2

Linear Elastic Stress-Strain Relations

2.1 Basic Equations

Consider a single layer of fiber-reinforced composite material as shown in Fig. 2.1. In this layer, the 1-2-3 orthogonal coordinate system is used where the directions are taken as follows:

1. The 1-axis is aligned with the fiber direction.
2. The 2-axis is in the plane of the layer and perpendicular to the fibers.
3. The 3-axis is perpendicular to the plane of the layer and thus also perpendicular to the fibers.

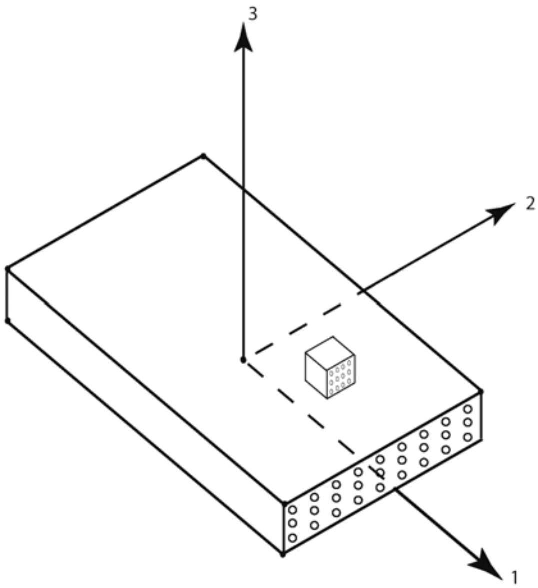

Fig. 2.1. A lamina illustrating the principle material coordinate system

The 1-direction is also called the *fiber direction*, while the 2- and 3-directions are called the *matrix directions* or the *transverse directions.* This 1-2-3 coordinate system is called the *principal material coordinate system.* The stresses and strains in the layer (also called a lamina) will be referred to the principal material coordinate system.

At this level of analysis, the strain or stress of an individual fiber or an element of matrix is not considered. The effect of the fiber reinforcement is smeared over the volume of the material. We assume that the two-material fiber-matrix system is replaced by a single homogeneous material. Obviously, this single material does not have the same properties in all directions. Such material with different properties in three mutually perpendicular directions is called an *orthotropic* material. Therefore, the layer (lamina) is considered to be orthotropic.

The stresses on a small infinitesimal element taken from the layer are illustrated in Fig. 2.2. There are three normal stresses σ_1, σ_2, and σ_3, and three shear stresses τ_{12}, τ_{23}, and τ_{13}. These stresses are related to the strains ε_1, ε_2, ε_3, γ_{12}, γ_{23}, and γ_{13} as follows (see [1]):

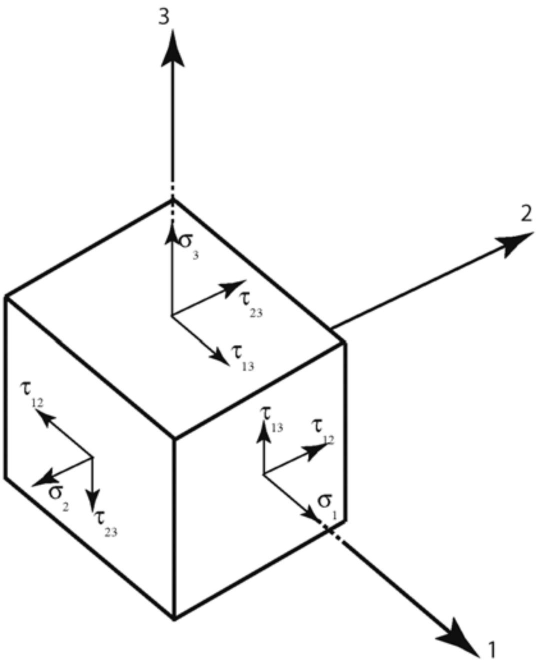

Fig. 2.2. An infinitesimal fiber-reinforced element showing the stresses

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} 1/E_1 & -\nu_{21}/E_2 & -\nu_{31}/E_3 & 0 & 0 & 0 \\ -\nu_{12}/E_1 & 1/E_2 & -\nu_{32}/E_3 & 0 & 0 & 0 \\ -\nu_{13}/E_1 & -\nu_{23}/E_2 & 1/E_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{23} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{12} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{Bmatrix} \tag{2.1}$$

In (2.1), E_1, E_2, and E_3 are the extensional moduli of elasticity along the 1, 2, and 3 directions, respectively. Also, ν_{ij} $(i, j = 1, 2, 3)$ are the different Poisson's ratios, while G_{12}, G_{23}, and G_{13} are the three shear moduli.

Equation (2.1) can be written in a compact form as follows:

$$\{\varepsilon\} = [S]\,\{\sigma\} \tag{2.2}$$

where $\{\varepsilon\}$ and $\{\sigma\}$ represent the 6×1 strain and stress vectors, respectively, and $[S]$ is called the *compliance matrix*. The elements of $[S]$ are clearly obtained from (2.1), i.e. $S_{11} = 1/E_1$, $S_{12} = -\nu_{21}/E_2, \ldots, S_{66} = 1/G_{12}$.

The inverse of the compliance matrix $[S]$ is called the stiffness matrix $[C]$ given, in general, as follows:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{31} & C_{32} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix} \tag{2.3}$$

In compact form (2.3) is written as follows:

$$\{\sigma\} = [C]\,\{\varepsilon\} \tag{2.4}$$

The elements of $[C]$ are not shown here explicitly but are calculated using the MATLAB function *OrthotropicStiffness* which is written specifically for this purpose.

It is shown (see [1]) that both the compliance matrix and the stiffness matrix are symmetric, i.e. $C_{21} = C_{12}$, $C_{23} = C_{32}$, $C_{13} = C_{31}$, and similarly for S_{21}, S_{23}, and S_{13}. Therefore, the following expressions can now be easily obtained:

$$\begin{aligned} C_{11} &= \frac{1}{S}(S_{22}S_{33} - S_{23}S_{23}) \\ C_{12} &= \frac{1}{S}(S_{13}S_{23} - S_{12}S_{33}) \\ C_{22} &= \frac{1}{S}(S_{33}S_{11} - S_{13}S_{13}) \\ C_{13} &= \frac{1}{S}(S_{12}S_{23} - S_{13}S_{22}) \end{aligned}$$

$$
\begin{aligned}
C_{33} &= \frac{1}{S}(S_{11}S_{22} - S_{12}S_{12}) \\
C_{23} &= \frac{1}{S}(S_{12}S_{13} - S_{23}S_{11}) \\
C_{44} &= \frac{1}{S_{44}} \\
C_{55} &= \frac{1}{S_{55}} \\
C_{66} &= \frac{1}{S_{66}} \\
S &= S_{11}S_{22}S_{33} - S_{11}S_{23}S_{23} - S_{22}S_{13}S_{13} - S_{33}S_{12}S_{12} + 2S_{12}S_{23}S_{13}
\end{aligned}
\tag{2.5}
$$

It should be noted that the material constants appearing in the compliance matrix in (2.1) are not all independent. This is clear since the compliance matrix is symmetric. Therefore, we have the following equations relating the material constants:

$$
\begin{aligned}
\frac{\nu_{12}}{E_1} &= \frac{\nu_{21}}{E_2} \\
\frac{\nu_{13}}{E_1} &= \frac{\nu_{31}}{E_3} \\
\frac{\nu_{23}}{E_2} &= \frac{\nu_{32}}{E_3}
\end{aligned}
\tag{2.6}
$$

The above equations are called the *reciprocity relations* for the material constants. It should be noted that the reciprocity relations can be derived irrespective of the symmetry of the compliance matrix – in fact we conclude that the compliance matrix is symmetric from using these relations. Thus it is now clear that there are nine independent material constants for an orthotropic material.

A material is called *transversely isotropic* if its behavior in the 2-direction is identical to its behavior in the 3-direction. For this case, $E_2 = E_3$, $\nu_{12} = \nu_{13}$, and $G_{12} = G_{13}$. In addition, we have the following relation:

$$
G_{23} = \frac{E_2}{2(1+\nu_{23})} \tag{2.7}
$$

It is clear that there are only five independent material constants (E_1, E_2, ν_{12}, ν_{23}, G_{12}) for a transversely isotropic material.

A material is called *isotropic* if its behavior is the same in all three 1-2-3 directions. In this case, $E_1 = E_2 = E_3 = E$, $\nu_{12} = \nu_{23} = \nu_{13} = \nu$, and $G_{12} = G_{23} = G_{13} = G$. In addition, we have the following relation:

$$
G = \frac{E}{2(1+\nu)} \tag{2.8}
$$

It is clear that there are only two independent material constants (E, ν) for an isotropic material.

At the other end of the spectrum, we have *anisotropic* materials – these materials have nonzero entries at the upper right and lower left portions of their compliance and stiffness matrices.

2.2 MATLAB Functions Used

The six MATLAB functions used in this chapter to calculate compliance and stiffness matrices are:

OrthotropicCompliance(E1, E2, E3, NU12, NU23, NU13, G12, G23, G13) – This function calculates the 6×6 compliance matrix for orthotropic materials. Its input are the nine independent material constants E_1, E_2, E_3, ν_{12}, ν_{23}, ν_{13}, G_{12}, G_{23}, and G_{13}.

OrthotropicStiffness(E1, E2, E3, NU12, NU23, NU13, G12, G23, G13) – This function calculates the 6×6 stiffness matrix for orthotropic materials. Its input are the nine independent material constants E_1, E_2, E_3, ν_{12}, ν_{23}, ν_{13}, G_{12}, G_{23}, and G_{13}.

TransverselyIsotropicCompliance(E1, E2, NU12, NU23, G12) – This function calculates the 6×6 compliance matrix for transversely isotropic materials. Its input are the five independent material constants E_1, E_2, ν_{12}, ν_{23}, and G_{12}.

TransverselyIsotropicStiffness(E1, E2, NU12, NU23, G12) – This function calculates the 6×6 stiffness matrix for transversely isotropic materials. Its input are the five independent material constants E_1, E_2, ν_{12}, ν_{23}, and G_{12}.

IsotropicCompliance(E, NU) – This function calculates the 6×6 compliance matrix for isotropic materials. Its input are the two independent material constants E and ν.

IsotropicStiffness(E, NU) – This function calculates the 6×6 stiffness matrix for isotropic materials. Its input are the two independent material constants E and ν.

The following is a listing of the MATLAB source code for each function:

```
function y = OrthotropicCompliance(E1,E2,E3,NU12,NU23,NU13,G12,G23,G13)
%OrthotropicCompliance   This function returns the compliance matrix
%                        for orthotropic materials. There are nine
%                        arguments representing the nine independent
%                        material constants. The size of the compliance
%                        matrix is 6 x 6.
y = [1/E1 -NU12/E1 -NU13/E1 0 0 0 ; -NU12/E1 1/E2 -NU23/E2 0 0 0 ;
    -NU13/E1 -NU23/E2 1/E3 0 0 0 ; 0 0 0 1/G23 0 0 ; 0 0 0 0 1/G13 0 ;
    0 0 0 0 0 1/G12];
```

```
function y = OrthotropicStiffness(E1,E2,E3,NU12,NU23,NU13,G12,G23,G13)
%OrthotropicStiffness   This function returns the stiffness matrix
%                       for orthotropic materials. There are nine
%                       arguments representing the nine independent
%                       material constants. The size of the stiffness
%                       matrix is 6 x 6.
x = [1/E1 -NU12/E1 -NU13/E1 0 0 0 ; -NU12/E1 1/E2 -NU23/E2 0 0 0 ;
    -NU13/E1 -NU23/E2 1/E3 0 0 0 ; 0 0 0 1/G23 0 0 ; 0 0 0 0 1/G13 0 ;
    0 0 0 0 0 1/G12];
y = inv(x);
```

```
function y = TransverselyIsotropicCompliance(E1,E2,NU12,NU23,G12)
%TransverselyIsotropicCompliance   This function returns the
%                                  compliance matrix for
%                                  transversely isotropic
%                                  materials. There are five
%                                  arguments representing the
%                                  five independent material
%                                  constants. The size of the
%                                  compliance matrix is 6 x 6.
y = [1/E1 -NU12/E1 -NU12/E1 0 0 0 ; -NU12/E1 1/E2 -NU23/E2 0 0 0 ;
    -NU12/E1 -NU23/E2 1/E2 0 0 0 ; 0 0 0 2*(1+NU23)/E2 0 0 ;
    0 0 0 0 1/G12 0 ; 0 0 0 0 0 1/G12];
```

```
function y = TransverselyIsotropicStiffness(E1,E2,NU12,NU23,G12)
%TransverselyIsotropicStiffness   This function returns the
%                                 stiffness matrix for
%                                 transversely isotropic
%                                 materials. There are five
%                                 arguments representing the
%                                 five independent material
%                                 constants. The size of the
%                                 stiffness matrix is 6 x 6.
x = [1/E1 -NU12/E1 -NU12/E1 0 0 0 ; -NU12/E1 1/E2 -NU23/E2 0 0 0 ;
   -NU12/E1 -NU23/E2 1/E2 0 0 0 ; 0 0 0 2*(1+NU23)/E2 0 0 ;
    0 0 0 0 1/G12 0 ; 0 0 0 0 0 1/G12];
y = inv(x);
```

```
function y = IsotropicCompliance(E,NU)
%IsotropicCompliance   This function returns the
%                      compliance matrix for isotropic
%                      materials. There are two
%                      arguments representing the
%                      two independent material
%                      constants. The size of the
%                      compliance matrix is 6 x 6.
y = [1/E -NU/E -NU/E 0 0 0 ; -NU/E 1/E -NU/E 0 0 0 ;
    -NU/E -NU/E 1/E 0 0 0 ; 0 0 0 2*(1+NU)/E 0 0 ;
    0 0 0 0 2*(1+NU)/E 0 ; 0 0 0 0 0 2*(1+NU)/E];
```

```
function y = IsotropicStiffness(E,NU)
%IsotropicStiffness   This function returns the
%                     stiffness matrix for isotropic
%                     materials. There are two
%                     arguments representing the
%                     two independent material
%                     constants. The size of the
%                     stiffness matrix is 6 x 6.
x = [1/E -NU/E -NU/E 0 0 0 ; -NU/E 1/E -NU/E 0 0 0 ;
    -NU/E -NU/E 1/E 0 0 0 ; 0 0 0 2*(1+NU)/E 0 0 ;
    0 0 0 0 2*(1+NU)/E 0 ; 0 0 0 0 0 2*(1+NU)/E];
y = inv(x);
```

Example 2.1

For an orthotropic material, derive expressions for the elements of the stiffness matrix C_{ij} directly in terms of the nine independent material constants.

Solution

Substitute the elements of $[S]$ from (2.1) into (2.5) along with using (2.6). This is illustrated in detail for C_{11} below. First evaluate the expression of S from (2.5) as follows:

$$
\begin{aligned}
S = \;& S_{11}S_{22}S_{33} - S_{11}S_{23}S_{23} - S_{22}S_{13}S_{13} - S_{33}S_{12}S_{12} + 2S_{12}S_{23}S_{13} \\
= \;& \frac{1}{E_1}\frac{1}{E_2}\frac{1}{E_3} - \frac{1}{E_1}\left(\frac{-\nu_{23}}{E_2}\right)\left(\frac{-\nu_{32}}{E_3}\right) \\
& - \frac{1}{E_2}\left(\frac{-\nu_{13}}{E_1}\right)\left(\frac{-\nu_{31}}{E_3}\right) - \frac{1}{E_3}\left(\frac{-\nu_{12}}{E_1}\right)\left(\frac{-\nu_{21}}{E_2}\right) \\
& + 2\left(\frac{-\nu_{12}}{E_1}\right)\left(\frac{-\nu_{23}}{E_2}\right)\left(\frac{-\nu_{31}}{E_3}\right) \\
= \;& \frac{1 - \nu_{23}\nu_{32} - \nu_{13}\nu_{31} - \nu_{12}\nu_{21} - 2\nu_{12}\nu_{23}\nu_{31}}{E_1E_2E_3} \\
= \;& \frac{1 - \nu_0}{E_1E_2E_3}
\end{aligned} \tag{2.9a}
$$

where ν_0 is given by

$$\nu_0 = \nu_{23}\nu_{32} + \nu_{13}\nu_{31} + \nu_{12}\nu_{21} + 2\nu_{12}\nu_{23}\nu_{31} \tag{2.9b}$$

Next, C_{11} is calculated as follows

$$C_{11} = \frac{1}{S}(S_{22}S_{33} - S_{23}S_{23})$$
$$= \frac{E_1E_2E_3}{1-\nu_0}\left[\frac{1}{E_2}\frac{1}{E_3} - \left(\frac{-\nu_{23}}{E_2}\right)\left(\frac{-\nu_{32}}{E_3}\right)\right]$$
$$= \frac{(1-\nu_{23}\nu_{32})\,E_1}{1-\nu_0} \tag{2.9c}$$

Similarly, the following expressions for the other elements of $[C]$ can be derived:

$$C_{12} = \frac{(\nu_{21}+\nu_{31}\nu_{23})\,E_1}{1-\nu_0} = \frac{(\nu_{12}+\nu_{32}\nu_{13})\,E_2}{1-\nu_0} \tag{2.9d}$$
$$C_{13} = \frac{(\nu_{31}+\nu_{21}\nu_{32})\,E_1}{1-\nu_0} = \frac{(\nu_{13}+\nu_{12}\nu_{23})\,E_3}{1-\nu_0} \tag{2.9e}$$
$$C_{22} = \frac{(1-\nu_{13}\nu_{31})\,E_2}{1-\nu_0} \tag{2.9f}$$
$$C_{23} = \frac{(\nu_{32}+\nu_{12}\nu_{31})\,E_2}{1-\nu_0} = \frac{(\nu_{23}+\nu_{21}\nu_{13})\,E_3}{1-\nu_0} \tag{2.9g}$$
$$C_{33} = \frac{(1-\nu_{12}\nu_{21})\,E_3}{1-\nu_0} \tag{2.9h}$$
$$C_{44} = G_{23} \tag{2.9i}$$
$$C_{55} = G_{13} \tag{2.9j}$$
$$C_{66} = G_{12} \tag{2.9k}$$

MATLAB Example 2.2

Consider a 60-mm cube made of graphite-reinforced polymer composite material that is subjected to a tensile force of 100 kN perpendicular to the fiber direction, directed along the 2-direction. The cube is free to expand or contract. Use MATLAB to determine the changes in the 60-mm dimensions of the cube. The material constants for graphite-reinforced polymer composite material are given as follows [1]:

$$E_1 = 155.0 \text{ GPa}, \qquad E_2 = E_3 = 12.10 \text{ GPa}$$
$$\nu_{23} = 0.458, \qquad \nu_{12} = \nu_{13} = 0.248$$
$$G_{23} = 3.20 \text{ GPa}, \qquad G_{12} = G_{13} = 4.40 \text{ GPa}$$

Solution

This example is solved using MATLAB. First, the normal stress in the 2-direction is calculated in GPa as follows:

```
>> sigma2 = 100/(60*60)

sigma2 =

    0.0278
```

The stress vector is set up next as follows:

```
>> sigma = [0  sigma2  0  0  0  0]

sigma =

    0  0.0278  0  0  0  0
```

The compliance matrix is then calculated using the MATLAB function *OrthotropicCompliance* as follows:

```
>> S = OrthotropicCompliance(155.0, 12.10, 12.10, 0.248, 0.458, 0.248,
4.40, 3.20, 4.40)

S =

    0.0065   -0.0016   -0.0016         0         0         0
   -0.0016    0.0826   -0.0379         0         0         0
   -0.0016   -0.0379    0.0826         0         0         0
         0         0         0    0.3125         0         0
         0         0         0         0    0.2273         0
         0         0         0         0         0    0.2273
```

The stress vector is adjusted to be a 6×1 column vector as follows:

```
>> sigma = sigma'

sigma =

         0
    0.0278
         0
         0
         0
         0
```

The strain vector is next obtained by applying (2.2) as follows:

```
>> epsilon = S*sigma

epsilon =

    -0.0000
     0.0023
    -0.0011
```

```
     0
     0
     0
```

Note that the strain in dimensionless. Note also that ε_{11} is very small but is not zero as it seems from the above result. To get the strain ε_{11} exactly, we need to use the `format` command to get more digits as follows:

```
>> format short e
>> epsilon

epsilon =

   -4.4444e-005
    2.2957e-003
   -1.0514e-003
              0
              0
              0
```

Finally, the change in length in each direction is calculated by multiplying the strain by the dimension in each direction as follows:

```
>> d1 = epsilon(1)*60

d1 =

   -2.6667e-003

>> d2 = epsilon(2)*60

d2 =

    1.3774e-001

>> d3 = epsilon(3)*60

d3 =

   -6.3085e-002
```

Notice that the change in the fiber direction is -2.6667×10^{-3} mm which is very small due to the fibers reducing the deformation in this direction. The minus sign indicates that there is a reduction in this dimension along the fibers. The change in the 2-direction is 0.13774 mm and is the largest change because the tensile force is along this direction. This change is positive indicating an extension in the dimension along this direction. Finally, the change in the 3-direction is -0.063085 mm. This change is minus since it indicates a reduction in the dimension along this direction.

Note that you can obtain online help from MATLAB on any of the MATLAB functions by using the `help` command. For example, to obtain help on the MATLAB function *OrthotropicCompliance*, use the `help` command as follows:

```
>> help OrthotropicCompliance

OrthotropicCompliance    This function returns the compliance matrix
                         for orthotropic materials. There are nine
                         arguments representing the nine independent
                         material constants. The size of the compliance
                         matrix is 6 x 6.
```

Note that we can use the MATLAB function *TransverselyIsotropicCompliance* instead of the MATLAB function *OrthotropicCompliance* in this example to obtain the same results. This is because the material constants for graphite-reinforced polymer composite material are the same in the 2- and 3-directions.

MATLAB Example 2.3

Repeat Example 2.2 if the cube is made of aluminum instead of graphite-reinforced polymer composite material. The material constants for aluminum are $E = 72.4\,\text{GPa}$ and $\nu = 0.300$. Use MATLAB.

Solution

This example is solved using MATLAB. First, the normal stress in the 2-direction is calculated in GPa as follows:

```
>> sigma2 = 100/(60*60)

sigma2 =

    0.0278
```

Next, the stress vector is setup directly as a column vector as follows:

```
>> sigma = [0 ; sigma2 ; 0 ; 0 ; 0 ; 0]

sigma =

         0
    0.0278
         0
         0
         0
         0
```

Since aluminum is an isotropic material, the compliance matrix for aluminum is calculated using the MATLAB function *IsotropicCompliance* as follows:

```
>> S = IsotropicCompliance(72.4, 0.3)

S =

    0.0138   -0.0041   -0.0041         0         0         0
   -0.0041    0.0138   -0.0041         0         0         0
   -0.0041   -0.0041    0.0138         0         0         0
         0         0         0    0.0359         0         0
         0         0         0         0    0.0359         0
         0         0         0         0         0    0.0359
```

Next, the strain vector is calculated using (2.2) as follows:

```
>> epsilon = S*sigma

epsilon =

1.0e-003 *

   -0.1151
    0.3837
   -0.1151
         0
         0
         0
```

Finally, the change in length in each direction is calculated by multiplying the strain by the dimension in each direction as follows:

```
>> d1 = epsilon(1)*60

d1 =

  -0.0069

>> d2 = epsilon(2)*60

d2 =

    0.0230

>> d3 = epsilon(3)*60

d3 =

  -0.0069
```

Notice that the change in the 1-direction is $-0.0069\,\text{mm}$. The minus sign indicates that there is a reduction in this dimension along 1-direction. The change in the 2-direction is $0.0230\,\text{mm}$ and is the largest change because the tensile force is along this direction. This change is positive indicating an extension in the dimension along this direction. Finally, the change in the 3-direction is $-0.0069\,\text{mm}$. This change is minus since it indicates a reduction in the dimension along this direction. Also, note that the changes along the 1- and 3-directions are identical since the material is isotropic and these two directions are perpendicular to the 2-direction in which the force is applied.

Problems

Problem 2.1

Derive (2.5) in detail.

Problem 2.2

Discuss the validity of the reciprocity relations given in (2.6).

Problem 2.3

Write the 6×6 compliance matrix for a transversely isotropic material directly in terms of the five independent material constants E_1, E_2, ν_{12}, ν_{23}, and G_{12}.

Problem 2.4

Derive expressions for the elements C_{ij} of the stiffness matrix for a transversely isotropic material directly in terms of the five independent material constants E_1, E_2, ν_{12}, ν_{23}, and G_{12}.

Problem 2.5

Write the 6×6 compliance matrix for an isotropic material directly in terms of the two independent material constants E and ν.

Problem 2.6

Write the 6×6 stiffness matrix for an isotropic material directly in terms of the two independent material constants E and ν.

MATLAB Problem 2.7

Consider a 40-mm cube made of glass-reinforced polymer composite material that is subjected to a compressive force of 150 kN perpendicular to the fiber direction, directed along the 3-direction. The cube is free to expand or contract. Use MATLAB to determine the changes in the 40-mm dimensions of the cube. The material constants for glass-reinforced polymer composite material are given as follows [1]:

$$\begin{aligned} E_1 &= 50.0 \text{ GPa}, & E_2 = E_3 &= 15.20 \text{ GPa} \\ \nu_{23} &= 0.428, & \nu_{12} = \nu_{13} &= 0.254 \\ G_{23} &= 3.28 \text{ GPa}, & G_{12} = G_{13} &= 4.70 \text{ GPa} \end{aligned}$$

MATLAB Problem 2.8

Repeat Problem 2.7 if the cube is made of aluminum instead of glass-reinforced polymer composite material. The material constants for aluminum are $E = 72.4$ GPa and $\nu = 0.300$. Use MATLAB.

MATLAB Problem 2.9

When a fiber-reinforced composite material is heated or cooled, the material expands or contracts just like an isotropic material. This is deformation that takes place independently of any applied load. Let ΔT be the change in temperature and let α_1, α_2, and α_3 be the coefficients of thermal expansion for the composite material in the 1, 2, and 3-directions, respectively. In this case, the stress-strain relation of (2.1) and (2.2) becomes as follows:

$$\begin{Bmatrix} \varepsilon_1 - \alpha_1 \Delta T \\ \varepsilon_2 - \alpha_2 \Delta T \\ \varepsilon_3 - \alpha_3 \Delta T \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{13} & S_{23} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{Bmatrix} \tag{2.10}$$

In terms of the stiffness matrix (2.10) becomes as follows:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 - \alpha_1 \Delta T \\ \varepsilon_2 - \alpha_2 \Delta T \\ \varepsilon_3 - \alpha_3 \Delta T \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix} \tag{2.11}$$

In (2.10) and (2.11), the strains ε_1, ε_2, and ε_3 are called the *total strains*, $\alpha_1 \Delta T$, $\alpha_2 \Delta T$, and $\alpha_3 \Delta T$ are called the *free thermal strains*, and $(\varepsilon_1 - \alpha_1 \Delta T)$, $(\varepsilon_2 - \alpha_2 \Delta T)$, and $(\varepsilon_3 - \alpha_3 \Delta T)$ are called the *mechanical strains*.

Consider now the cube of graphite-reinforced polymer composite material of Example 2.2 but without the tensile force. Suppose the cube is heated 30°C above some reference state. Given $\alpha_1 = -0.01800 \times 10^{-6}/°\text{C}$ and $\alpha_2 = \alpha_3 = 24.3 \times 10^{-6}/°\text{C}$, use MATLAB to determine the changes in length of the cube in each one of the three directions.

Problem 2.10

Consider the effects of moisture strains in this problem. Let ΔM be the change in moisture and let β_1, β_2, and β_3 be the coefficients of moisture expansion in the 1, 2, and 3-directions, respectively. In this case, the free moisture strains are $\beta_1 \Delta M$, $\beta_2 \Delta M$, and $\beta_3 \Delta M$ in the 1, 2, and 3-directions, respectively. Write the stress-strain equations in this case that correspond to (2.10) and (2.11). In your equations, superimpose both the free thermal strains and the free moisture strains.