On the Design Process of Tensile Structures

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Summary. *The influence of the development of computer programmes and automatic generation of cable nets and membrane structures will be shown in some examples. The main interest is laying on new evaluation methods of cable nets and membrane structure and the design process of membrane structures, integrating the material behaviour of coated fabric.*

Key words: Design process, cable nets, membrane structures, inflated structures

1 Introduction

The design process of pretensioned structures such as cable nets and membrane structures is influenced by the development of computational methods. While the first methods of form finding had been physical modelling with fabric, wire nets or soap films, today several numerical methods of form finding are developed based on the force density method [1,2], the principle of minimal surfaces [3,4] using dynamic relaxation [5,6] or other approaches in fulfilling the three-dimensional equilibrium. Further process has been carried out in the form finding with an anisotropic stress distribution [7]. All methods have in common that no material laws are necessary finding an equilibrium of the three dimensional shape for given stress distributions, boundary conditions and supports. These shapes of equilibrium should ensure in the built structure a homogeneous distribution of the tension stresses. In reality the material behaviour, process of cutting patterns, manufacturing and pretensioning on site influencing the stress distribution, wrinkles and regions of over stress are obvious, can be seen and measured.

The design process of pretensioned structures needed to be extended taking into account evaluation methods for shapes of equilibrium in relation the material behaviour and process of prestensioning. More realistic modelling of membrane structures is necessary including the strips in width, orientation of the fabric and seams for analysing the load charring behaviour. The process of reassembling flatten strips had already been proposed for a rotational symmetric hat type tent [8]. The process of form finding can be embedded in a design process including cutting pattern und structural behaviour under external loads. The load bearing behaviour

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can be evaluated by redundancy, flexibility or a stiffness value in relation to the curvature and the elastic strain of the materials.

2 State of the Art

The design process of tension structures such as double curved cable nets or membrane structures such as tents, air support halls or airships can be divided into form finding, static analysis and cutting pattern. The result of the form finding is a shape of equilibrium for a certain stress distribution and boundary conditions. The shape of equilibrium ensures the geometry of the double curved surface which has only tension and avoids compression in the surface. From this geometry the structural behaviour is exanimated and the cutting pattern is made of. The flattening of the double curved surface is a geometrical process without considering the stress distribution and the material behaviour. In the recent development of cutting pattern methods the stress distribution is taking into account [9,10]. The analysis of the structural behaviour is carried out without the influence of the width of the strips, the seams, the orientation of the fabric and the process of pretensioning. The separation of the structural behaviour and the cutting pattern leads in built structures to highly inhomogeneous stress distributions which can be seen in wrinkles and measured in stresses which are two times higher than required. The difference in the stress distribution and geometry between the numerical found shape of equilibrium and the real structure causes in the non consistent design procedure see Fig. 1.

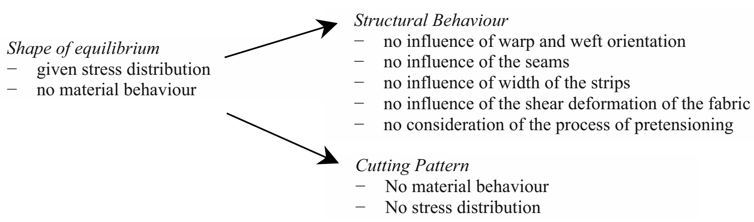


Fig. 1. Common design process of membrane structures

3 Enhanced Design Process of Tension Structures

An enhanced design concept will be based on 5 design steps defining the shape of equilibrium, generating the cutting pattern, reassembling and pretensioning the cutting pattern, the structural analysis of the reassembled structure and the evaluation of the structural behaviour. The material behaviour is considered in the last three steps: flattening the shape of equilibrium, reassembling and load bearing behaviour. The length and the width of the strips has an influence to the shear deformation of the coated fabric. The orthotropic behaviour of coated fabric influences the process of pretension and the stress distribution in the reassembled structure. The numerical process allows after evaluation modifications to reach better results in the reassemble structures considering stress distribution and deformations.

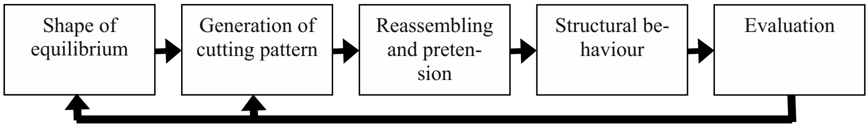


Fig. 2. Enhanced design process of membrane structures [11]

3.1 Shape of Equilibrium

The development of Computer Aided Geometric Design (CAGD) marked the start of changes in geometry endorsing new and free forms. This generation of double curved 3-dimensional surfaces is restricted by few limitations. Theoretically there are an unlimited number of forms to be numerical generated and represented. However, the manufacture and realization of such double curved surfaces are subject to numerous boundary conditions and restrictions. Using cables and membranes for the load transfer only tension forces can be carried, the cables and membranes can not withstand bending moments and compression forces in a global point of view. The structures have to be pretensioned activating the geometric stiffness or to be able carrying compression forces by reducing the pretension. The shape of equilibrium defines a pretensioned geometry of a doubled curved surface for a cable net or a membrane structure. The relation between the tension stress, geometry and equilibrium allows three possibilities to introduce the tension into the membranes and influences the shape of equilibrium, see Fig. 3.

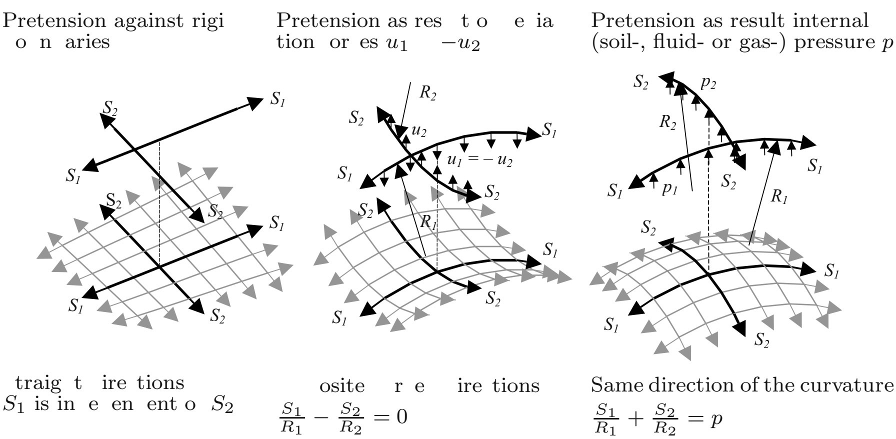


Fig. 3. Relation between tension forces and curvature

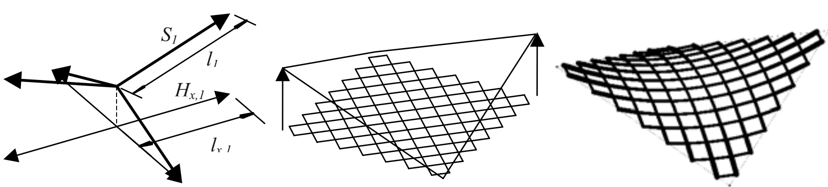
The pretension against rigid boundaries enables plain tension structures, tension structures with single curvatures and double curved tension structures if the direction of the cables or yarns is along the evolution line of a hyper parabola. The tension forces are independent from each other in this case.

The tension forces in surfaces with negative Gaussian curvature result of the deviation forces at the nodes and this leads to a relation between tension forces and curvature. Fulfilling the equilibrium at each node the tension forces are related to the radius of curvature in the both directions. Equal forces in both directions require the same curvature of the cables. The ratio of tension forces and radius of curvature is constant by meaning the higher the forces the lower the curvature to ensure equilibrium.

Stabilising the membranes with internal pressure leads to surfaces with positive Gaussian curvature and a dependency between the internal pressure, the tension forces and the curvature. The tension forces are directly related to the internal pressure and the lower the curvature the higher the forces.

Cable nets with square meshes are cinematic systems, the thin membrane without bending stiffness is statically determined. In both cases the double curved surface is a result of the three dimensional equilibrium at each node for given tension forces in a cable net or at each point for a given stress distribution in a membrane. The equilibrium is fulfilled without taking into account the material behaviour and is influenced by the boundary conditions such as high points, boundary cables or rigid boundaries.

For cable nets the numerical solution is based on the constant ratio of cable force and length in the first step. The ratio of cable force and length is described as force density [12] and the shape of equilibrium is calculated from a plane net with a square gird by moving the nodes in the third direction, forced by the fixed points and boundaries which don't lie in the same plane as the cable net. Depending on the change in length from the cable links in the plane into the three dimensional surface the cable forces changes, the longer the cables the higher the forces. The result is a doubled curved surface with a steady change in the forces along each cable related to the change of curvature of the surface. This method can also be exceed to cable nets which are statically indeterminate such as nets with triangle meshes because of the constant ratio of force/length. Both forces and length of the links are free parameter searching for the three dimensional equilibrium.



$$\frac{S}{l} = \frac{H_x}{l_x} = \frac{V}{l_z} = const.$$

Plane net with square meshes and constant forces in each link

Shape of equilibrium change in forces

Fig. 4. Shape of equilibrium fulfilling vertical equilibrium

Fulfilling the equilibrium in the tangential plane at each knot allows adjusting the link length and leads to constant forces in each cable. The cables are oriented along geodesic lines onto the surface and the angles are not constant at the nodes between crossing cables.

In membranes the equilibrium has to be fulfilled at each point of the surface. Plane state of stress assumed shapes of equilibrium are also the result of a given stress

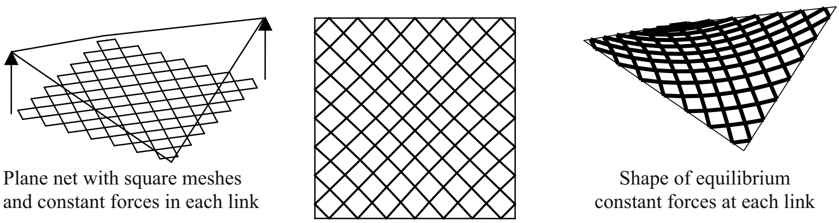


Fig. 5. Shape of equilibrium fulfilling vertical and tangential equilibrium

distribution. In general and in covariant description [13] the equilibrium normal to the surface is written as:

$$\sigma^{\alpha\beta} b_{\alpha\beta} = 0 \quad , \quad \text{with } b_{\alpha\beta} \text{ as tensor of curvature}$$

Related to main axis

$$\sigma^{11} b_{11} + \sigma^{12} b_{12} + \sigma^{21} b_{21} + \sigma^{22} b_{22} = 0$$

The orientation of the coordinate system in direction of the principle stresses ($\sigma^{12} = \sigma^{21} = 0$) or principle curvature ($b_{12} = b_{21} = 0$)

$$\sigma^{11} b_{11} + \sigma^{22} b_{22} = 0$$

Tension stresses in both principle directions $\sigma^{11} \geq 0$ und $\sigma^{22} \geq 0$ requires a negative Gaussian curvature, with $b_{11} = \frac{1}{R_1}$ and $b_{22} = -\frac{1}{R_2}$ the equilibrium normal to the surface results in

$$\frac{\sigma^{11}}{R_1} - \frac{\sigma^{22}}{R_2} = 0$$

The equilibrium in the tangential plane of the point can be written in covariant description as

$$\sigma^{\alpha\beta}_{|\beta} = 0$$

Assuming the stress is constant at a certain point leads to

$$\sigma^{\alpha\beta} = \sigma g^{\alpha\beta} \quad \text{with } g^{\alpha\beta} \text{ as metric tensor}$$

Substituted

$$(\sigma g^{\alpha\beta})_{|\beta} = 0 \Rightarrow \sigma_{|\beta} g^{\alpha\beta} + \underbrace{\sigma g^{\alpha\beta}_{|\beta}}_{\Rightarrow 0} = 0$$

and results in

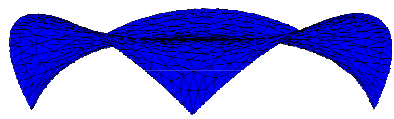
$$\sigma_{|\beta} g^{\alpha\beta} = 0$$

The metric tensor has a certain value at each point in a double curved surface; this means the deviation of the stress has to be zero. This requires a constant stress distribution also to neighboured points and describes the hydrostatic state of stress. Therefore has to be $\sigma_{11} = \sigma_{22} = \mathit{constant}$ and with

$$\frac{\sigma_{11}}{R_1} - \frac{\sigma^{22}}{R_2} = 0 \Rightarrow \left(\frac{R_2 - R_1}{R_1 \cdot R_2} \right) = 0 \quad \text{and } R_1 = R_2$$

The tension stress in the surface is isotropic and homogeneous by meaning the stresses are at each point and in each direction constant and this is named as hydrostatic state of stress. The stress can be set as a constant value and reduces the description of shapes of equilibrium to the geometrical problem searching for the minimal surface by given boundary conditions. Physical models of minimal surfaces are soap films, in earlier times one of the few methods describing double curved surfaces which are at each point under tension.

Soap film [14]



Numerical solution of the minimal surface [15]

Fig. 6. Minimal surfaces as soap film and the numerical solution

3.2 Cutting Pattern

The shapes of equilibrium are characterized by no material behaviour or by the material behaviour of soap films without shear resistance. The real shape of the tensioned structures is influenced by the material behaviour and the difference between the shape of equilibrium and the materialized, pretensioned shape resulting in the non existing shear stiffness of a cable net, the orthotropic behaviour of coated fabric or the relatively high shear stiffness of foils. Known from the globe is the fact that double curved surfaces cannot be flattened without distortion. Furthermore the fabric is manufactured in width up to max. 5 m and this requires the assembling of the whole cover with patches or strips of a certain length and width. The common way of generating the cutting pattern from the shape of equilibrium is described in four steps. The shape of equilibrium is cut into strips mostly using geodesic lines for the cutting lines. The whole structure is then divided into double curved strips. These strips are flattened with different methods such a paper strip method or minimizing the strain energy while flattening the strips. The compensation as final step is necessary to introduce the tension forces by elongation of the fabric. All strips have to be decreased in width and length in relation to the stress and strain behaviour of the fabric in the built structure.

Differences in geometry and stresses between the shape of equilibrium and the built structure are caused by the orientation of the fabric, the shear deformation of the fabric, the stiffness of the seams und the process of pretension. Reducing the mistakes in the cutting pattern which can be seen in wrinkles and can be measured in local stress peaks is possible by taking into account the jamming condition of the coated fabric. The load carrying compounds in a fabric are the yarns which are protected by the coating. In a woven fabric warp and fill will kept in place if the tension stress acts in direction of the yarns. Shear forces lead to a rotation of warp and fill against each other up to an angle when the yarns touch each other. The

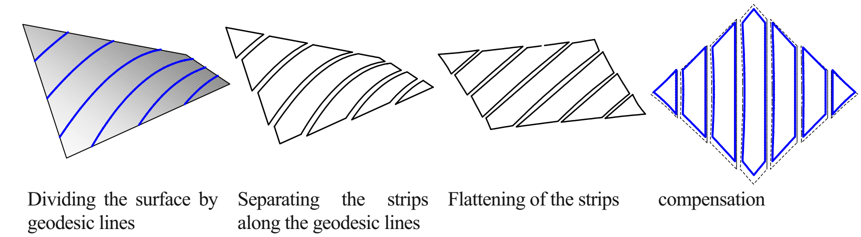


Fig. 7. Generation of cutting pattern [16]

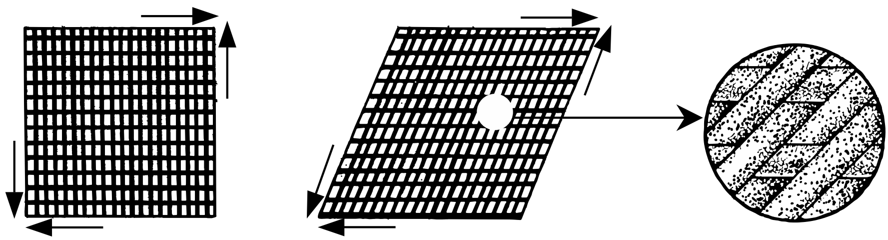


Fig. 8. Shear deformation of woven fabric [17]

maximum shear deformation is depended by the thickness of the yarns, the distance of the yarns and the flexibility of the coating. If the rotation of the yarns is larger than the required distortion to flatten the doubled curved strips the flattening is only a process of strainless deformation.

If the process is invert and still definite needs further examination because the manufacturing of membrane structures is from the flat and assembled strips into the double curved and pretensioned structure. Already known is the shear deformation which is used to build double curved surfaces with cable nets. The cable net can be put onto the doubled curved surface just by changing the angles between the cables; the distance between the nodes is kept as constant. The rotation of the two layers of cables against each other is related to the curvatures of the surface.

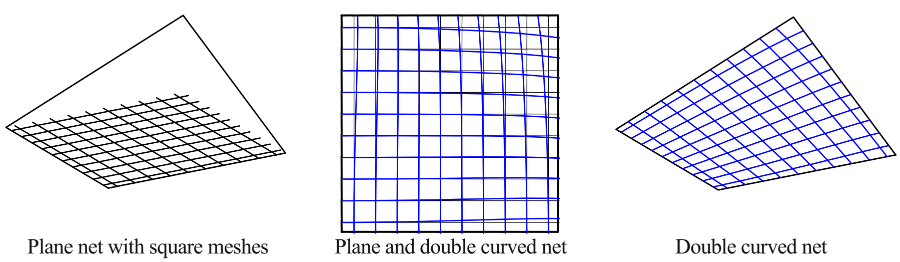


Fig. 9. Shear deformation from the plane into the double curved net

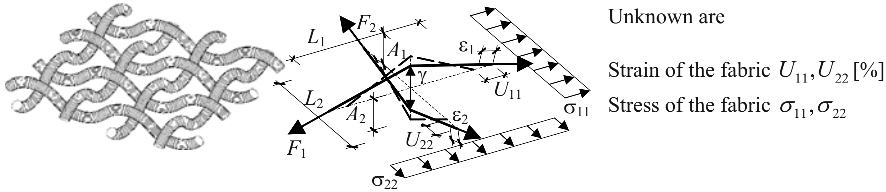


Fig. 10. Model describing the behaviour of a woven fabric

3.3 Reassembling and Pretensioning

The tension forces can only be introduced into cable nets or membranes by elastic strain of the cables and coated fabric. The numerical process of reassembly requires the description of the material behaviour in which both the change of the geometry and the elastic strain is considered. The change in geometry is for cable nets mostly the in plane shear deformation reaching the double curved surface. The change of geometry in woven fabric is related to the elongation of the yarns. The simple model is useful enough describing the behaviour of a woven fabric, developed in 1978 [18], refined and tested in 1987 [19] and finally numerical transferred in 2003 [20].

Neglecting the influence of the coating the behaviour of a woven fabric can be described by the

- Geometry of the fabric such as thickness and distance of the yarns (warp A_1, L_1 and inclination $m_1 = A_1/L_1$, fill A_2, L_2 and inclination $m_2 = A_2/L_2$)
- Stress-strain-behaviour of each yarn (warp F_1, ε_1, fill F_2, ε_2)
- The change in the thickness of the fabric (γ) and
- The equilibrium of the deviation forces at each knot

The ratio of unstrained to strained length is described by:

$$\mu_1 = 1 + U_{11} \text{ and } \mu_2 = 1 + U_{22}$$

With the ratio of undeformed and deformed inclination of

$$k_1 = A_1/\mu_1 \text{ and } k_2 = A_2/\mu_2$$

is the elastic strain of the yarns

$$\varepsilon_1 - \mu_1 \frac{\sqrt{1 + k_1^2 m_1^2}}{\sqrt{1 + m_1^2}} + 1 = 0 \quad \text{ and } \varepsilon_2 - \mu_2 \frac{\sqrt{1 + k_2^2 m_2^2}}{\sqrt{1 + m_2^2}} + 1 = 0$$

The constrain of the distance between the yarns at the knots is

$$k_1 \mu_1 A_1 + k_2 \mu_2 A_2 - A_1 - A_2 - \gamma F_1 \frac{k_1 m_1}{\sqrt{1 + k_1^2 m_1^2}} = 0$$

Equilibrium of the yarn

$$F_2 \frac{k_2 m_2}{\sqrt{1 + k_2^2 m_2^2}} - F_1 \frac{k_1 m_1}{\sqrt{1 + k_1^2 m_1^2}} = 0$$

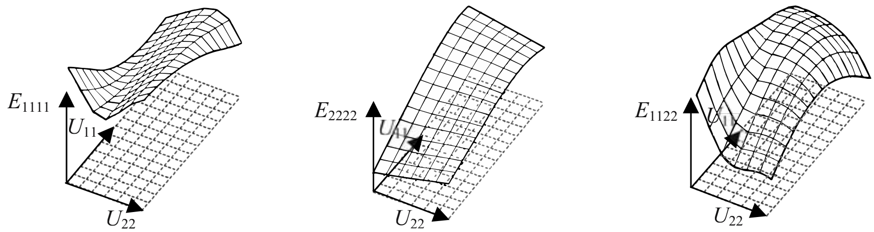


Fig. 11. Young's Moduli and Poisson ratio as function of the fabric strain, PVC coated fabric [19]

This set of 4 equations serves a non linear system of equations for the four unknown values $\varepsilon_1, \varepsilon_2, k_1, k_2$. After solving the equations the stresses of the fabric can be defined directly by

$$\sigma_{11} = \frac{1}{L_2}\left(\frac{F_1}{\sqrt{1 + k_1^2 m_1^2}}\right) \quad \text{and} \quad \sigma_{22} = \frac{1}{L_1}\left(\frac{F_2}{\sqrt{1 + k_2^2 m_2^2}}\right)$$

The calculated strains and stresses enable to define the stiffness E_{1111}, E_{2222} and E_{1122}. The elastic stiffness are non linear and closely related to the strain ratio in warp and fill direction. Even the Poisson ratio E_{1122} is non linear and depending to the strain ratio of the yarns.

The numerical process of reassembling enables taking into account the behaviour of the fabric, the influence of the seams and the distribution of the tension stress through the whole surface. The plane strips have to be remeshed, sewed together and pretensioned by moving the sewed structure into defined boundaries, moving support points into their position after reassembling or putting internal pressure onto the system. The stress distribution and geometry of the sewed and pretensioned structure is different from the assumed stress distribution of the shape of equilibrium. The differences are depending on the curvature of the surface, the orientation of the strips in relation to the main curvature, the torsion of the strips, the distortion of the load transfer along the seams, the stiffness of the seams, the assumed compensation of the flatten strips, the width of the strips, the of the surface, the shear deformation of yarns and in the shown example of the load transfer between the boundary cables and fabric, see Fig. 12.

In the shown example the stress distributions varies in a single strip and changes from strip to strip. Relatively low tension stress in the middle strip can been see as result of less compensation. The influence of the stiffness of the seams can be shown in the difference between deformation in vertical direction comparing the geometry of the shape of equilibrium and reassembled and pretensioned structure. For the shown example the difference is app. 20% of the span. The antimetric deformation is caused by the inhomogeneous stress distribution in the cross section along the high points. The tension stress perpendicular are unsteady, low stress leads to high vertical deformations and high stress kept the fabric down which can clearly seen in the up and down of the differences.

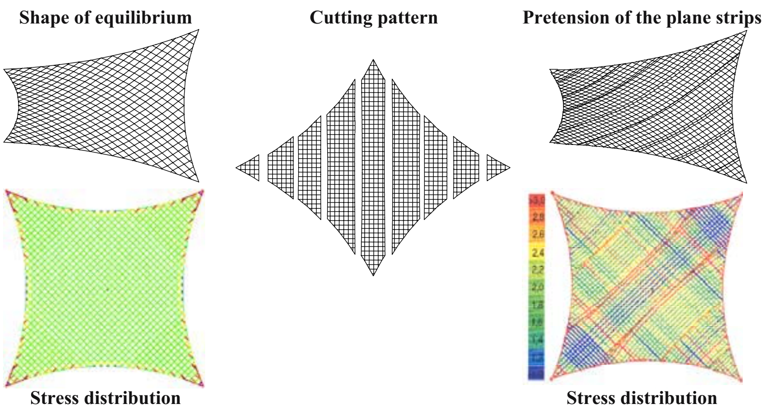


Fig. 12. Influence of the cutting pattern to the stress distribution of a membrane [21]

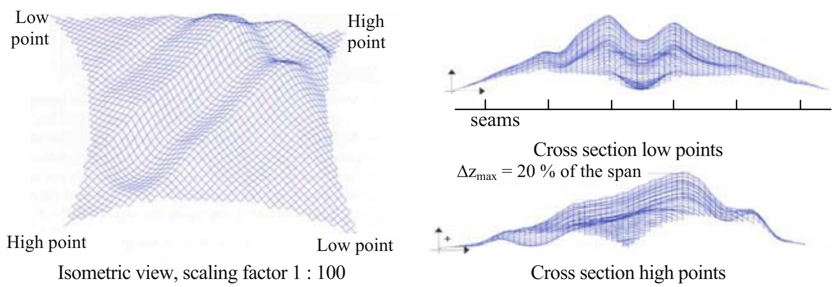


Fig. 13. Difference in z–direction between the reassemble shape and the shape of equilibrium [21]

3.4 Load Bearing Behaviour

The load bearing behaviour is in general depended on:

- The flexibility of the structures including masts, bending elements such as arches, stay cables, anchorages and foundations
- The height of the pretension related to external loads
- The orientation of the cables or yarns related to the main curvature of the surface
- The curvature of the surface and
- The stress – strain – behaviour of the material

The stability of the cable net or membrane structures is depending on the pretension. In structures with negative Gaussian curvature the pretension is only to reduce the deformation. The slag of the spanning direction causes a change in the system but no instability.

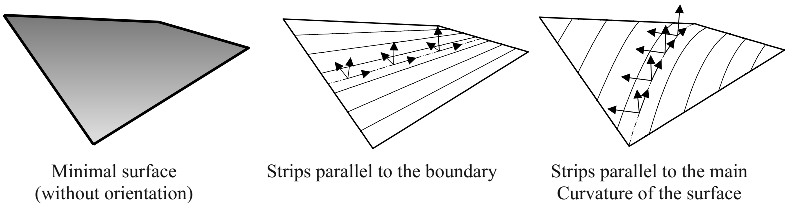


Fig. 14. Orientation of the strips in relation to the curvature of the surface

The isotropic and homogeneous stress in minimal surfaces allows at first any kind of orientation of the cables or yarns on the surface but the curvature of the cable layers or yarns, the elastic strain under external loads, the strainless deformations and the shear deformation is depending on the orientation. In the shown example, see Fig. 14, the yarns are straight if the strips are parallel to the boundaries and the torsion of each strip is high. Compared to the orientation parallel to the main curvature of the surface the torsion of the strips is zero if the centre line of the strip is equal to a line of main curvature.

Although both membranes have the same shape of equilibrium the load carrying behaviour is totally different. The membrane with straight yarns has large deformations under a constant and uniform distributed load, the stresses in warp and fill will increase. The membrane with the yarns oriented to the main curvature carries load be increasing the stresses in the sag directions and decreasing the forces in the span direction. The deformation is compared to the membrane with straight yarns very low.

Curvature and elastic stiffness have different influences to the load bearing behaviour of the membranes. Membranes with large curvature react under external loads highly linear, increasing and decreasing of the stresses are nearly independent of the elastic stiffness if the spans and the materials have been chooses for usual membranes and loads in Europe. Only if the elastic stiffness is relatively low the behaviour is starting to get non linear because of the large elastic strain. The vertical deformations are large because of the total length of the yarns spanning between the supports or boundary cables. The opposite behaviour occurs for membranes with low curvature by meaning the change of the stresses under external load is non linear and the deformation are decreasing nearly linear by increasing the elastic stiffness. Membranes with less curvature are more sensitive to changes in the elastic stiffness considering the change in stress and deformation than membranes with a higher curvature.

3.5 Evaluation

In relation to the influence of the curvature and the elastic stiffness to the load bearing behaviour a stiffness value can be defined for any doubled curved surface, developed by R. Blum [17].

For a constant and uniformly distributed load the equilibrium normal to the surface is written in covariant description as following:

$$\sigma^{\alpha\beta} b_{\alpha\beta} = p^3 \tag{1}$$

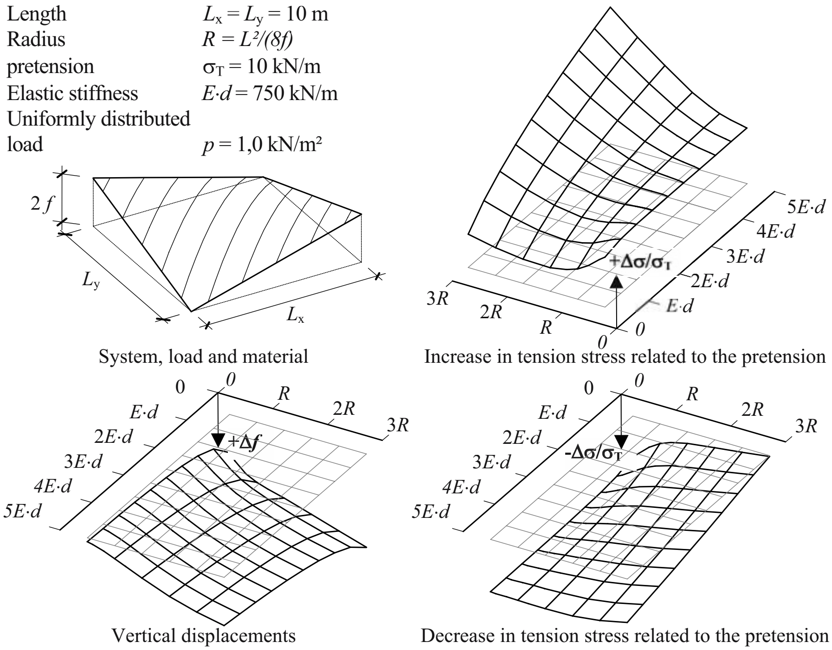


Fig. 15. Influence of curvature and elastic stiffness to the stresses and deformation

The change of the stresses caused by external loads is:

$$[\sigma^{\alpha\beta} + \Delta\sigma^{\alpha\beta}] \cdot [b_{\alpha\beta} + \Delta b_{\alpha\beta}] = p^3 \tag{2}$$

Multiplication of the terms and neglecting terms of high order result in:

$$\Delta\sigma^{\alpha\beta} b_{\alpha\beta} + \Delta b_{\alpha\beta}\sigma^{\alpha\beta} = p^3 \tag{3}$$

Linear elastic behaviour of the material lead to

$$\Delta\sigma^{\alpha\beta} = n^{\alpha\beta\delta\gamma} \cdot \Delta\varepsilon_{\delta\gamma} \tag{4}$$

The deviation of the curvature can be approximately seen as the displacement in vertical direction

$$\Delta b_{\alpha\beta} = u^3_{|\alpha,\beta} \tag{5}$$

The change in the elastic strain is approximately multiplication of the curvature with the vertical displacement:

$$\Delta\varepsilon_{\delta\gamma} = b_{\delta\gamma} \cdot u^3 \tag{6}$$

(6) substituted (4) and with (5) is the change of the stresses

$$\Delta\sigma^{\alpha\beta} = \frac{n^{\alpha\beta\delta\gamma} \cdot b_{\delta\gamma}}{n^{\alpha\beta\delta\gamma} \cdot b_{\alpha\beta} \cdot b_{\delta\gamma}} \cdot p^3$$

with (4), (5) and (6) in (2) follows

$$n^{\alpha\beta\delta\gamma} b_{\delta\gamma} u^3 b_{\alpha\beta} + u^3_{|\alpha,\beta} \cdot \sigma^{\alpha\beta} = p^3$$

Assuming only vertical loads, allows setting the 2. term to zero and the vertical displacement is

$$u^3 = \frac{1}{n^{\alpha\beta\delta\gamma} \cdot b_{\alpha\beta} \cdot b_{\delta\gamma}} \cdot p^3$$

In both equations, the change of the stresses and the vertical displacement, the denominator is the same and a product of the elastic stiffness and the curvature of the surface. The lower this product is the higher the vertical deformations will be. Therefore this term describes the stiffness of the surface and is named as

$$D = n^{\alpha\beta\delta\gamma} \cdot b_{\alpha\beta} \cdot b_{\delta\gamma}$$

Expanded and the orientation of the coordinate system in direction of the principle stresses ($\sigma^{12} = \sigma^{21} = 0$) or principle curvature ($b_{12} = b_{21} = 0$) leads to:

$$D = \frac{n^{1111}}{R_1^2} + \frac{n^{2222}}{R_2^2} = \frac{R_2^2 \cdot n^{1111} + R_1^2 \cdot n^{2222}}{R_1^2 \cdot R_2^2}$$

Flexibility ellipsoids

Two aspects have to be mentioned considering the load bearing behaviour of cable nets or membrane structures, the stiffness of a three-dimensional shape and the possibility of pretensioning the structure in relation to the material behaviour and the stiffness. There exists an analogy between net calculation in geodesy and the analysis of membranes [22,23] and leads to new aspects describing the load carrying capacity of structures. Flexibility can be seen as the deformation of each node loaded by a rotating unit load and leads to flexibility ellipsoids showing the three dimensional deformation of the node.

The in plane stiffness of the cable net or membranes has a large influence to the possibility of pretension because this allows to change plane two dimensional flat strips into a three dimensional surface without wrinkles. The advantage of cinematic cable nets and membranes is the ability to distribute the forces during the process of pretensioning nearly homogeneous by tensioning only boundary cables or lifting high points. The ability of a double curved cable net distributing forces which are acting at the edges or boundaries homogeneous through the net can be described by redundancy.

The comparison between three different types of cable nets will give an example for the application of flexibility ellipsoids in evaluation of the structural behaviour. In geometry three homogeneous nets are exiting which can be transformed in double curved cable nets. Each net has the same tension forces and the stiffness per meter. The net with hexagonal meshes has only nodes with three links and leads to an equilibrium of each node under pretension only if the forces in all links are the same. The shape is then comparable with a minimal surface. The high degree of kinematics makes these nets very flexible and the stiffness can be mostly influenced by the height of the pretension forces. The net with square meshes has still no in plane shear stiffness but if the cables are arranged in the direction of the main curvature this net has even less deformation for a uniformly distributed load compared to the net with triangle meshes. The net with hexagonal meshes is the one with the highest flexibility; the flexibility of the net with triangle meshes is in direction normal to the net surface nearly the same compared to the net with square meshes. The in plane stiffness reduces theses deformation of the net with triangle meshes.

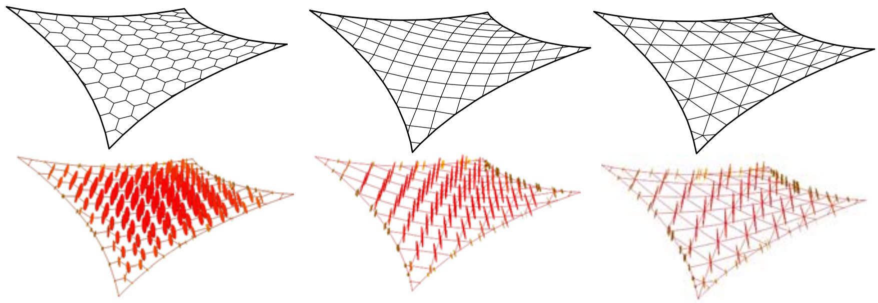


Fig. 16. Flexibility ellipsoids of pretensioned nets with different meshes

Redundancy

A common definition of redundancy is the availability of more functional system components than necessary for meeting the requirements. For example the statically indeterminacy is a measure for redundancy of structures by the meaning of safety against failure. In cinematic structures such as cable nets this definition has to be extended by taking into account the geometrical stiffness. Well known is the fact that each node has 3 independent possibilities of movement in space structure, the normal force of each element and the boundary forces are giving the necessary equations for equilibrium. If the number of independent movements of the nodes is equal to the number of equations resulting from the element forces and boundary forces the structure is statically determined.

Redundancy is more than the statically indeterminacy, which can be shown by the considerations of Dieter Stroebel [23] The number of indeterminacy is distributed among all elements in relation the geometry of the structure, the stiffness of the elements and the relatively elongation of each element within the structure. The elastic elongation of each element is influenced by stiffness of the connected elements. Focusing on one element means an extremely stiff surrounding structure compared to the element the change of length causes directly an increase of the force. The opposite is a very flexible structure which allows the elongation of the element without influencing the forces.

This behaviour can be described by the *elastic redundancy* in terms between 0 and 1. An elastic redundancy of 0 means no forces arise if the length an single element is changed. An elastic redundancy of 1 says changing the length leads to a force depending on the elastic stiffness.

Cinematic structures can be stabilised by tension forces. The tension forces built up a resistance if the structure is deformed under load known as geometrical stiffness. The three components of the stabilising tension forces lead to three additional

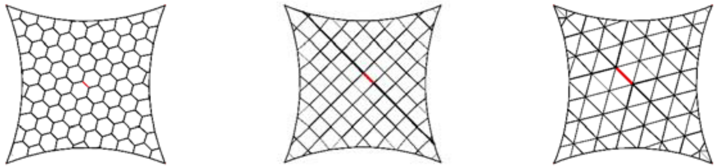


Fig. 17. Change of forces by shortening one link of 0,5% of its length

equations for each element in a pretensioned structure and an additional geometric redundancy. The geometric redundancy of 3 says no influence of the geometric stiffness is required; the geometric redundancy lower than 3 describes the part of the geometric stiffness necessary for stabilisation and higher than 3 means the element is unstable and needed to be stabilized

For the three cable nets the elastic and geometric redundancy can be analysed for the links and gives information to the influence of manufacturing errors, the possibility of pretensioning and the height of the tension forces related to the deformation. The change of the 0,5% of the length of one link causes in the net with the hexagonal meshes no changes in the forces, shown by the elastic redundancy close to zero. The opposite can be seen in the net with the triangle meshes, the change in the length causes in that element an increasing force.

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