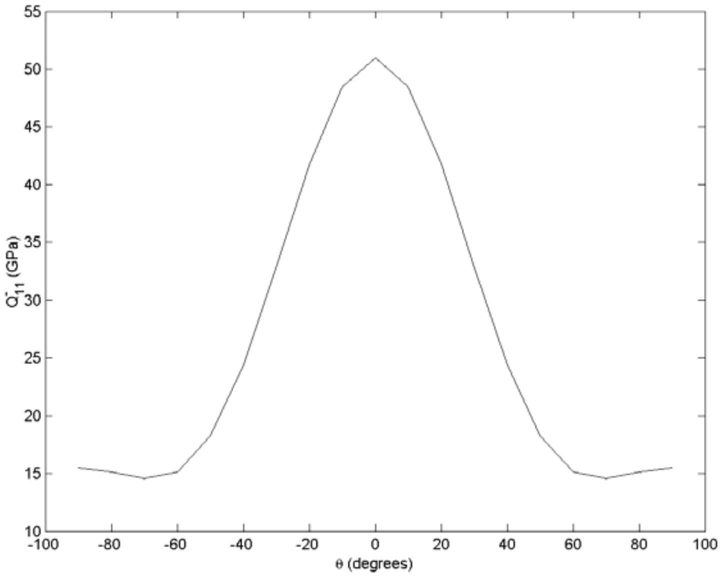

Fig. Variation of $\bar{Q}_{11}$ versus θ for Problem 5.10

```
>> y2 = [Q1(1,2) Q2(1,2) Q3(1,2) Q4(1,2) Q5(1,2) Q6(1,2) Q7(1,2)
        Q8(1,2) Q9(1,2) Q10(1,2) Q11(1,2) Q12(1,2) Q13(1,2) Q14(1,2)
        Q15(1,2) Q16(1,2) Q17(1,2) Q18(1,2) Q19(1,2)]
```

```
y2 =

  Columns 1 through 14

    3.9380    5.3777    9.0230   13.1683   15.8740   15.8740
    13.1683    9.0230    5.3777    3.9380    5.3777    9.0230
    13.1683   15.8740


  Columns 15 through 19

    15.8740   13.1683    9.0230    5.3777    3.9380

>> plot(x,y2)
>> xlabel('\theta (degrees)');
>> ylabel('Q^{-}_{12} (GPa)');
```

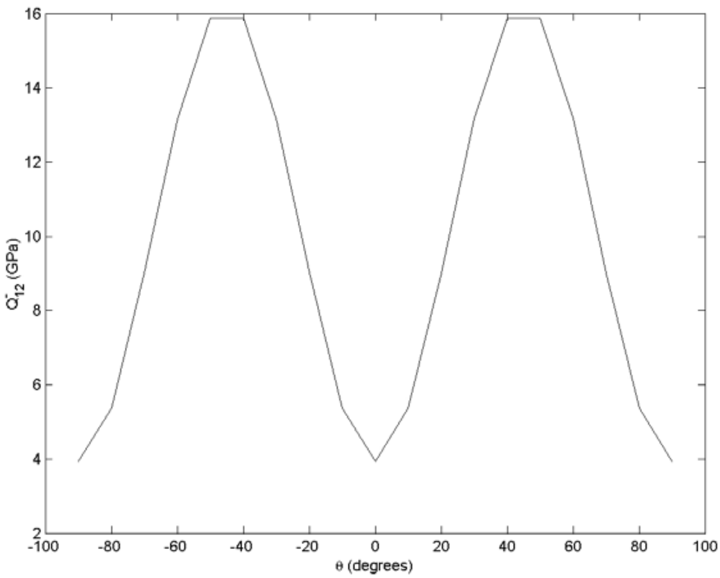

Fig. Variation of $\bar{Q}_{12}$ versus θ for Problem 5.10

```
>> y3 = [Q1(1,3) Q2(1,3) Q3(1,3) Q4(1,3) Q5(1,3) Q6(1,3) Q7(1,3)
        Q8(1,3) Q9(1,3) Q10(1,3) Q11(1,3) Q12(1,3) Q13(1,3) Q14(1,3)
        Q15(1,3) Q16(1,3) Q17(1,3) Q18(1,3) Q19(1,3)]
```

```
y3 =

  Columns 1 through 14

     0.0000    1.8406    0.7118   -4.7121  -13.2692  -21.6877
   -26.0285  -23.5283  -13.9810         0   13.9810   23.5283
    26.0285   21.6877

  Columns 15 through 19

    13.2692    4.7121   -0.7118   -1.8406   -0.0000

>> plot(x,y3)
>> xlabel('\theta (degrees)');
>> ylabel('Q^{-}_{16} (GPa)');
```

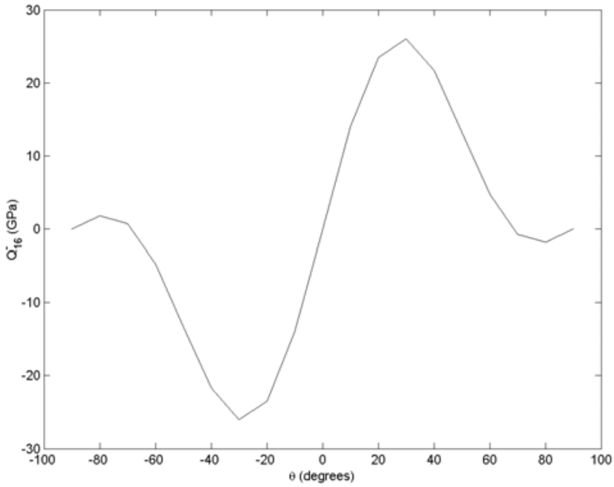

Fig. Variation of $\bar{Q}_{16}$ versus θ for Problem 5.10

```
>> y4 = [Q1(2,2) Q2(2,2) Q3(2,2) Q4(2,2) Q5(2,2) Q6(2,2) Q7(2,2)
        Q8(2,2) Q9(2,2) Q10(2,2) Q11(2,2) Q12(2,2) Q13(2,2) Q14(2,2)
        Q15(2,2) Q16(2,2) Q17(2,2) Q18(2,2) Q19(2,2)]

y4 =

  Columns 1 through 14

   51.0003   48.4903   41.7630   32.8959   24.3981   18.2343
   15.1478   14.5714   15.1348   15.5041   15.1348   14.5714
   15.1478   18.2343
```

```
  Columns 15 through 19

    24.3981   32.8959   41.7630   48.4903   51.0003

>> plot(x,y4)
>> xlabel('\theta (degrees)');
>> ylabel('Q^{-}_{22} (GPa)');
```

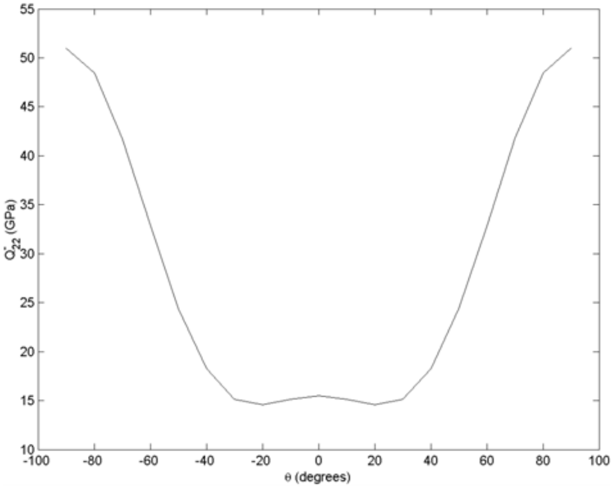

Fig. Variation of $\bar{Q}_{22}$ versus θ for Problem 5.10

```
>> y5 = [Q1(2,3) Q2(2,3) Q3(2,3) Q4(2,3) Q5(2,3) Q6(2,3) Q7(2,3)
        Q8(2,3) Q9(2,3) Q10(2,3) Q11(2,3) Q12(2,3) Q13(2,3) Q14(2,3)
        Q15(2,3) Q16(2,3) Q17(2,3) Q18(2,3) Q19(2,3)]

y5 =

  Columns 1 through 14

    -0.0000  -13.9810  -23.5283  -26.0285  -21.6877  -13.2692
    -4.7121    0.7118    1.8406         0   -1.8406   -0.7118
     4.7121   13.2692

  Columns 15 through 19

     21.6877   26.0285   23.5283   13.9810    0.0000

>> plot(x,y5)
>> xlabel('\theta (degrees)');
>> ylabel('Q^{-}_{26} (GPa)');
```

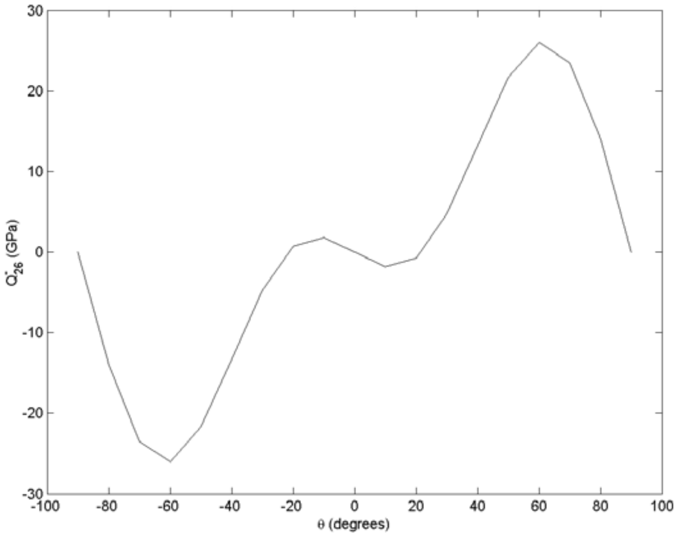

Fig. Variation of $\bar{Q}_{26}$ versus θ for Problem 5.10

```
>> y6 = [Q1(3,3) Q2(3,3) Q3(3,3) Q4(3,3) Q5(3,3) Q6(3,3) Q7(3,3)
        Q8(3,3) Q9(3,3) Q10(3,3) Q11(3,3) Q12(3,3) Q13(3,3) Q14(3,3)
        Q15(3,3) Q16(3,3) Q17(3,3) Q18(3,3) Q19(3,3)]

y6 =

  Columns 1 through 14

    4.7000     7.5793    14.8700    23.1606    28.5719    28.5719
    23.1606    14.8700     7.5793     4.7000     7.5793    14.8700
    23.1606    28.5719

  Columns 15 through 19

    28.5719    23.1606    14.8700     7.5793     4.7000

>> plot(x,y6)
>> xlabel('\theta (degrees)');
>> ylabel('Q^{-}_{66} (GPa)');
```

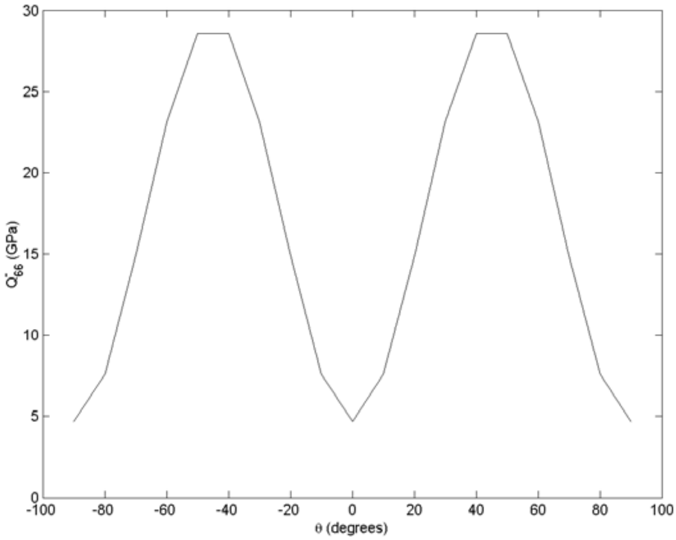

Fig. Variation of $\bar{Q}_{66}$ versus θ for Problem 5.10

Problem 5.11

When $\theta = 0°$, we have $[T] = [T]^{-1} = [I]$, where $[I]$ is the identity matrix. Therefore, we have;

$$[\bar{S}] = [T]^{-1}[S][T] = [I][S][I] = [S]$$
$$[\bar{Q}] = [T]^{-1}[Q][T] = [I][Q][I] = [Q]$$

Problem 5.12

For isotropic materials, we showed in Problem 4.3 that $[S]$ is given by:

$$[S] = \begin{bmatrix} \frac{1}{E} & \frac{-\nu}{E} & 0 \\ \frac{-\nu}{E} & \frac{1}{E} & 0 \\ 0 & 0 & \frac{2(1+\nu)}{E} \end{bmatrix}$$

Therefore, we have:

$$S_{11} = \frac{1}{E}$$
$$S_{12} = \frac{-\nu}{E}$$

$$S_{22} = \frac{1}{E}$$
$$S_{16} = 0$$
$$S_{26} = 0$$
$$S_{66} = \frac{2(1+\nu)}{E}$$

Substitute the above equations into (5.16) from the book to obtain:

$$\begin{aligned}
\bar{S}_{11} &= \frac{1}{E}m^4 + \left[\frac{-2\nu}{E} + \frac{2(1+\nu)}{E}\right] n^2m^2 + \frac{1}{E}n^4 \\
&= \frac{1}{E}\left(m^2+n^2\right)^2 \\
&= \frac{1}{E} \\
\bar{S}_{12} &= \left[\frac{1}{E} + \frac{1}{E} - \frac{2(1+\nu)}{E}\right] n^2m^2 - \frac{\nu}{E}\left(n^4+m^4\right) \\
&= -\frac{\nu}{E}\left(m^2+n^2\right)^2 \\
&= -\frac{\nu}{E} \\
\bar{S}_{22} &= \frac{1}{E} \text{ (derivation similar to } \bar{S}_{11}\text{).} \\
\bar{S}_{16} &= \left[\frac{2}{E} - \frac{2\nu}{E} - \frac{2(1+\nu)}{E}\right] nm^3 - \left[\frac{2}{E} - \frac{2\nu}{E} - \frac{2(1+\nu)}{E}\right] n^3m \\
&= 0 - 0 \\
&= 0 \\
\bar{S}_{26} &= 0 \text{ (derivation similar to } \bar{S}_{16}\text{).} \\
\bar{S}_{66} &= 2\left[\frac{2}{E} + \frac{2}{E} + \frac{4\nu}{E} - \frac{2(1+\nu)}{E}\right] n^2m^2 + \frac{2(1+\nu)}{E}\left(n^4+m^4\right) \\
&= \frac{2(1+\nu)}{E}\left(m^2+n^2\right)^2 \\
&= \frac{2(1+\nu)}{E}
\end{aligned}$$

Therefore, we have now the following equation;

$$[\bar{S}] = [S] = \begin{bmatrix} \dfrac{1}{E} & \dfrac{-\nu}{E} & 0 \\ \dfrac{-\nu}{E} & \dfrac{1}{E} & 0 \\ 0 & 0 & \dfrac{2(1+\nu)}{E} \end{bmatrix}$$

Problem 5.13

We can follow the same approach used in solving Problem 5.12 while using the result of Problem 5.5. Alternatively, we can follow a shorter approach by using Problem 5.4 and taking the inverse of $[\bar{S}]$ as follows:

From Problem 5.12, we have:

$$[\bar{S}] = \begin{bmatrix} \dfrac{1}{E} & \dfrac{-\nu}{E} & 0 \\ \dfrac{-\nu}{E} & \dfrac{1}{E} & 0 \\ 0 & 0 & \dfrac{2(1+\nu)}{E} \end{bmatrix}$$

and from Problem 5.5 we obtain:

$$[\bar{Q}] = [\bar{S}]^{-1} = \begin{bmatrix} \dfrac{1}{E} & \dfrac{-\nu}{E} & 0 \\ \dfrac{-\nu}{E} & \dfrac{1}{E} & 0 \\ 0 & 0 & \dfrac{2(1+\nu)}{E} \end{bmatrix}^{-1} = \begin{bmatrix} \dfrac{E}{1-\nu^2} & \dfrac{\nu E}{1-\nu^2} & 0 \\ \dfrac{\nu E}{1-\nu^2} & \dfrac{E}{1-\nu^2} & 0 \\ 0 & 0 & \dfrac{E}{2(1+\nu)} \end{bmatrix} = [Q]$$

See also Problem 4.4.

Problem 5.14

```
>> S = ReducedCompliance(50.0, 15.20, 0.254, 4.70)

S =

    0.0200   -0.0051         0
   -0.0051    0.0658         0
         0         0    0.2128

>> S1 = Sbar(S,0)
```

```
S1 =

    0.0200   -0.0051         0
   -0.0051    0.0658         0
         0         0    0.2128

>> sigma = [100e-3 ; 0 ; 0]

sigma =

    0.1000
         0
         0

>> epsilon = S1*sigma

epsilon =

    0.0020
   -0.0005
         0

>> deltax = 50*epsilon(1)

deltax =

    0.1000

>> deltay = 50*epsilon(2)

deltay =

   -0.0254

>> gammaxy = epsilon(3)

gammaxy =

     0

>> dx = 50 + deltax

dx =

   50.1000
```

```
>> dy = 50 + deltay

dy =

   49.9746

>> S2 = Sbar(S, 45)

S2 =

    0.1253   -0.0875   -0.0229
   -0.0875    0.1253   -0.0229
   -0.0114   -0.0114    0.0480

>> epsilon = S2*sigma

epsilon =

    0.0125
   -0.0087
   -0.0011

>> deltax = 50*epsilon(1)

deltax =

    0.6265

>> deltay = 50*epsilon(2)

deltay =

   -0.4374

>> dx = 50 + deltax

dx =

   50.6265

>> dy = 50 + deltay

dy =

   49.5626

>> gammaxy = epsilon(3)
```

```
gammaxy =

   -0.0011

>> S3 = Sbar(S, -45)

S3 =

    0.1253   -0.0875    0.0229
   -0.0875    0.1253    0.0229
    0.0114    0.0114    0.0480

>> epsilon = S3*sigma

epsilon =

    0.0125
   -0.0087
    0.0011

>> deltax = 50*epsilon(1)

deltax =

    0.6265

>> deltay = 50*epsilon(2)

deltay =

   -0.4374

>> dy = 50 + deltay

dy =

   49.5626

>> dx = 50 + deltax

dx =

   50.6265

>> gammaxy = epsilon(3)

gammaxy =

    0.0011
```

Problem 5.15

Using the result of Problem 4.10, we have:

$$\begin{Bmatrix} \varepsilon_1 - \alpha_1 \Delta T - \beta_1 \Delta M \\ \varepsilon_2 - \alpha_2 \Delta T - \beta_2 \Delta M \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}$$

Now, we need to transform the above equation from the 1-2-3 coordinate system to the x-y-z global coordinate system. The above equation can be rewritten as follows where we have introduced a factor of 1/2 for the engineering shear strain:

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \frac{1}{2}\gamma_{12} \end{Bmatrix} - \begin{Bmatrix} \alpha_1 \Delta T \\ \alpha_2 \Delta T \\ \frac{0}{2} \end{Bmatrix} - \begin{Bmatrix} \beta_1 \Delta M \\ \beta_2 \Delta M \\ \frac{0}{2} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}$$

Next, we substitute the following transformation relations along with (5.2) and (5.6) into the above equation:

$$\begin{Bmatrix} \alpha_1 \Delta T \\ \alpha_2 \Delta T \\ \frac{0}{2} \end{Bmatrix} = [T] \begin{Bmatrix} \alpha_x \Delta T \\ \alpha_y \Delta T \\ \frac{1}{2}\alpha_{xy} \Delta T \end{Bmatrix}$$

$$\begin{Bmatrix} \beta_1 \Delta M \\ \beta_2 \Delta M \\ \frac{0}{2} \end{Bmatrix} = [T] \begin{Bmatrix} \beta_x \Delta M \\ \beta_y \Delta M \\ \frac{1}{2}\beta_{xy} \Delta M \end{Bmatrix}$$

Therefore, we obtain the desired relation as follows (after grouping the terms together and using (5.11)):

$$\begin{Bmatrix} \varepsilon_x - \alpha_x \Delta T - \beta_x \Delta M \\ \varepsilon_y - \alpha_y \Delta T - \beta_y \Delta M \\ \gamma_{xy} - \alpha_{xy} \Delta T - \beta_{xy} \Delta M \end{Bmatrix} = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26} \\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}$$

Taking the inverse of the above relation, we obtain the second desired results as follows:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x - \alpha_x \Delta T - \beta_x \Delta M \\ \varepsilon_y - \alpha_y \Delta T - \beta_y \Delta M \\ \gamma_{xy} - \alpha_{xy} \Delta T - \beta_{xy} \Delta M \end{Bmatrix}$$

Problem 6.1

From an elementary course on mechanics of materials, we have the following equation:

$$\nu_{xy} = -\frac{\varepsilon_y}{\varepsilon_x}$$

We also have the following two equations that can be obtained from (5.10):

$$\varepsilon_y = \bar{S}_{12}\sigma_x$$
$$\varepsilon_x = \bar{S}_{11}\sigma_x$$

Substitute the above two equations into the first equation above to obtain the desired relation:

$$\nu_{xy} = -\frac{\bar{S}_{12}}{\bar{S}_{11}} = \frac{\nu_{12}\left(n^4 + m^4\right) - \left(1 + \frac{E_1}{E_2} - \frac{E_1}{G_{12}}\right) n^2 m^2}{m^4 + \left(\frac{E_1}{G_{12}} - 2\nu_{12}\right) n^2 m^2 + \frac{E_1}{E_2} n^2}$$

where we have used (5.16) from Chap. 5.

Problem 6.2

From an elementary course on mechanics of materials, we have the following equation:

$$\varepsilon_y = \frac{\sigma_y}{E_y}$$

We also have the following equation that can be obtained from (5.10):

$$\varepsilon_y = \bar{S}_{22}\sigma_y$$

Comparing the above two equation, we obtain the desired result as follows:

$$E_y = \frac{1}{\bar{S}_{22}} = \frac{E_2}{m^4 + \left(\frac{E_2}{G_{12}} - 2\nu_{21}\right) n^2 m^2 + \frac{E_2}{E_1} n^4}$$

where we have used (5.16) from Chap. 5.

Problem 6.3

From an elementary course on mechanics of materials, we have the following equation:

$$\nu_{yx} = -\frac{\varepsilon_x}{\varepsilon_y}$$

We also have the following two equations that can be obtained from (5.10):

$$\varepsilon_y = \bar{S}_{22}\sigma_y$$
$$\varepsilon_x = \bar{S}_{12}\sigma_y$$

Substitute the above two equations into the first equation above to obtain the desired relation:

$$\nu_{yx} = -\frac{\bar{S}_{12}}{\bar{S}_{22}} = \frac{\nu_{21}\left(n^4 + m^4\right) - \left(1 + \frac{E_2}{E_1} - \frac{E_2}{G_{12}}\right) n^2 m^2}{m^4 + \left(\frac{E_2}{G_{12}} - 2\nu_{21}\right) n^2 m^2 + \frac{E_2}{E_1} n^2}$$

where we have used (5.16) from Chap. 5.

Problem 6.4

From an elementary course on mechanics of materials, we have the following equation:

$$\gamma_{xy} = \frac{\tau_{xy}}{G_{xy}}$$

We also have the following equation which can be obtained from (5.10):

$$\gamma_{xy} = \bar{S}_{66} \tau_{xy}$$

Comparing the above two equation, we obtain the desired result as follows:

$$G_{xy} = \frac{1}{\bar{S}_{66}} = \frac{G_{12}}{n^4 + m^4 + 2\left(\frac{2G_{12}}{E_1}\left(1 + 2\nu_{12}\right) + \frac{2G_{12}}{E_2} - 1\right) n^2 m^2}$$

where we have used (5.16) from Chap. 5.

Problem 6.5

```
>> Ex1 = Ex(50.0, 15.20, 0.254, 4.70, -90)

Ex1 =

   15.2000

>> Ex2 = Ex(50.0, 15.20, 0.254, 4.70, -80)

Ex2 =

   14.7438
```

```
>> Ex3 = Ex(50.0, 15.20, 0.254, 4.70, -70)

Ex3 =

   13.7932

>> Ex4 = Ex(50.0, 15.20, 0.254, 4.70, -60)

Ex4 =

   13.1156

>> Ex5 = Ex(50.0, 15.20, 0.254, 4.70, -50)

Ex5 =

   13.2990

>> Ex6 = Ex(50.0, 15.20, 0.254, 4.70, -40)

Ex6 =

   14.8715

>> Ex7 = Ex(50.0, 15.20, 0.254, 4.70, -30)

Ex7 =

   18.7440

>> Ex8 = Ex(50.0, 15.20, 0.254, 4.70, -20)

Ex8 =

   26.7217

>> Ex9 = Ex(50.0, 15.20, 0.254, 4.70, -10)

Ex9 =

   40.3275
```

```
>> Ex10 = Ex(50.0, 15.20, 0.254, 4.70, 0)

Ex10 =

    50

>> Ex11 = Ex(50.0, 15.20, 0.254, 4.70, 10)

Ex11 =

   40.3275

>> Ex12 = Ex(50.0, 15.20, 0.254, 4.70, 20)

Ex12 =

   26.7217

>> Ex13 = Ex(50.0, 15.20, 0.254, 4.70, 30)

Ex13 =

   18.7440

>> Ex14 = Ex(50.0, 15.20, 0.254, 4.70, 40)

Ex14 =

   14.8715

>> Ex15 = Ex(50.0, 15.20, 0.254, 4.70, 50)

Ex15 =

   13.2990

>> Ex16 = Ex(50.0, 15.20, 0.254, 4.70, 60)

Ex16 =

   13.1156

>> Ex17 = Ex(50.0, 15.20, 0.254, 4.70, 70)

Ex17 =

   13.7932
```

```
>> Ex18 = Ex(50.0, 15.20, 0.254, 4.70, 80)

Ex18 =

   14.7438


>> Ex19 = Ex(50.0, 15.20, 0.254, 4.70, 90)

Ex19 =

   15.2000

>> y1 = [Ex1 Ex2 Ex3 Ex4 Ex5 Ex6 Ex7 Ex8 Ex9 Ex10 Ex11 Ex12 Ex13 Ex14
        Ex15 Ex16 Ex17 Ex18 Ex19]

y1 =

  Columns 1 through 14

    15.2000   14.7438   13.7932   13.1156   13.2990   14.8715
    18.7440   26.7217   40.3275   50.0000   40.3275   26.7217
    18.7440   14.8715

  Columns 15 through 19

    13.2990   13.1156   13.7932   14.7438   15.2000

>> x = [-90 -80 -70 -60 -50 -40 -30 -20 -10 0 10 20 30 40 50 60 70
       80 90]

x =

   -90   -80   -70   -60   -50   -40   -30   -20   -10     0    10
    20    30    40    50    60    70    80    90

>> plot(x,y1)
>> xlabel('\theta (degrees)');
>> ylabel('E_x (GPa)');
```

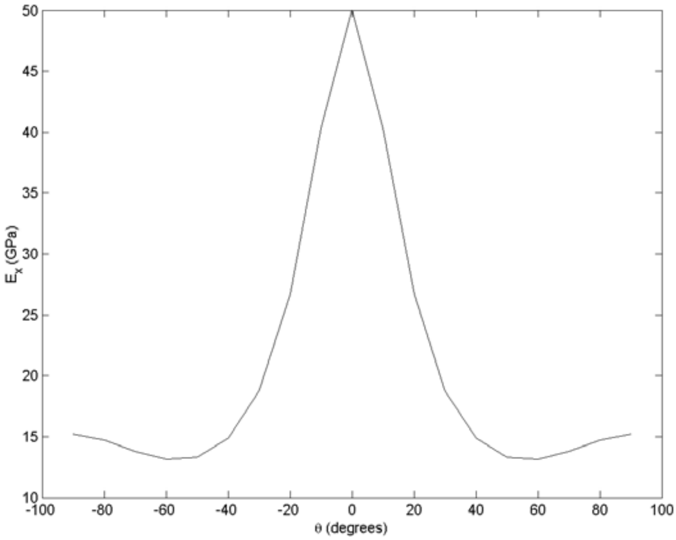

Fig. Variation of E_x versus θ for Problem 6.5

```
>> NUxy1 = NUxy(50.0, 15.20, 0.254, 4.70, -90)

NUxy1 =

    0.0772

>> NUxy2 = NUxy(50.0, 15.20, 0.254, 4.70, -80)

NUxy2 =

    0.1218

>> NUxy3 = NUxy(50.0, 15.20, 0.254, 4.70, -70)


NUxy3 =

    0.2162

>> NUxy4 = NUxy(50.0, 15.20, 0.254, 4.70, -60)

NUxy4 =

    0.3046
```

```
>> NUxy5 = NUxy(50.0, 15.20, 0.254, 4.70, -50)

NUxy5 =

    0.3665

>> NUxy6 = NUxy(50.0, 15.20, 0.254, 4.70, -40)

NUxy6 =

    0.4015

>> NUxy7 = NUxy(50.0, 15.20, 0.254, 4.70, -30)

NUxy7 =

    0.4108
>> NUxy8 = NUxy(50.0, 15.20, 0.254, 4.70, -20)

NUxy8 =

    0.3878

>> NUxy9 = NUxy(50.0, 15.20, 0.254, 4.70, -10)

NUxy9 =

    0.3180

>> NUxy10 = NUxy(50.0, 15.20, 0.254, 4.70, 0)

NUxy10 =

    0.2540

>> NUxy11 = NUxy(50.0, 15.20, 0.254, 4.70, 10)

NUxy11 =

    0.3180

>> NUxy12 = NUxy(50.0, 15.20, 0.254, 4.70, 20)

NUxy12 =

    0.3878
```

```
>> NUxy13 = NUxy(50.0, 15.20, 0.254, 4.70, 30)

NUxy13 =

    0.4108

>> NUxy14 = NUxy(50.0, 15.20, 0.254, 4.70, 40)

NUxy14 =

    0.4015

>> NUxy15 = NUxy(50.0, 15.20, 0.254, 4.70, 50)

NUxy15 =

    0.3665


>> NUxy16 = NUxy(50.0, 15.20, 0.254, 4.70, 60)

NUxy16 =

    0.3046

>> NUxy17 = NUxy(50.0, 15.20, 0.254, 4.70, 70)

NUxy17 =

    0.2162

>> NUxy18 = NUxy(50.0, 15.20, 0.254, 4.70, 80)

NUxy18 =

    0.1218

>> NUxy19 = NUxy(50.0, 15.20, 0.254, 4.70, 90)

NUxy19 =

    0.0772

>> y2 = [NUxy1 NUxy2 NUxy3 NUxy4 NUxy5 NUxy6 NUxy7 NUxy8 NUxy9 NUxy10
        NUxy11 NUxy12 NUxy13 NUxy14 NUxy15 NUxy16 NUxy17 NUxy18 NUxy19]
```

```
y2 =

  Columns 1 through 14

    0.0772    0.1218    0.2162    0.3046    0.3665    0.4015
    0.4108    0.3878    0.3180    0.2540    0.3180    0.3878
    0.4108    0.4015

  Columns 15 through 19

    0.3665    0.3046    0.2162    0.1218    0.0772

>> plot(x,y2)
>> xlabel('\theta (degrees)');
>> ylabel('\nu_{xy}');
```

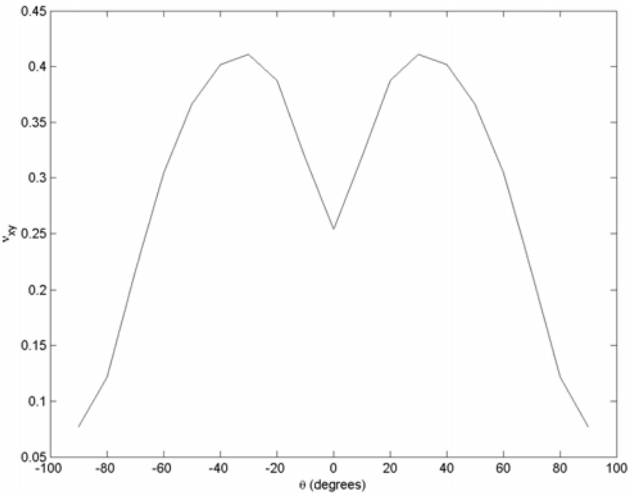

Fig. Variation of ν_{xy} versus θ for Problem 6.5

```
>> Ey1 = Ey(50.0, 15.20, 0.254, 4.70, -90)

Ey1 =

    50

>> Ey2 = Ey(50.0, 15.20, 0.254, 4.70, -80)

Ey2 =

   41.4650
```

```
>> Ey3 = Ey(50.0, 15.20, 0.254, 4.70, -70)

Ey3 =

   28.5551

>> Ey4 = Ey(50.0, 15.20, 0.254, 4.70, -60)

Ey4 =

   20.4127

>> Ey5 = Ey(50.0, 15.20, 0.254, 4.70, -50)

Ey5 =

   16.2331

>> Ey6 = Ey(50.0, 15.20, 0.254, 4.70, -40)

Ey6 =

   14.3773

>> Ey7 = Ey(50.0, 15.20, 0.254, 4.70, -30)

Ey7 =

   13.9114
>> Ey8 = Ey(50.0, 15.20, 0.254, 4.70, -20)

Ey8 =

   14.2660

>> Ey9 = Ey(50.0, 15.20, 0.254, 4.70, -10)

Ey9 =

   14.8932

>> Ey10 = Ey(50.0, 15.20, 0.254, 4.70, 0)

Ey10 =

   15.2000
```

```
>> Ey11 = Ey(50.0, 15.20, 0.254, 4.70, 10)

Ey11 =

   14.8932

>> Ey12 = Ey(50.0, 15.20, 0.254, 4.70, 20)

Ey12 =

   14.2660

>> Ey13 = Ey(50.0, 15.20, 0.254, 4.70, 30)

Ey13 =

   13.9114

>> Ey14 = Ey(50.0, 15.20, 0.254, 4.70, 40)

Ey14 =

   14.3773

>> Ey15 = Ey(50.0, 15.20, 0.254, 4.70, 50)

Ey15 =

   16.2331

>> Ey16 = Ey(50.0, 15.20, 0.254, 4.70, 60)

Ey16 =

   20.4127

>> Ey17 = Ey(50.0, 15.20, 0.254, 4.70, 70)

Ey17 =

   28.5551

>> Ey18 = Ey(50.0, 15.20, 0.254, 4.70, 80)

Ey18 =

   41.4650
```

```
>> Ey19 = Ey(50.0, 15.20, 0.254, 4.70, 90)

Ey19 =

    50

>> y3 = [Ey1 Ey2 Ey3 Ey4 Ey5 Ey6 Ey7 Ey8 Ey9 Ey10 Ey11 Ey12 Ey13 Ey14
        Ey15 Ey16 Ey17 Ey18 Ey19]

y3 =

  Columns 1 through 14

    50.0000   41.4650   28.5551   20.4127   16.2331   14.3773
    13.9114   14.2660   14.8932   15.2000   14.8932   14.2660
    13.9114   14.3773

  Columns 15 through 19

    16.2331   20.4127   28.5551   41.4650   50.0000

>> plot(x,y3)
>> xlabel('\theta (degrees)');
>> ylabel('E_y (GPa)');
```

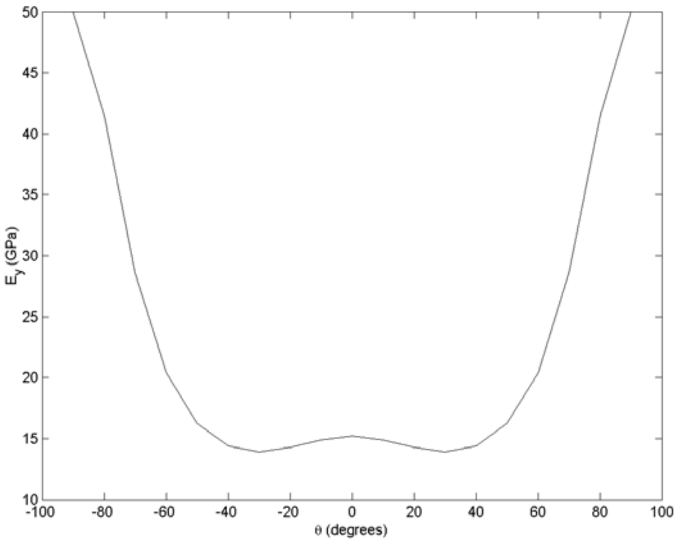

Fig. Variation of E_y versus θ for Problem 6.5

```
>> NUyx1 = NUyx(50.0, 15.20, 0.254, 4.70, -90)

NUyx1 =

    0.8355

>> NUyx2 = NUyx(50.0, 15.20, 0.254, 4.70, -80)

NUyx2 =

    0.7873

>> NUyx3 = NUyx(50.0, 15.20, 0.254, 4.70, -70)

NUyx3 =

    0.7112

>> NUyx4 = NUyx(50.0, 15.20, 0.254, 4.70, -60)

NUyx4 =

    0.6495

>> NUyx5 = NUyx(50.0, 15.20, 0.254, 4.70, -50)

NUyx5 =

    0.5928

>> NUyx6 = NUyx(50.0, 15.20, 0.254, 4.70, -40)

NUyx6 =

    0.5295

>> NUyx7 = NUyx(50.0, 15.20, 0.254, 4.70, -30)

NUyx7 =

    0.4529

>> NUyx8 = NUyx(50.0, 15.20, 0.254, 4.70, -20)

NUyx8 =

    0.3655
```

```
>> NUyx9 = NUyx(50.0, 15.20, 0.254, 4.70, -10)

NUyx9 =

    0.2871

>> NUyx10 = NUyx(50.0, 15.20, 0.254, 4.70, 0)

NUyx10 =

    0.2540

>> NUyx11 = NUyx(50.0, 15.20, 0.254, 4.70, 10)

NUyx11 =

    0.2871

>> NUyx12 = NUyx(50.0, 15.20, 0.254, 4.70, 20)

NUyx12 =

    0.3655

>> NUyx13 = NUyx(50.0, 15.20, 0.254, 4.70, 30)

NUyx13 =

    0.4529

>> NUyx14 = NUyx(50.0, 15.20, 0.254, 4.70, 40)

NUyx14 =

    0.5295

>> NUyx15 = NUyx(50.0, 15.20, 0.254, 4.70, 50)

NUyx15 =

    0.5928

>> NUyx16 = NUyx(50.0, 15.20, 0.254, 4.70, 60)

NUyx16 =

    0.6495
```

```
>> NUyx17 = NUyx(50.0, 15.20, 0.254, 4.70, 70)

NUyx17 =

    0.7112

>> NUyx18 = NUyx(50.0, 15.20, 0.254, 4.70, 80)

NUyx18 =

    0.7873

>> NUyx19 = NUyx(50.0, 15.20, 0.254, 4.70, 90)

NUyx19 =

    0.8355

>> y4 = [NUyx1 NUyx2 NUyx3 NUyx4 NUyx5 NUyx6 NUyx7 NUyx8 NUyx9 NUyx10
        NUyx11 NUyx12 NUyx13 NUyx14 NUyx15 NUyx16 NUyx17 NUyx18 NUyx19]


y4 =

 Columns 1 through 14

    0.8355    0.7873    0.7112    0.6495    0.5928    0.5295
    0.4529    0.3655    0.2871    0.2540    0.2871    0.3655
    0.4529    0.5295

  Columns 15 through 19

    0.5928    0.6495    0.7112    0.7873    0.8355

>> plot(x,y4)
>> xlabel('\theta (degrees)');
>> ylabel('\nu_{yx}');
```

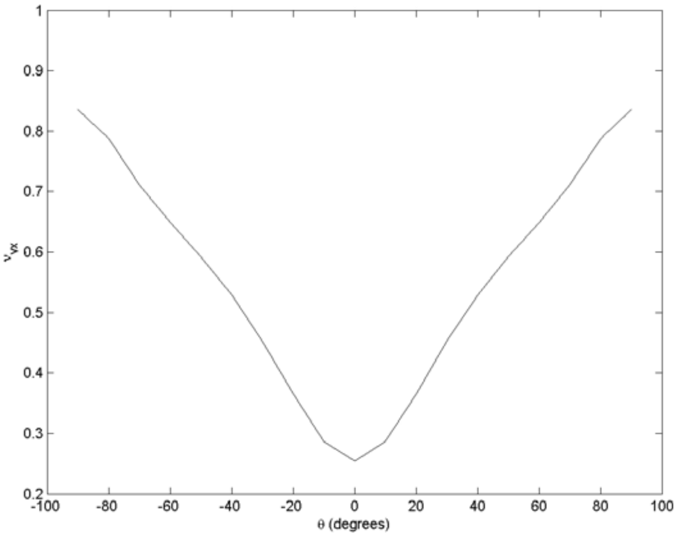

Fig. Variation of ν_{yx} versus θ for Problem 6.5

```
>> Gxy1 = Gxy(50.0, 15.20, 0.254, 4.70, -90)

Gxy1 =

    4.7000

>> Gxy2 = Gxy(50.0, 15.20, 0.254, 4.70, -80)

Gxy2 =

    5.0226

>> Gxy3 = Gxy(50.0, 15.20, 0.254, 4.70, -70)

Gxy3 =

    6.0790

>> Gxy4 = Gxy(50.0, 15.20, 0.254, 4.70, -60)

Gxy4 =

    7.9902
```

```
>> Gxy5 = Gxy(50.0, 15.20, 0.254, 4.70, -50)

Gxy5 =

   10.0531

>> Gxy6 = Gxy(50.0, 15.20, 0.254, 4.70, -40)

Gxy6 =

   10.0531

>> Gxy7 = Gxy(50.0, 15.20, 0.254, 4.70, -30)

Gxy7 =

    7.9902

>> Gxy8 = Gxy(50.0, 15.20, 0.254, 4.70, -20)

Gxy8 =

    6.0790

>> Gxy9 = Gxy(50.0, 15.20, 0.254, 4.70, -10)

Gxy9 =

    5.0226

>> Gxy10 = Gxy(50.0, 15.20, 0.254, 4.70, 0)

Gxy10 =

    4.7000

>> Gxy11 = Gxy(50.0, 15.20, 0.254, 4.70, 10)

Gxy11 =

    5.0226

>> Gxy12 = Gxy(50.0, 15.20, 0.254, 4.70, 20)

Gxy12 =

    6.0790
```

```
>> Gxy13 = Gxy(50.0, 15.20, 0.254, 4.70, 30)

Gxy13 =

    7.9902

>> Gxy14 = Gxy(50.0, 15.20, 0.254, 4.70, 40)

Gxy14 =

   10.0531

>> Gxy15 = Gxy(50.0, 15.20, 0.254, 4.70, 50)

Gxy15 =

   10.0531

>> Gxy16 = Gxy(50.0, 15.20, 0.254, 4.70, 60)

Gxy16 =

    7.9902

>> Gxy17 = Gxy(50.0, 15.20, 0.254, 4.70, 70)

Gxy17 =

    6.0790

>> Gxy18 = Gxy(50.0, 15.20, 0.254, 4.70, 80)

Gxy18 =

    5.0226

>> Gxy19 = Gxy(50.0, 15.20, 0.254, 4.70, 90)

Gxy19 =

    4.7000

>> y5 = [Gxy1 Gxy2 Gxy3 Gxy4 Gxy5 Gxy6 Gxy7 Gxy8 Gxy9 Gxy10 Gxy11
        Gxy12 Gxy13 Gxy14 Gxy15 Gxy16 Gxy17 Gxy18 Gxy19]
```

```
y5 =

  Columns 1 through 14

    4.7000     5.0226     6.0790     7.9902    10.0531    10.0531
    7.9902     6.0790     5.0226     4.7000     5.0226     6.0790
    7.9902    10.0531

  Columns 15 through 19

    10.0531     7.9902     6.0790     5.0226     4.7000

>> plot(x,y5)
>> xlabel('\theta (degrees)');
>> ylabel('G_{xy} (GPa)');
```

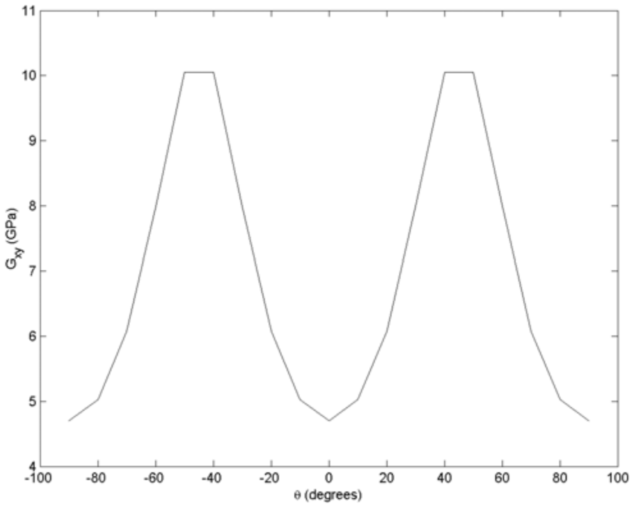

Fig. Variation of G_{xy} versus θ for Problem 6.5

Problem 6.6

From (5.10), we have:

$$\gamma_{xy} = \bar{S}_{16}\sigma_x$$
$$\varepsilon_x = \bar{S}_{11}\sigma_x$$

Substitute the above two equations into (6.6) to obtain the desired result as follows:

$$\eta_{xy,x} = \frac{\bar{S}_{16}}{\bar{S}_{11}}$$

Similarly, from (5.10) again, we have:

$$\gamma_{xy} = \bar{S}_{26}\sigma_y$$
$$\varepsilon_y = \bar{S}_{22}\sigma_y$$

Substitute the above two equation into (6.7) to obtain the desired result as follows:

$$\eta_{xy,y} = \frac{\bar{S}_{26}}{\bar{S}_{22}}$$

Problem 6.7

From (5.10), we have:

$$\varepsilon_x = \bar{S}_{16}\tau_{xy}$$
$$\gamma_{xy} = \bar{S}_{66}\tau_{xy}$$

Substitute the above two equations into (6.10) to obtain the desired result as follows:

$$\eta_{x,xy} = \frac{\bar{S}_{16}}{\bar{S}_{66}}$$

Similarly, from (5.10) again, we have:

$$\varepsilon_y = \bar{S}_{26}\tau_{xy}$$
$$\gamma_{xy} = \bar{S}_{66}\tau_{xy}$$

Substitute the above two equations into (6.11) to obtain the desired result as follows:

$$\eta_{y,xy} = \frac{\bar{S}_{26}}{\bar{S}_{66}}$$

Problem 6.8

Continuing with the commands from Example 6.3, we obtain:

```
>> Etaxxy1 = Etaxxy(S1)

Etaxxy1 =

 -7.7070e-017
```

```
>> Etaxxy2 = Etaxxy(S2)

Etaxxy2 =

   -0.2192

>> Etaxxy3 = Etaxxy(S3)

Etaxxy3 =

   -0.4244

>> Etaxxy4 = Etaxxy(S4)

Etaxxy4 =

   -0.4970

>> Etaxxy5 = Etaxxy(S5)

Etaxxy5 =

    0.1268

>> Etaxxy6 = Etaxxy(S6)

Etaxxy6 =

    1.3271

>> Etaxxy7 = Etaxxy(S7)

Etaxxy7 =

    1.2187

>> Etaxxy8 = Etaxxy(S8)

Etaxxy8 =

    0.7457

>> Etaxxy9 = Etaxxy(S9)

Etaxxy9 =

    0.3457
```

```
>> Etaxxy10 = Etaxy(S10)

Etaxxy10 =

     0

>> Etaxxy11 = Etaxxy(S11)

Etaxxy11 =

   -0.3457

>> Etaxxy12 = Etaxxy(S12)

Etaxxy12 =

   -0.7457

>> Etaxxy13 = Etaxxy(S13)

Etaxxy13 =

   -1.2187

>> Etaxxy14 = Etaxxy(S14)

Etaxxy14 =

   -1.3271

>> Etaxxy15 = Etaxxy(S15)

Etaxxy15 =

   -0.1268

>> Etaxxy16 = Etaxxy(S16)

Etaxxy16 =

    0.4970

>> Etaxxy17 = Etaxxy(S17)

Etaxxy17 =

    0.4244
```

```
>> Etaxxy18 = Etaxxy(S18)

Etaxxy18 =

    0.2192

>> Etaxxy19 = Etaxxy(S19)

Etaxxy19 =

  7.7070e-017

>> y8 = [Etaxxy1 Etaxxy2 Etaxxy3 Etaxxy4 Etaxxy5 Etaxxy6 Etaxxy7
        Etaxxy8 Etaxxy9 Etaxxy10 Etaxxy11 Etaxxy12 Etaxxy13 Etaxxy14
        Etaxxy15 Etaxxy16 Etaxxy17 Etaxxy18 Etaxxy19]

y8 =

  Columns 1 through 14

    -0.0000   -0.2192   -0.4244   -0.4970    0.1268    1.3271
     1.2187    0.7457    0.3457         0   -0.3457   -0.7457   -1.2187
    -1.3271

  Columns 15 through 19

    -0.1268    0.4970    0.4244    0.2192    0.0000

>> plot(x,y8)
>> xlabel('\theta (degrees)');
>> ylabel('\eta_{x,xy}');

>> Etayxy1 = Etayxy(S1)

Etayxy1 =

  1.1813e-016

>> Etayxy2 = Etayxy(S2)

Etayxy2 =

    0.3457

>> Etayxy3 = Etayxy(S3)

Etayxy3 =

    0.7457
```

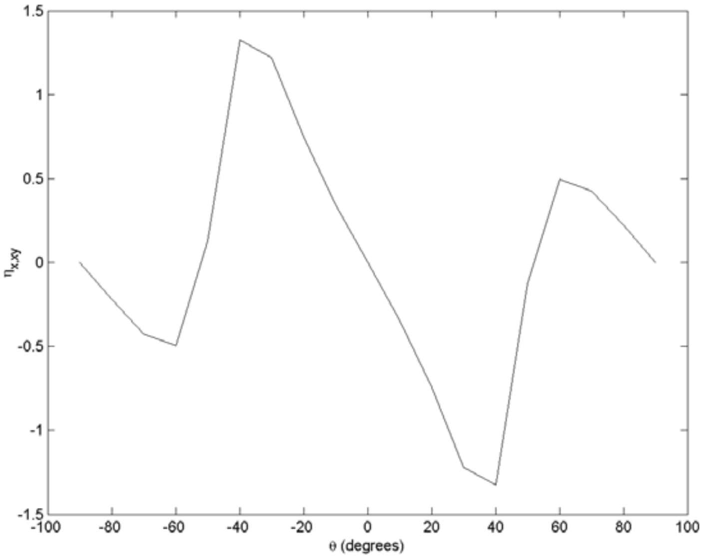

Fig. Variation of $\eta_{x,xy}$ versus θ for Problem 6.8

```
>> Etayxy4 = Etayxy(S4)

Etayxy4 =

    1.2187

>> Etayxy5 = Etayxy(S5)

Etayxy5 =

    1.3271

>> Etayxy6 = Etayxy(S6)

Etayxy6 =

    0.1268

>> Etayxy7 = Etayxy(S7)

Etayxy7 =

   -0.4970
```

```
>> Etayxy8 = Etayxy(S8)

Etayxy8 =

   -0.4244

>> Etayxy9 = Etayxy(S9)

Etayxy9 =

   -0.2192

>> Etayxy10 = Etayxy(S10)

Etayxy10 =

     0

>> Etayxy11 = Etayxy(S11)

Etayxy11 =

    0.2192

>> Etayxy12 = Etayxy(S12)

Etayxy12 =

    0.4244

>> Etayxy13 = Etayxy(S13)

Etayxy13 =

    0.4970

>> Etayxy14 = Etayxy(S14)

Etayxy14 =

   -0.1268

>> Etayxy15 = Etayxy(S15)

Etayxy15 =

   -1.3271
```

```
>> Etayxy16 = Etayxy(S16)

Etayxy16 =

   -1.2187

>> Etayxy17 = Etayxy(S17)

Etayxy17 =

   -0.7457

>> Etayxy18 = Etayxy(S18)

Etayxy18 =

   -0.3457

>> Etayxy19 = Etayxy(S19)

Etayxy19 =

 -1.1813e-016

>> y9 = [Etayxy1 Etayxy2 Etayxy3 Etayxy4 Etayxy5 Etayxy6 Etayxy7
        Etayxy8 Etayxy9 Etayxy10 Etayxy11 Etayxy12 Etayxy13 Etayxy14
        Etayxy15 Etayxy16 Etayxy17 Etayxy18 Etayxy19]

y9 =

  Columns 1 through 14

     0.0000    0.3457    0.7457    1.2187    1.3271    0.1268
    -0.4970   -0.4244   -0.2192         0    0.2192    0.4244
     0.4970   -0.1268

  Columns 15 through 19

    -1.3271   -1.2187   -0.7457   -0.3457   -0.0000

>> plot(x,y9)
>> xlabel('\theta {degrees}');
>> ylabel('\eta_{y,xy}');
```

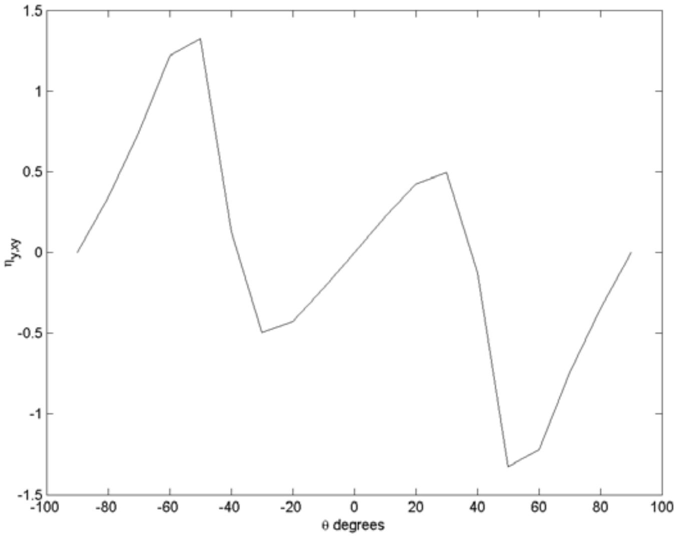

Fig. Variation of $\eta_{y,xy}$ versus θ for Problem 6.8

Problem 6.9

```
>> S = ReducedCompliance(50.0, 15.20, 0.254, 4.70)

S =

    0.0200   -0.0051         0
   -0.0051    0.0658         0
         0         0    0.2128

>> S1 = Sbar(S, -90)

S1 =

    0.0658   -0.0051   -0.0000
   -0.0051    0.0200    0.0000
   -0.0000    0.0000    0.2128
>> S2 = Sbar(S, -80)

S2 =

    0.0740   -0.0147   -0.0451
   -0.0147    0.0310    0.0608
   -0.0226    0.0304    0.1935
```

```
>> S3 = Sbar(S, -70)

S3 =

    0.0945   -0.0391   -0.0664
   -0.0391    0.0594    0.0959
   -0.0332    0.0479    0.1447

>> S4 = Sbar(S, -60)

S4 =

    0.1161   -0.0669   -0.0515
   -0.0669    0.0932    0.0912
   -0.0258    0.0456    0.0892

>> S5 = Sbar(S, -50)

S5 =

    0.1268   -0.0850   -0.0056
   -0.0850    0.1188    0.0507
   -0.0028    0.0254    0.0529

>> S6 = Sbar(S, -40)

S6 =

    0.1188   -0.0850    0.0507
   -0.0850    0.1268   -0.0056
    0.0254   -0.0028    0.0529

>> S7 = Sbar(S, -30)

S7 =

    0.0932   -0.0669    0.0912
   -0.0669    0.1161   -0.0515
    0.0456   -0.0258    0.0892

>> S8 = Sbar(S, -20)

S8 =

    0.0594   -0.0391    0.0959
   -0.0391    0.0945   -0.0664
    0.0479   -0.0332    0.1447
>> S9 = Sbar(S, -10)
```

```
S9 =

    0.0310   -0.0147    0.0608
   -0.0147    0.0740   -0.0451
    0.0304   -0.0226    0.1935

>> S10 = Sbar(S, 0)

S10 =

    0.0200   -0.0051         0
   -0.0051    0.0658         0
         0         0    0.2128

>> S11 = Sbar(S, 10)

S11 =

    0.0310   -0.0147   -0.0608
   -0.0147    0.0740    0.0451
   -0.0304    0.0226    0.1935

>> S12 = Sbar(S, 20)

S12 =

    0.0594   -0.0391   -0.0959
   -0.0391    0.0945    0.0664
   -0.0479    0.0332    0.1447

>> S13 = Sbar(S, 30)

S13 =

    0.0932   -0.0669   -0.0912
   -0.0669    0.1161    0.0515
   -0.0456    0.0258    0.0892

>> S14 = Sbar(S, 40)

S14 =

    0.1188   -0.0850   -0.0507
   -0.0850    0.1268    0.0056
   -0.0254    0.0028    0.0529
```

```
>> S15 = Sbar(S, 50)

S15 =

    0.1268   -0.0850    0.0056
   -0.0850    0.1188   -0.0507
    0.0028   -0.0254    0.0529

>> S16 = Sbar(S, 60)

S16 =

    0.1161   -0.0669    0.0515
   -0.0669    0.0932   -0.0912
    0.0258   -0.0456    0.0892

>> S17 = Sbar(S, 70)

S17 =

    0.0945   -0.0391    0.0664
   -0.0391    0.0594   -0.0959
    0.0332   -0.0479    0.1447

>> S18 = Sbar(S, 80)

S18 =

    0.0740   -0.0147    0.0451
   -0.0147    0.0310   -0.0608
    0.0226   -0.0304    0.1935

>> S19 = Sbar(S, 90)

S19 =

    0.0658   -0.0051    0.0000
   -0.0051    0.0200   -0.0000
    0.0000   -0.0000    0.2128

>> Etaxyx1 = Etaxyx(S1)

Etaxyx1 =

 -2.6414e-016

>> Etaxyx2 = Etaxyx(S2)
```

```
Etaxyx2 =

   -0.6095

>> Etaxyx3 = Etaxyx(S3)

Etaxyx3 =

   -0.7031

>> Etaxyx4 = Etaxyx(S4)

Etaxyx4 =

   -0.4437

>> Etaxyx5 = Etaxyx(S5)

Etaxyx5 =

   -0.0444

>> Etaxyx6 = Etaxyx(S6)

Etaxyx6 =

    0.4269

>> Etaxyx7 = Etaxyx(S7)

Etaxyx7 =

    0.9779

>> Etaxyx8 = Etaxyx(S8)

Etaxyx8 =

    1.6138

>> Etaxyx9 = Etaxyx(S9)

Etaxyx9 =

    1.9599

>> Etaxyx10 = Etaxyx(S10)
```

```
Etaxyx10 =

     0

>> Etaxyx11 = Etaxyx(S11)

Etaxyx11 =

   -1.9599

>> Etaxyx12 = Etaxyx(S12)

Etaxyx12 =

   -1.6138

>> Etaxyx13 = Etaxyx(S13)

Etaxyx13 =

   -0.9779

>> Etaxyx14 = Etaxyx(S14)

Etaxyx14 =

   -0.4269

>> Etaxyx15 = Etaxyx(S15)

Etaxyx15 =

    0.0444

>> Etaxyx16 = Etaxyx(S16)

Etaxyx16 =

    0.4437

>> Etaxyx17 = Etaxyx(S17)

Etaxyx17 =

    0.7031
```

```
>> Etaxyx18 = Etaxyx(S18)

Etaxyx18 =

    0.6095

>> Etaxyx19 = Etaxyx(S19)

Etaxyx19 =

  2.6414e-016

>> x = [-90 -80 -70 -60 -50 -40 -30 -20 -10 0 10 20 30 40 50 60 70
       80 90]

x =

   -90   -80   -70   -60   -50   -40   -30   -20   -10     0    10
   20    30    40    50    60    70    80    90

>> y1 = [Etaxyx1 Etaxyx2 Etaxyx3 Etaxyx4 Etaxyx5 Etaxyx6 Etaxyx7
        Etaxyx8 Etaxyx9 Etaxyx10 Etaxyx11 Etaxyx12 Etaxyx13 Etaxyx14
        Etaxyx15 Etaxyx16 Etaxyx17 Etaxyx18 Etaxyx19]

y1 =

  Columns 1 through 14

    -0.0000   -0.6095   -0.7031   -0.4437   -0.0444    0.4269
     0.9779   1.6138    1.9599   0   -1.9599   -1.6138
    -0.9779   -0.4269

  Columns 15 through 19

     0.0444    0.4437    0.7031    0.6095    0.0000

>> plot(x,y1)
>> xlabel('\theta {degrees}');
>> ylabel('\eta_{xy,x}');
```

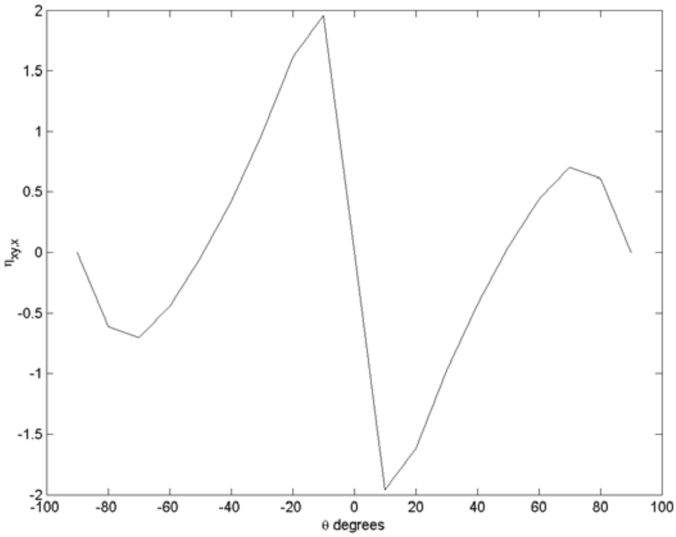

Fig. Variation of $\eta_{xy,\,x}$ versus θ for Problem 6.9

```
>> Etaxyy1 = Etaxyy(S1)

Etaxyy1 =

  1.1492e-015

>> Etaxyy2 = Etaxyy(S2)

Etaxyy2 =

    1.9599

>> Etaxyy3 = Etaxyy(S3)

Etaxyy3 =

    1.6138

>> Etaxyy4 = Etaxyy(S4)

Etaxyy4 =

    0.9779
```

```
>> Etaxyy5 = Etaxyy(S5)

Etaxyy5 =

    0.4269

>> Etaxyy6 = Etaxyy(S6)

Etaxyy6 =

   -0.0444

>> Etaxyy7 = Etaxyy(S7)

Etaxyy7 =

   -0.4437

>> Etaxyy8 = Etaxyy(S8)

Etaxyy8 =

   -0.7031

>> Etaxyy9 = Etaxyy(S9)

Etaxyy9 =

   -0.6095

>> Etaxyy10 = Etaxyy(S10)

Etaxyy10 =

     0

>> Etaxyy11 = Etaxyy(S11)

Etaxyy11 =

    0.6095

>> Etaxyy12 = Etaxyy(S12)

Etaxyy12 =

    0.7031
```

```
>> Etaxyy13 = Etaxyy(S13)

Etaxyy13 =

    0.4437

>> Etaxyy14 = Etaxyy(S14)

Etaxyy14 =

    0.0444

>> Etaxyy15 = Etaxyy(S15)


Etaxyy15 =

   -0.4269

>> Etaxyy16 = Etaxyy(S16)

Etaxyy16 =

   -0.9779

>> Etaxyy17 = Etaxyy(S17)

Etaxyy17 =

   -1.6138

>> Etaxyy18 = Etaxyy(S18)

Etaxyy18 =

   -1.9599

>> Etaxyy19 = Etaxyy(S19)

Etaxyy19 =

 -1.1492e-015

>> y2 = [Etaxyy1 Etaxyy2 Etaxyy3 Etaxyy4 Etaxyy5 Etaxyy6 Etaxyy7
        Etaxyy8 Etaxyy9 Etaxyy10 Etaxyy11 Etaxyy12 Etaxyy13 Etaxyy14
        Etaxyy15 Etaxyy16 Etaxyy17 Etaxyy18 Etaxyy19]
```

```
y2 =

  Columns 1 through 14

     0.0000     1.9599     1.6138     0.9779     0.4269    -0.0444
    -0.4437    -0.7031    -0.6095    0     0.6095     0.7031
     0.4437     0.0444

  Columns 15 through 19

    -0.4269    -0.9779    -1.6138    -1.9599    -0.0000

>> plot(x,y2)
>> xlabel('\theta {degrees}');
>> ylabel('\eta_{xy,y}');
```

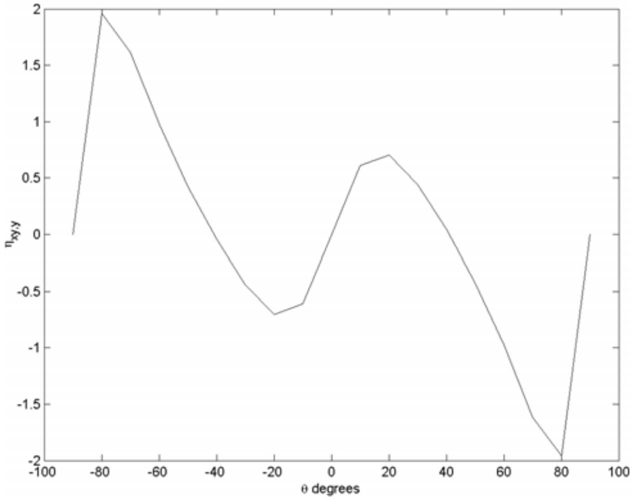

Fig. Variation of $\eta_{xy,\,y}$ versus θ for Problem 6.9

Problem 6.10

Continuing with the commands from Problem 6.9, we obtain:

```
>> Etaxxy1 = Etaxxy(S1)

Etaxxy1 =

 -8.1673e-017
```

```
>> Etaxxy2 = Etaxxy(S2)

Etaxxy2 =

   -0.2333

>> Etaxxy3 = Etaxxy(S3)

Etaxxy3 =

   -0.4591

>> Etaxxy4 = Etaxxy(S4)

Etaxxy4 =

   -0.5779

>> Etaxxy5 = Etaxxy(S5)

Etaxxy5 =

   -0.1064

>> Etaxxy6 = Etaxxy(S6)

Etaxxy6 =

    0.9581

>> Etaxxy7 = Etaxxy(S7)

Etaxxy7 =

    1.0226

>> Etaxxy8 = Etaxxy(S8)

Etaxxy8 =

    0.6626

>> Etaxxy9 = Etaxxy(S9)

Etaxxy9 =

    0.3142
```

```
>> Etaxxy10 = Etaxxy(S10)

Etaxxy10 =

     0

>> Etaxxy11 = Etaxxy(S11)

Etaxxy11 =

   -0.3142

>> Etaxxy12 = Etaxxy(S12)

Etaxxy12 =

   -0.6626

>> Etaxxy13 = Etaxxy(S13)

Etaxxy13 =

   -1.0226

>> Etaxxy14 = Etaxxy(S14)

Etaxxy14 =

   -0.9581

>> Etaxxy15 = Etaxxy(S15)

Etaxxy15 =

    0.1064

>> Etaxxy16 = Etaxxy(S16)

Etaxxy16 =

    0.5779

>> Etaxxy17 = Etaxxy(S17)

Etaxxy17 =

    0.4591
```

```
>> Etaxxy18 = Etaxxy(S18)

Etaxxy18 =

    0.2333

>> Etaxxy19 = Etaxxy(S19)

Etaxxy19 =

  8.1673e-017

>> y3 = [Etaxxy1 Etaxxy2 Etaxxy3 Etaxxy4 Etaxxy5 Etaxxy6 Etaxxy7
        Etaxxy8 Etaxxy9 Etaxxy10 Etaxxy11 Etaxxy12 Etaxxy13 Etaxxy14
        Etaxxy15 Etaxxy16 Etaxxy17 Etaxxy18 Etaxxy19]

y3 =

  Columns 1 through 14

    -0.0000   -0.2333   -0.4591   -0.5779   -0.1064    0.9581
     1.0226    0.6626    0.3142         0   -0.3142   -0.6626
    -1.0226   -0.9581

  Columns 15 through 19

     0.1064    0.5779    0.4591    0.2333    0.0000

>> plot(x,y3)
>> xlabel('\theta {degrees}');
>> ylabel('\eta_{x,xy}');

>> Etayxy1 = Etayxy(S1)

Etayxy1 =

  1.0803e-016

>> Etayxy2 = Etayxy(S2)

Etayxy2 =

    0.3142

>> Etayxy3 = Etayxy(S3)

Etayxy3 =

    0.6626
```

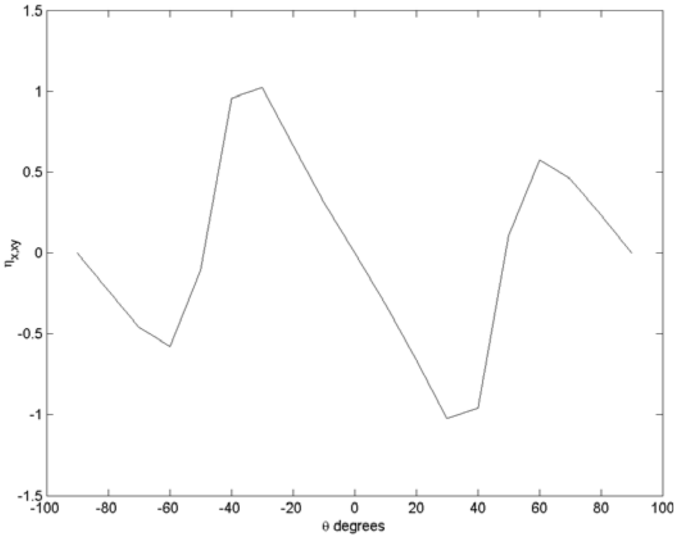

Fig. Variation of $\eta_{x,\,xy}$ versus θ for Problem 6.10

```
>> Etayxy4 = Etayxy(S4)

Etayxy4 =

    1.0226

>> Etayxy5 = Etayxy(S5)

Etayxy5 =

    0.9581

>> Etayxy6 = Etayxy(S6)

Etayxy6 =

   -0.1064

>> Etayxy7 = Etayxy(S7)

Etayxy7 =

   -0.5779
```

```
>> Etayxy8 = Etayxy(S8)

Etayxy8 =

   -0.4591

>> Etayxy9 = Etayxy(S9)

Etayxy9 =

   -0.2333

>> Etayxy10 = Etayxy(S10)

Etayxy10 =

     0

>> Etayxy11 = Etayxy(S11)

Etayxy11 =

    0.2333

>> Etayxy12 = Etayxy(S12)

Etayxy12 =

    0.4591

>> Etayxy13 = Etayxy(S13)

Etayxy13 =

    0.5779

>> Etayxy14 = Etayxy(S14)

Etayxy14 =

    0.1064

>> Etayxy15 = Etayxy(S15)

Etayxy15 =

   -0.9581
```

```
>> Etayxy16 = Etayxy(S16)

Etayxy16 =

   -1.0226

>> Etayxy17 = Etayxy(S17)

Etayxy17 =

   -0.6626

>> Etayxy18 = Etayxy(S18)

Etayxy18 =

   -0.3142

>> Etayxy19 = Etayxy(S19)

Etayxy19 =

   -1.0803e-016

>> y4 = [Etayxy1 Etayxy2 Etayxy3 Etayxy4 Etayxy5 Etayxy6 Etayxy7
        Etayxy8 Etayxy9 Etayxy10 Etayxy11 Etayxy12 Etayxy13 Etayxy14
        Etayxy15 Etayxy16 Etayxy17 Etayxy18 Etayxy19]

y4 =

  Columns 1 through 14

     0.0000    0.3142    0.6626    1.0226    0.9581   -0.1064
    -0.5779   -0.4591   -0.2333         0    0.2333    0.4591
     0.5779    0.1064

  Columns 15 through 19

    -0.9581   -1.0226   -0.6626   -0.3142   -0.0000

>> plot(x,y4)
>> xlabel('\theta {degrees}');
>> ylabel('\eta_{y,xy}');
```

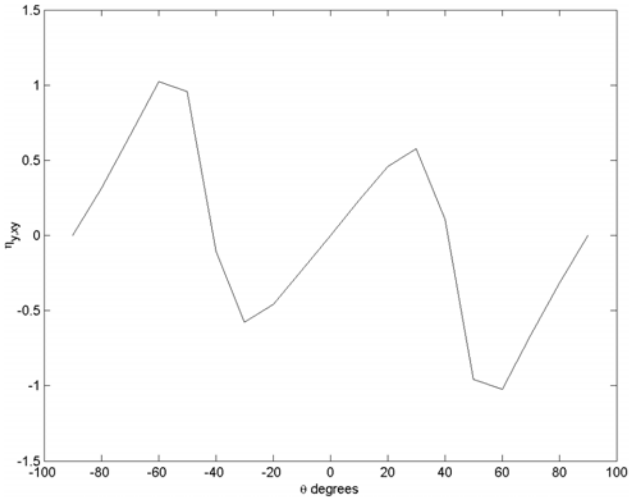

Fig. Variation of $\eta_{y,\,xy}$ versus θ for Problem 6.10

Problem 7.1

```
EDU>> epsilon1 = Strains(500e-6,0,0,0,0,0,-0.300)

epsilon1 =

  1.0e-003 *

    0.5000
         0
         0

EDU>> epsilon2 = Strains(500e-6,0,0,0,0,0,-0.150)

epsilon2 =

  1.0e-003 *

    0.5000
         0
         0

EDU>> epsilon3 = Strains(500e-6,0,0,0,0,0,0)

epsilon3 =

  1.0e-003 *
```

```
    0.5000
         0
         0

EDU>> epsilon4 = Strains(500e-6,0,0,0,0,0,0.150)

epsilon4 =

  1.0e-003 *

    0.5000
         0
         0

EDU>> epsilon5 = Strains(500e-6,0,0,0,0,0,0.300)

epsilon5 =

  1.0e-003 *

    0.5000
         0
         0

EDU>> Q = ReducedStiffness(50.0, 15.2, 0.254, 4.70)

Q =

   51.0003    3.9380         0
    3.9380   15.5041         0
         0         0    4.7000

EDU>> Qbar1 = Qbar(Q,0)

Qbar1 =

   51.0003    3.9380         0
    3.9380   15.5041         0
         0         0    4.7000

EDU>> Qbar2 = Qbar(Q,90)

Qbar2 =

   15.5041    3.9380   -0.0000
    3.9380   51.0003    0.0000
   -0.0000    0.0000    4.7000
```

```
EDU>> Qbar3 = Qbar(Q,90)

Qbar3 =

   15.5041    3.9380   -0.0000
    3.9380   51.0003    0.0000
   -0.0000    0.0000    4.7000

EDU>> Qbar4 = Qbar(Q,0)

Qbar4 =

   51.0003    3.9380         0
    3.9380   15.5041         0
         0         0    4.7000

EDU>> sigma1a = Qbar1*epsilon1*1e3

sigma1a =

   25.5001
    1.9690
         0

EDU>> sigma1b = Qbar1*epsilon2*1e3

sigma1b =

   25.5001
    1.9690
         0

EDU>> sigma2a = Qbar2*epsilon2*1e3

sigma2a =

    7.7520
    1.9690
   -0.0000

EDU>> sigma2b = Qbar2*epsilon3*1e3

sigma2b =

    7.7520
    1.9690
   -0.0000
```

```
EDU>> sigma3a = Qbar3*epsilon3*1e3

sigma3a =

    7.7520
    1.9690
   -0.0000

EDU>> sigma3b = Qbar3*epsilon4*1e3

sigma3b =

    7.7520
    1.9690
   -0.0000


EDU>> sigma4a = Qbar4*epsilon4*1e3

sigma4a =

   25.5001
    1.9690
         0

EDU>> sigma4b = Qbar4*epsilon5*1e3

sigma4b =

   25.5001
    1.9690
         0

EDU>> y = [0.300 0.150 0.150 0 0 -0.150 -0.150 -0.300]

y =

    0.3000    0.1500    0.1500    0    0   -0.1500
   -0.1500   -0.3000

EDU>> x = [sigma4b(1) sigma4a(1) sigma3b(1) sigma3a(1) sigma2b(1)
          sigma2a(1) sigma1b(1) sigma1a(1)]

x =

   25.5001   25.5001    7.7520    7.7520    7.7520    7.7520
   25.5001   25.5001
```

```
EDU>> plot(x,y)
EDU>> xlabel('\sigma_x (MPa)')
EDU>> ylabel('z (mm)')
```

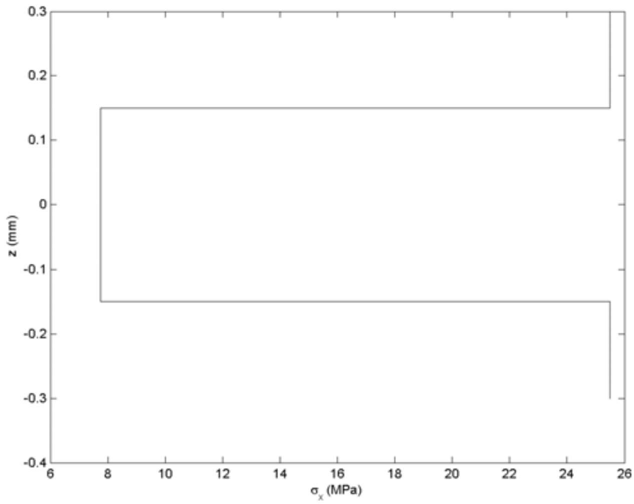

Fig. Variation of σ_x versus z for Problem 7.1

```
EDU>> x = [sigma4b(2) sigma4a(2) sigma3b(2) sigma3a(2) sigma2b(2)
          sigma2a(2) sigma1b(2) sigma1a(2)]

x =

    1.9690    1.9690    1.9690    1.9690    1.9690    1.9690
    1.9690    1.9690

EDU>> plot(x,y)
EDU>> ylabel('z (mm)')
EDU>> xlabel('\sigma_y (MPa)')

EDU>> x = [sigma4b(3) sigma4a(3) sigma3b(3) sigma3a(3) sigma2b(3)
          sigma2a(3) sigma1b(3) sigma1a(3)]

x =

  1.0e-015 *

         0    0    -0.2102    -0.2102    -0.2102    -0.2102
         0    0
```

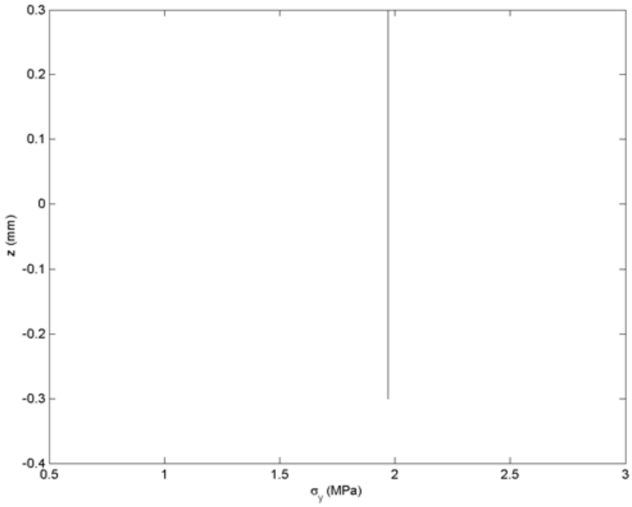

Fig. Variation of σ_y versus z for Problem 7.1

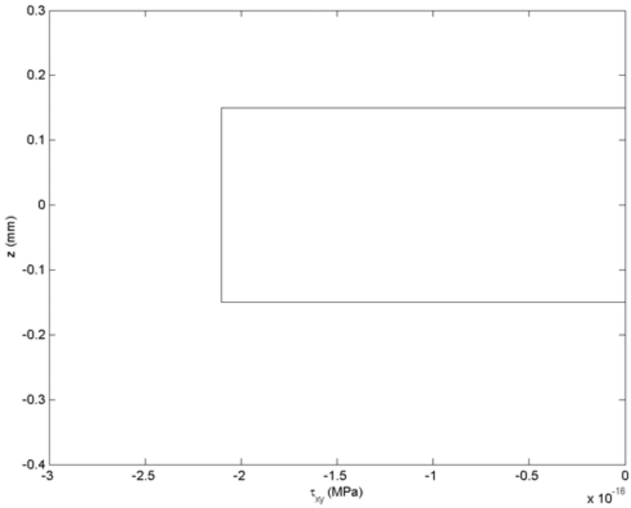

Fig. Variation of τ_{xy} versus z for Problem 7.1

```
EDU>> plot(x,y)
EDU>> ylabel('z (mm)')
EDU>> xlabel('\tau_{xy} (MPa)')

EDU>> Nx = 0.150e-3 * (sigma1a(1) + sigma2a(1) + sigma3a(1) +
      sigma4a(1))
```

```
Nx =

    0.0100

EDU>> Ny = 0.150e-3 * (sigma1a(2) + sigma2a(2) + sigma3a(2) +
           sigma4a(2))

Ny =

    0.0012

EDU>> Nxy = 0.150e-3 * (sigma1a(3) + sigma2a(3) + sigma3a(3) +
            sigma4a(3))

Nxy =

 -6.3064e-020

EDU>> Mx = sigma1a(1)*((-0.150e-3)^2 - (0.300e-3)^2) + sigma2a(1)*(0 -
           (-0.150e-3)^2) + sigma3a(1)*((0.150e-3)^2 - 0) +
           sigma4a(1)*((0.300e-3)^2 - (0.150e-3)^2)
Mx =

     0

EDU>> My = sigma1a(2)*((-0.150e-3)^2 - (0.300e-3)^2) + sigma2a(2)*(0
           - (-0.150e-3)^2) + sigma3a(2)*((0.150e-3)^2 - 0) +
           sigma4a(2)*((0.300e-3)^2 - (0.150e-3)^2)

My =

     0

EDU>> Mxy = sigma1a(3)*((-0.150e-3)^2 - (0.300e-3)^2) + sigma2a(3)*(0
            - (-0.150e-3)^2) + sigma3a(3)*((0.150e-3)^2 - 0) +
            sigma4a(3)*((0.300e-3)^2 - (0.150e-3)^2)

Mxy =

     0

EDU>> T1 = T(0)

T1 =

     1     0     0
     0     1     0
     0     0     1
```

```
EDU>> T2 = T(90)

T2 =

    0.0000    1.0000    0.0000
    1.0000    0.0000   -0.0000
   -0.0000    0.0000   -1.0000

EDU>> T3 = T(90)

T3 =

    0.0000    1.0000    0.0000
    1.0000    0.0000   -0.0000
   -0.0000    0.0000   -1.0000

EDU>> T4 = T(0)

T4 =

     1     0     0
     0     1     0
     0     0     1

EDU>> eps1a = T1*epsilon1

eps1a =

  1.0e-003 *

    0.5000
         0
         0

EDU>> eps1b = T1*epsilon2

eps1b =

  1.0e-003 *

    0.5000
         0
         0

EDU>> eps2a = T2*epsilon2
```

```
eps2a =

  1.0e-003 *

    0.0000
    0.5000
   -0.0000

EDU>> eps2b = T2*epsilon3

eps2b =

  1.0e-003 *

    0.0000
    0.5000
   -0.0000

EDU>> eps3a = T3*epsilon3

eps3a =

  1.0e-003 *

    0.0000
    0.5000
   -0.0000

EDU>> eps3b = T3*epsilon4

eps3b =

  1.0e-003 *

    0.0000
    0.5000
   -0.0000

EDU>> eps4a = T4*epsilon4

eps4a =

  1.0e-003 *

    0.5000
         0
         0
```

```
EDU>> eps4b = T4*epsilon5

eps4b =

  1.0e-003 *

    0.5000
         0
         0

EDU>> sig1 = T1*sigma1a

sig1 =

   25.5001
    1.9690
         0

EDU>> sig2 = T2*sigma2a

sig2 =

    1.9690
    7.7520
   -0.0000

EDU>> sig3 = T3*sigma3a

sig3 =

    1.9690
    7.7520
   -0.0000

EDU>> sig4 = T4*sigma4a

sig4 =

   25.5001
    1.9690
         0
```

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Contents of the Accompanying CD-ROM

The accompanying CD-ROM includes two folders as follows:

1. *M-Files.* This folder includes the 44 MATLAB functions written specifically to be used with this book. In order to use them they should be copied to the working directory in your MATLAB folder on the hard disk or you can set the MATLAB path to the correct folder that includes these files.
2. Solutions to most of the problems in the book. Specifically, detailed solutions are included to all the problem of the first six chapters.

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