3
Mechanical modelling of solid woven fabric composites

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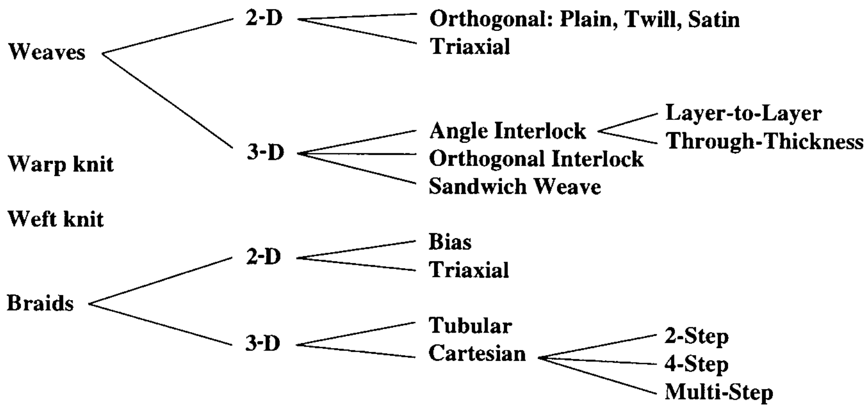
3.1 Introduction

Solid woven fabric composites represent a class of advanced composites which are reinforced by 2-D or 3-D woven preforms [1]. These materials offer new and exciting opportunities for tailoring the microstructure to specific thermomechanical applications in the fields of aerospace, marine, medicine and sports technology. The variables under control include fibre and matrix materials, yarn placement, yarn size and type. Together with this emerging ability to engineer composite materials comes the need to develop *computationally efficient* micromechanics models that can predict, with *sufficient accuracy*, the effect of the microstructural details on the internal and macroscopic behaviour of these new materials. Computational efficiency is indispensable because there are many parameters that must be varied in the course of engineering a composite material. This chapter addresses the issue of developing micromechanical models for solid woven fabric composites. In the future, it is probably inevitable that the optimization of the microstructure of a woven fabric composite will require the marriage of such micromechanical models and optimization algorithms.

3.2 Review on solid woven fabric composites

3.2.1 Introduction

This section provides a survey of the literature. First, an overview of woven fabric composites is presented. Solid woven preforms vary considerably in terms of fibre orientation, entanglement and geometry. Second, in order to exploit the advantages of these composites fully, it is important to create a link between the microstructural geometry and the thermomechanical performance [2]. In the past decade, a variety of micromechanical models have been employed to study the overall thermo-elastic behaviour of orthogonal *2-D woven fabric composites* based on the properties of the constituents



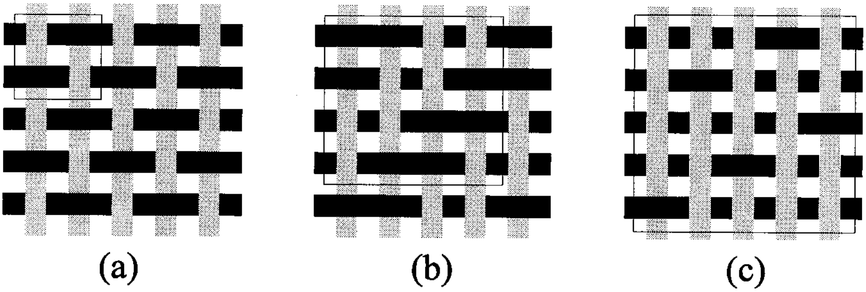
3.1 Classification of textile preforms for composite structures [3].

and the fabric architecture. Some of these models also provide the opportunity to address strength properties. A review will assist in defining possible modelling strategies for *3-D woven fabric composites.*

3.2.2 Classification

Fibre reinforcement constitutes the structural backbone of a composite. The classification by Cox and Flanagan [3] of various textile preforms is reproduced in Fig. 3.1. The left column classifies textile preforms according to the machines and processes used to produce them. The major textile-forming techniques for composite reinforcements are weaving, knitting and braiding. Further, it is possible to make a distinction between the dimensionality of the textile preform. Following the definition of Cox [3], the division into 2-D and 3-D textile structures is determined by whether the fibre preform can transport an important load (higher than the load carried by the matrix alone) in two or three linearly independent directions.

In general, an orthogonal *2-D woven fabric* is made by weaving yarns together. A yarn is a continuous strand of textile fibres. The fabric is produced on a loom that interlaces yarns at right angles to one another [2–8]. The lengthwise yarns are called warps, while the yarns that are shuttled across the loom are called fillings or wefts. The individual yarns in the warp and filling directions are also called an end and a pick, respectively. The interlacing of the yarns causes yarn undulation or yarn crimp. The weave type is determined by the method of interlacing both sets of yarns. Figure 3.2 shows three basic constructions: plain, twill and satin weave. Even in rather simple woven fabrics, there are important geometric differences between the warp and the weft direction. Those differences are the result



3.2 Basic weave constructions: (a) plain, (b) twill and (c) 5HS satin weave. The black box represents the fabric unit cell.

of numerous constructional and process parameters such as weaving density, warp tension, weft tension and beating motion.

The term 'hybrid' is used to describe fabrics containing more than one type of fibre material. Hybrid fabrics are attractive preforms for structural materials for two major reasons. First, these fabrics supply an even wider variety of material selection for designers. They offer the potential of improved composites' mechanical properties, weight saving or excellent impact resistance. Second, a more cost-effective use of expensive fibres can be obtained by replacing them partially with less expensive fibres. Hybrid fabrics are woven from fibrous materials such as glass, aramid, carbon, boron, ceramics and natural fibres.

Advances in textile manufacturing technology are rapidly expanding the number and complexity of *3-D woven preforms*. By changing the traditional weaving technique to produce 2-D fabrics, it is now possible to achieve a much higher degree of integration in the thickness direction of the textile. The two major classes of solid 3-D weaving are through-thickness angle interlock weaving [10] and orthogonal interlock weaving [1–3]. Angle interlock 3-D woven fabrics can be produced on a dobby loom or a jacquard loom. The warp yarns can now enter more than one layer of weft yarns. Other textile structures with laid-in straight yarns are also possible. By changing the number of layers, the pattern of repeat and the position of the laid-in yarns, an almost infinite number of geometric variations becomes possible. In an orthogonal interlock 3-D weave, the yarns are placed in three mutually orthogonal directions. These fabrics are produced principally by the multiple warp weaving method. Matrix-rich regions are created in composites reinforced with a 3-D woven orthogonal preform.

In general, solid woven fabrics offer the advantages of handleability, dimensional stability, improved impact and damage resistance. However, these advantages are obtained at the cost of reduced stiffness and strength properties owing to the undulation of the yarns. There is thus a significant need to model the mechanical behaviour of these composites.

3.2.3 Micromechanical models

Considering the actual importance of 2-D woven fabric composites in the family of structural composites, the mechanical analyses of these composites are now extensively reviewed and presented. Most of the published data are related to stiffness properties of plain weave laminae. There are few publications on the internal stress distribution and on the damage and strength analysis problem of general woven fabric composites. The possible extension of the different micromechanical models to analyse 3-D woven fabric composites will be discussed. It should also be stressed here that in this rapidly evolving field of study any review will soon be incomplete. New results are always being presented or printed.

Models of Ishikawa and Chou

In the 1980s, an extensive amount of work on the thermo-mechanical modelling of 2-D woven fabric composites was done by Ishikawa and Chou. They developed and presented *three analytical 1-D elastic models* [11–13]. These models are known as the mosaic model, the fibre crimp model and the bridging model. The classical lamination theory forms the basic analytical tool for these developments [14].

The models of Ishikawa and Chou are labelled 1-D models because they only consider the undulation of the yarns in the loading direction. Notice the total *absence* of any geometric analysis. That is, the actual yarn cross-sectional shape or the presence of a gap between adjacent yarns is not considered. Therefore, no predictions are made for the out-of-plane yarn orientation and the fibre volume fraction. Moreover, these models consider balanced closed weaves only, whereas in practice the fabric can be unbalanced and open. Since the classical laminated plate theory is the basis of each model *only the in-plane elastic properties* are predicted. The elastic models were extended to analyse the thermal properties, hybrid fabrics and the *knee behaviour* under uniaxial tensile loading along the filling direction only. However, an extension to treat 3-D woven preforms is not useful because of the geometric simplifications and the limitation to predicting only in-plane properties.

Models of N. Naik, Shembekar and Ganesh

N. Naik and Shembekar have developed *2-D elastic models* for a 2-D non-hybrid plain weave fabric composite [15]. These models are essentially an extension of the 1-D models of Ishikawa and Chou. However, these 2-D models take into account the undulation of both warp and weft yarns, the presence of a possible gap between adjacent yarns, the real cross-section of the yarn and the possible unbalanced nature of the plain fabric lamina. The representative unit cell is discretized into slices along or across the loading direction. These slices are further divided into different elements such as straight cross-ply or unidirectional regions, undulated cross-ply or unidirectional regions and pure matrix elements. In the analysis of Naik and Shembekar, two schemes for combining the in-plane stiffness matrices of the different elements are used: parallel–series and series–parallel. In the parallel–series (PS) model, the elements are first assembled in parallel across the loading direction with the isostrain assumption (adding the stiffness matrices, weighted by their volume fractions). Then, those multi-elements are assembled in series along the loading direction with the isostress assumption. In the second scheme, all the infinitesimal elements of a section along the loading direction are assembled with an isostress assumption (adding the compliance matrices, weighted by their volume fractions). Then, all the sections along the loading direction are assembled with an isostrain condition. Such a scheme is called a series–parallel (SP) model. Both schemes yield a full 2-D stiffness matrix for the plain woven fabric composite. A full mathematical treatment of the problem has been presented in reference [16]. Based on experimental work, the PS model is recommended for the prediction of all in-plane elastic constants. Out-of-plane properties *cannot* be predicted. Hence, the extension of the model to treat 3-D woven preforms is not useful.

Recently, Naik and Ganesh have presented an extension of their thermoelastic models to include the prediction of *failure in plain weave composites under on-axis static tensile loading* [17,18]. The load is assumed along the filling direction. Different stages of failure such as warp yarn transverse failure, filling yarn shear failure, filling yarn transverse failure, pure matrix element failure and filling yarn longitudinal failure are considered. The newness of the model lies in the calculation procedure for the stresses in the matrix and yarn elements. However, this is exactly where the model is most confusing. A lot of effort has been spent on describing material non-linearities, geometric non-linearities and geometric effects of matrix element failures, while the available information on the stress prediction procedure is inadequate. The failure analysis is then carried out by comparing the local element stresses or strains with the admissible values of stress or strain. The Tsai–Wu failure criterion [19] is used to predict the failure in the filling yarn elements. The maximum stress and strain criteria are used to predict the failure in the warp yarn and matrix elements. If an element fails, the stiffness of that element is reduced (degraded stiffness). The final failure of the unit cell laminate is assumed to have occurred if the fibres in the filling yarn are broken.

In conclusion, some more practical drawbacks and disadvantages of the strength model of Naik are provided. First, the stress model lacks logic and simplicity (when and why is the PS model to be preferred over the SP model?). Second, only on-axis uniaxial tensile loads can be considered along the warp or weft direction. Third, the model does not account for thermal stresses which are known to be important in the stress and strength analysis of fibre composites. Finally, only a non-hybrid 2-D plain weave composite can be considered in the present analysis.

Model of Hahn and Pandy

The *3-D thermo-elastic model* of Hahn and Pandy [20] for non-hybrid plain fabric composites is simple in concept and mathematical implementation. This model is essentially an extension of the 2-D models of Naik. The geometric model accounts for the undulation of warp and weft yarns, the actual yarn cross-section and the presence of a gap between adjacent yarns. The yarn undulations are sinusoidal and described with shape functions. The gap between two neighbouring yarns, however, is introduced by terminating the yarn at the start of the gap. Hence, for large gaps the yarn cross-section becomes quasi-rectangular, which is not realistic.

In the thermo-elastic model, the strain is assumed to be uniform throughout the composite unit cell. Therefore the effective stiffness of the woven fabric composite is obtained as a volume average of the local stiffness properties of yarn and matrix elements. This is a so-called isostrain model. Closed-form expressions are provided for the 3-D effective elastic moduli and effective thermal expansion constants for a 2-D plain weave composite.

The model has the advantage of being simple and easy to use. The isostrain model can very easily be applied to analyse complex 3-D woven fabric composites. However, some disadvantages are here provided. First, the accuracy of the isostrain model still remains to be verified through more experimental verification of *all* 3-D elastic constants. It will be further shown in this chapter that the isostrain technique is not capable of accurately predicting *all* 3-D elastic constants [21]. Second, the model can certainly not be extended to solve the stress analysis problem accurately, and hence cannot be used for strength predictions.

Model of R. Naik

Recently, a micromechanics analysis tool labelled TexCad was developed by R. Naik to calculate the thermo-elastic properties along with damage and strength estimates for woven fabric composites [22]. This tool can be used to analyse non-hybrid plain weave and satin weave composites. It discretely models the yarn centreline paths within the repeating unit cell by assuming a sinusoidal undulation of the yarns. The 3-D effective stiffness matrix is computed by a yarn discretization scheme (which subdivides each yarn into smaller, piecewise straight yarn slices) that assumed an isostrain state within the unit cell. Hence, as in the Hahn and Pandy model, the isostrain model is applied. In the calculation for the strength, TexCad uses a curved beam-on-elastic-foundation model for yarn crimp regions together with an incremental approach in which stiffness properties for the failed yarn slices are reduced, based on the predicted yarn slice failure mode. Only on-axis tensile loadings and in-plane shear loadings were modelled and reported. Certainly, the most questionable assumption in this strength model is the calculation of the local stress fields in yarn and matrix slices based on the isostrain assumption. Basically, TexCad is well documented and easy to use. It is a thorough implementation of the isostrain approach which could be extended easily to analyse complex 3-D woven fabric composites. It will perform stiffness and failure analyses as correctly as can be expected for an isostrain model.

Model of Paumelle, Hassim and Léné

Paumelle *et al.* [23,24] developed a finite element method for analysing plain weave fabric composite structures. The periodic medium homogenization method is implemented. Basically, by applying periodic boundary conditions on the surface of the unit cell and by solving six elementary loading conditions on the unit cell, the complete set of elastic moduli of the homogenized structure can be computed. At the same time, the method provides a good approximation of the local distribution of force and stress fields acting in the composite components and at their interface. These microscopic stress fields give a strong indication of the types of damage that will occur. To the best of our knowledge, Paumelle *et al.* have not yet reported an extension of this finite element model to predict damage propagation or to analyse 3-D woven preforms. Moreover, outlined below are some problems encountered in a practical finite element analysis of solid woven fabric composites.

First, this approach requires large computer calculation power and memory because of the 3-D nature and the complexity (size) of the yarn architecture. Second, a correct finite element model includes the generation of the fabric geometry and the finite element mesh of nodes and elements. Most of the time spent is related to the creation and the verification of a correct geometric model [25]. Finally, there are major problems in analysing and interpreting the results in a 3-D domain of a rather complex geometry [26].

Model of Blackketter

Here, we will discuss in some detail the research work of Blackketter [27]. In our opinion, this work is certainly one of the first and most important efforts to model damage propagation in 2-D woven composites. The approach could also be applied to 3-D woven fabric composites.

Blackketter constructed a simplified 3-D unit cell of a single ply non-hybrid plain weave graphite/epoxy composite. From this description 3-D finite element models were generated. Twenty node isoparametric hexahedron elements were used in generating the finite element meshes. Limits on element refinement were imposed by the computational time required for solution. An incremental iterating finite element algorithm was developed to analyse loading response. Each iteration or load step required about 30 real-time minutes using a VAX8800 computer. Tension and shear loadings were modelled. The finite element model included capacities to model non-linear constitutive material behaviour and a scheme to estimate the effects of damage propagation by stiffness reduction. Results from this analysis compared extremely well with experimental stress–strain data. It was concluded that the non-linear stress–strain behaviour of the woven fabric composite is principally caused by damage propagation rather than by plastic deformation of the matrix.

Let us describe now the damage propagation model as developed by Blackketter *et al*. At each Gaussian integration point (27 Gaussian quadrature integration points over each volume element), damage or failure was detected by comparing the actual stress state with a failure criterion. To simulate damage at an integration point, the local stiffness properties were reduced. Therefore, each element in the model can contain both intact and failed Gaussian integration points. After the occurrence of damage, the volume considered was capable of sustaining only reduced loads and stresses had to be redistributed to surrounding volumes.

It is important to select an appropriate failure criterion for the matrix and yarn materials. For the isotropic matrix material a maximum normal stress criterion was used to detect damage. If the principal stress exceeded the strength, the tensile modulus was reduced to 1% of its initial value. The shear modulus was reduced to 20% of its initial value. After failure, the matrix was no longer isotropic. For the transversely isotropic yarns, it is necessary to account both for the type of damage and the orientation of that damage. Blackketter compared the actual stresses in the local coordinate system (123) with the respective ultimate strengths. This is a maximum stress criterion. The 1-axis corresponds to the longitudinal yarn direction. Table 3.1 presents the different failure modes and the stiffness reduction factors used by Blackketter. Each Gaussian integration point was allowed to fail in one or all modes. Finally, catastrophic failure of the unit cell was characterized by large displacements compared with the previous values.

The analysis by Blackketter of graphite/epoxy plain weave fabric composites has shown that by carefully modelling the fabric geometry, using

Table 3.1. Stiffness reduction scheme for the UD yarn elements, according to Blackketter [27]

Failure mode	Mechanical property and degradation factors					
	E_{11}	E_{22}	E_{33}	G_{23}	G_{31}	G_{12}
1 Longitudinal tension σ_{11}	0.01	0.01	0.01	0.01	0.01	0.01
2 Transverse tension σ_{22}	1.0	0.01	1.0	1.0	0.2	0.2
3 Transverse tension σ_{33}	1.0	1.0	0.01	1.0	0.2	0.2
4 Transverse shear τ_{23}	1.0	0.01	0.01	0.01	0.01	0.01
5 Longitudinal shear τ_{13}	1.0	1.0	0.01	1.0	0.01	1.0
6 Longitudinal shear τ_{12}	1.0	0.01	1.0	1.0	1.0	0.01

correct constituent stiffness/strength data and by applying an appropriate stiffness reduction scheme, it is possible to predict the stress–strain behaviour of woven fabric composites. The same ideas could certainly be applied to analyse 3-D woven fabric composites. However, Blackketter does not discuss in detail the limitations of the finite element modelling technique (meshing or calculation time).

Models of Whitcomb

Whitcomb and coworkers [28–30] have studied the effect of the yarn architecture on the predicted elastic moduli and stresses in plain weave composites. The work is restricted to linear elastic analysis. Three-dimensional finite element models were used. Only simple plain weaves were studied because these offer sufficient complexity for the task. A refined model of the complete unit cell would require immense amounts of computer memory and calculation time. However, by exploiting the geometric and material symmetries in the simple unit cell, it was sufficient to analyse 1/32 of the size of the complete plain weave unit cell. Twenty node isoparametric hexahedral elements were used. Two different yarn architectures were investigated. The first was the 'translated architecture' where the complete yarn is created by keeping the cross-section vertical along the yarn path. The second was the 'extruded architecture' wherein the yarn cross-section is always placed perpendicular to the yarn path. The extruded yarn architecture requires a more complex mesh generation.

Whitcomb and coworkers also analysed progressive failure of plain weave fabric composites under in-plane tensile loading using a 3-D finite element model. The mechanical loading was parallel to one of the yarn directions. Thermal loading or thermal residual stresses were not considered. The effects of various characteristics of the finite element model on predicted behaviour were examined. There is no 'right' way to model damage evolution that is also practical [30]. The most simple method to account for damage is to modify the constitutive matrix of the damaged finite element. Therefore, the failure analysis becomes a series of linear analyses. A maximum stress criterion was used to evaluate the damage of the matrix and yarn elements. Withcomb and coworkers have applied and compared three different techniques to modify the constitutive matrix after damage. First, the total constitutive matrix was reduced to essentially zero when any of the allowable strengths was exceeded. In the second technique, only specific rows and columns of the constitutive matrix were reduced according to the damage mode. Third, specific engineering moduli were reduced. This is the reduction scheme developed previously by Blackketter. Essentially, it was concluded that the predicted strength decreased considerably with increased waviness of the yarns. The modification technique of the constitutive matrix has a major effect on the predicted stress–strain curve. However, more numerical experiments are necessary to establish guidelines for an accurate failure analysis. No final conclusions have been given yet concerning the different reduction schemes. No extension is made to treat 3-D woven preforms.

3.2.4 Conclusions

In the past 15 years, a variety of different micromechanical approaches has been developed to study the effective behaviour of 2-D woven fabric composites. Tables 3.2 and 3.3 summarize those micromechanical models. Basically, the literature review reveals that considerable work addressing the effects of fabric architecture on the *effective elastic and thermal expansion properties* was done. However, this work has not been systematic or exhaustive in general. Research has been too focused on material systems based on plain weave fabrics, limited ranges of fibre volume fractions and specific material thermo-elastic properties. Second, the *stress and strength analyses* are still in their infancy. Here, research has focused on specific loading directions, knee behaviour and damage mechanisms. There is certainly a need for reliable strength models. Finally, the extension of the models to consider *3-D preforms* can only be achieved in a few cases (Tables 3.2 and 3.3).

In the *analytical methods* we observe a predominant use of the isostrain assumption to predict the effective thermo-elastic and strength properties. No data are available to verify the accuracy of this approximation. Moreover, most researchers have concentrated only on the primary determinant of mechanical and physical properties, namely the geometric orientation of the yarns. The idea that other geometric effects or boundary conditions could have an influence on the prediction of effective properties of woven fabric composites was ignored. The well-established *finite element method*

Table 3.2. Modelling review of woven fabric composites: analytical mechanical models. First four columns indicate whether the full stiffness matrix *C*, the thermal expansion coefficient α and the strength are predicted, and whether FEM is used. The last column indicates whether this model can be extended to 3-D-woven fabric composites

Model	*C*	α	Strength	FEM	Limitations	To 3-D
Chou & Ishikawa: mosaic model, 1985	Y	Y	N	N	No yarn undulation In-plane properties Isostrain/isostress	N
Chou & Ishikawa: crimp model, 1985	Y	Y	Knee	N	Plain weave Undulation in one yarn system In-plane properties Isostress	N
Chou & Ishikawa: bridging model, 1985	Y	Y	Knee	N	Satin weave Undulation in one yarn system In-plane properties Isostress	N
N.K. Naik *et al.*, 1992–1995	Y	Y	Y	N	Non-hybrid plain weave In-plane properties Mixed isostress/isostrain On-axis tensile load	N
Hahn & Pandey, 1993	Y	Y	N	N	Non-hybrid plain weave Isostrain	Y
R. Naik, 1995	Y	Y	Y	N	Non-hybrid plain and satin Isostrain On-axis tensile and shear loads	Y

Table 3.3. Modelling review of woven fabric composites: numerical mechanical models

Model	*C*	α	Strength	FEM	Limitations	To 3-D
Paumelle *et al.*, 1992	Y	N	Stress	Y	Computational time Non-hybrid plain weave No damage or strength model	N
Blackketter *et al.*, 1993	Y	N	Y	Y	Computational time Non-hybrid plain weave On-axis in-plane load	Y/N
Whitcomb and coworker, 1993	Y	N	Y	Y	Computational time Non-hybrid plain weave On-axis in-plane load	N

was employed by most researchers to compute the local stress fields accurately and to predict the strength of woven fabric composites. A comprehensive stiffness reduction scheme for damage modelling has been offered by Blackketter [27]. While it is possible to use mesh generation programs to alter the geometry and while changing constituent properties is very simple, the cost in computing time for a parametric study is significant. It is becoming increasingly clear that approaches other than the finite element method are needed to develop computationally efficient analysis tools for solid woven fabric composites. As mentioned previously, this is because of the large number of parameters that must be varied in identifying an optimal yarn architecture.

In conclusion, the current models for 2-D woven fabric composites all have limited applicability, in that either they are not *sufficiently accurate* to predict the local stress fields in yarn and matrix phases, or they are not *computationally efficient*. The struggle between accuracy and computational efficiency is a continuous one. Nowadays, the ability to engineer not only the material composition but also the internal yarn geometry of 2-D and 3-D solid woven fabric composites gives the designer of a composite material unmatched control over the material. Exerting that control intelligently, however, requires a body of theory. The objective of micromechanical modelling should always be to develop both accurate and computationally efficient approaches that can predict the behaviour directly, given the material composition and the internal yarn geometry.

3.3 Elastic model: the complementary energy model

3.3.1 Introduction

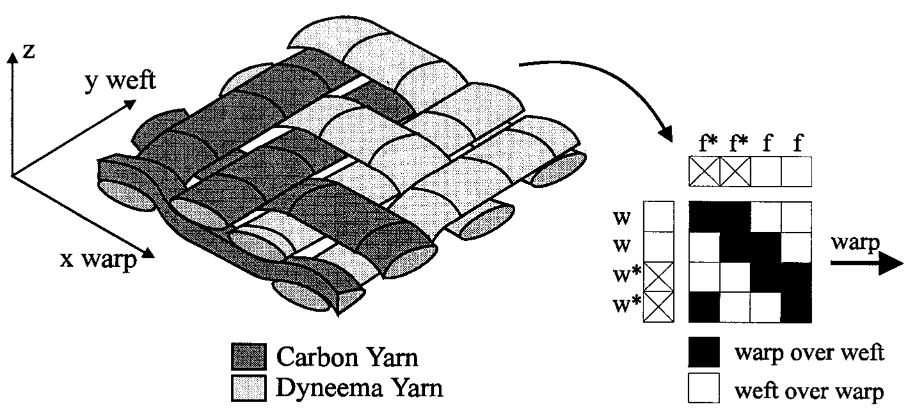
In this section, we focus on the recent development at the Katholieke Universiteit Leuven of the complementary energy model for 2-D woven fabric composites [32–34]. This model is included here because it is a pertinent example of the vigour that exists in textile composites research. The model captures both the 'orientation effect' and the 'position effect', important features of the actual heterogeneous composite material. Currently, the model is being extended to the mechanical analysis of 3-D woven fabric composites, and of braided fabric composites.

It should be stressed that the yarn architecture of a solid woven fabric composite is rather precisely *determined* by the textile processing route. Woven fabric composites provide new opportunities for tailoring the yarn architecture to specific applications. This is in contrast to random or unidirectional composites, where the precise control of the fibre orientation and spatial distribution is difficult and only *statistically* macroscopic arrangements are possible. Hence, for the mechanical modelling of random or unidirectional composites the geometric characteristics are taken into consideration in a certain *average sense* (fibre level), whereas for solid woven fabric composites the yarn interlacings, curvatures and locations, should be taken into account, reflecting the *actual geometry* (yarn level).

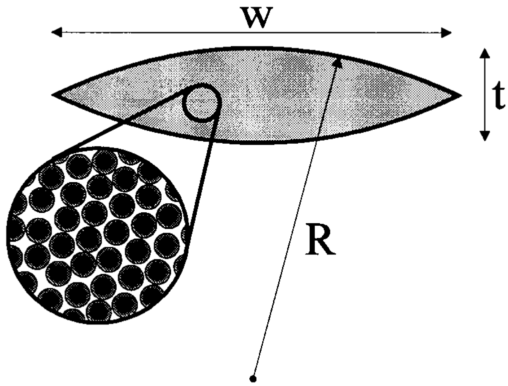
3.3.2 Geometric model

Since the mechanical properties of woven fabric composites have a very strong dependence upon the reinforcing yarn geometry, it is essential to create a geometric concept or scheme for describing the fibre architecture. The model deals with a perfect, regular, one-layer fabric composite. Hence, the presence of voids, the misorientation of yarns and the nesting of fabric layers are neglected, just as they are neglected in most other fabric composite models [35].

The woven fabric is treated as an assembly of unit cells (Fig. 3.2). By definition, the unit cell is the smallest repeating pattern in the structure. Figure 3.3 shows an example of a checkerboard pattern for a complex fabric, namely a hybrid carbon/Dyneema® twill weave. The rows of the board represent the warp yarns, while the columns are the filling or weft yarns. At an interlacing point, the square is coloured black if the warp yarn runs over the weft yarn. The main complexity arises from the fact that the fabrics considered here can contain two different warp and weft yarn types. First, the extension is necessary to describe hybrid weave styles. Of course, the extension is also needed when using special fibres in woven constructions such as optical glass fibres or shape-memory alloy fibres with particular sizes.



3.3 Extended checkerboard pattern of a hybrid 2 × 2 twill weave unit cell. The w and f type yarns are Dyneema® fibres, and the w* and f* type yarns are carbon fibres.



3.4 Lenticular yarn cross-section. W is width of yarn, t is yarn thickness, R is radius of curvature.

The most detailed geometric analysis would consider the path of each single fibre in the unit cell. The greatest practical problem, however, is caused by the fact that the complete set of input data necessary for such a detailed geometric description is very large and difficult to quantify. Therefore, the geometric analysis is carried out on the yarn level. We assume that all individual fibres in the yarn run in the same direction as the yarn. The intra-yarn fibre volume fraction or fibre packing density K, defined as the fibre to yarn area ratio, is assumed to be a constant for the woven fabric composite. Interlacing of the yarns and processing of the composite leads to thread or yarn flattening. On the basis of microscopic observations, a lenticular shape was selected to describe the cross-sectional shape of the yarn (Fig. 3.4).

The geometric characteristics of a hybrid weave can be subdivided in *three groups* (Table 3.4). The first group, the *know group*, contains those parameters that are supplied by the weaving company. All the parameters that one has to measure on a real woven fabric composite are put together in the second group, the *measure group*. This fabric information can be obtained by microscopic observation of warp and weft sections of the fabric composite. The aspect ratio f of the yarn, defined as the width w over the thickness t of the lenticular yarn cross-section, is the most important one. The crimp parameter h_f describes the undulation of the filling yarns. Of course, the undulation of the yarns in the warp direction is related to this parameter because the increase of undulation in one direction of the fabric reduces the undulation in the other direction [36]. Finally, the third group, the *calculate group*, contains all values that are calculated from the previous parameters, using formulas based on simple geometric considerations.

Table 3.4. Classification of the geometric parameters

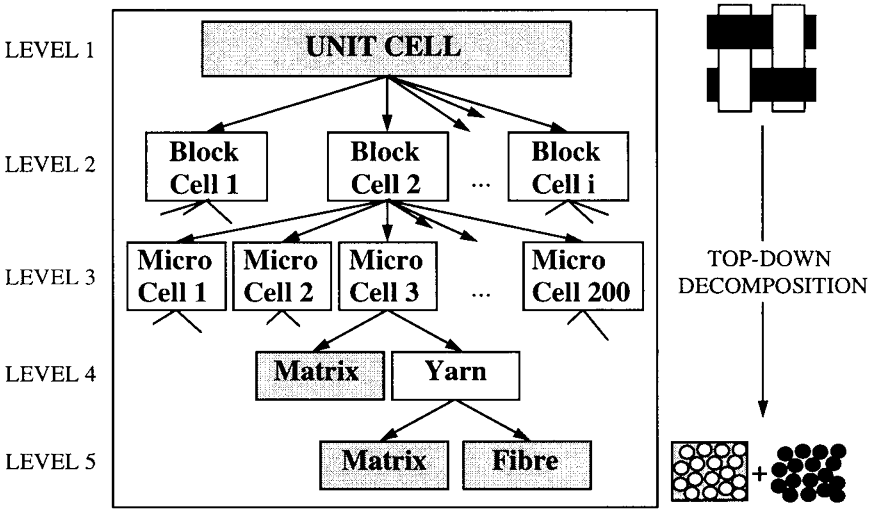
Know group	N_f	Number of fibres in the yarn
	d	Diameter of the fibre
	p	Yarn spacing
Measure group	f	Aspect ratio of the yarns
	h_f	Crimp parameter for the filling yarn
	D	Thickness of the composite
	K	Fibre packing density
Calculate group	$h_{w,}h_{w*}$	Crimp parameter for the warp yarns
	t	Thickness of the yarn cross-section
	w	Width of the yarn cross-section
	β	Orientation of the yarn
	V_f	Fibre volume fraction

Certainly, the most important output of the calculation procedure is the fibre volume fraction, the orientation of the yarns and the fractional volume of each cell. These data are the basis for a modelling of mechanical properties. Moreover, the geometric model as such is most useful in determining some textile properties as fabric thickness, but also in determining the allowable microstructural states of fabrics. A custom Microsoft Excel® application, called TexComp, has been developed to perform all geometric calculations [34].

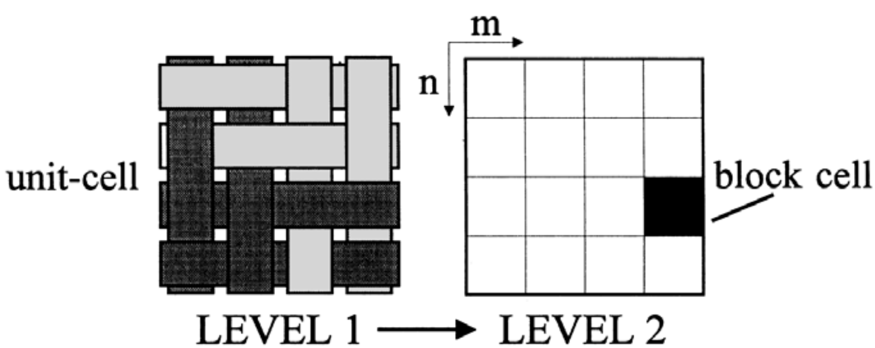
3.3.3 Multilevel decomposition scheme

The geometric model treats a woven fabric composite unit cell, shown in Fig. 3.5, as a hierarchical system that can be decomposed. Two major motivations are here formulated. First, the calculation and bookkeeping of geometric data should become a simple task. It is easy to calculate the geometric parameters that fully describe the yarn architecture only based on the presented 'know' and 'measure' group. Second, a logic and simple geometric meshing of the unit cell is essential for the computation of the mechanical properties. Basically, the composite unit cell level (1) is split up into block cells or macro-cells (2), micro-cells (3), matrix and yarn layers (4) and matrix and fibres (5). This five-level decomposition scheme could be considered as an 'intelligent mesh generator' for 2-D woven fabric composites. A logic extension towards 3-D woven preforms and to braids is currently being carried out.

The block partition of the unit cell consists of discretizing the unit cell in a number of rectangular *block cells*. At each crossover zone of a warp yarn and a weft yarn, one 'building block' is defined. The size of each block can easily be computed as a function of yarn spacings p, yarn widths w and com-



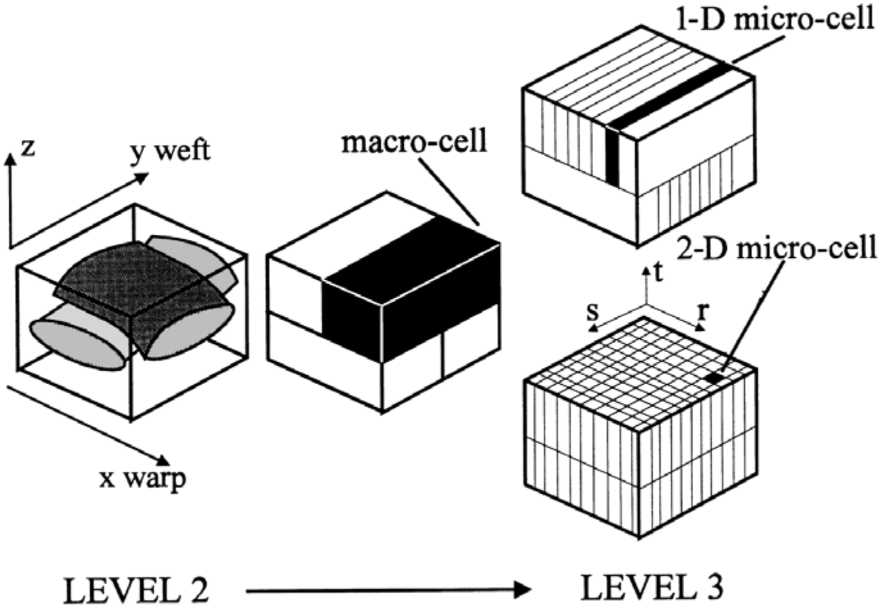
3.5 Multilevel decomposition of woven fabric composites: top-down.



3.6 Schematic of a hybrid twill weave unit cell (level 1) and a block cell (level 2).

posite thickness *D*. As can be seen in Fig. 3.6, we need 16 block cells to create the unit cell of a 2 × 2 hybrid twill fabric composite.

Basically, each block cell is uniquely identified by four macro-cells (Fig. 3.7). That is, at each crossover point of a warp and a weft yarn, one needs two macro-cells in one layer to define the path for the warp yarn and two in the other layer for the weft yarn. However, it should be stressed that the two layers of macro-cells, which are always present in the unit cell, yield a correct description of the geometry (for example, the fibre volume fraction is correct). In order to describe a general 2-D weave geometry, a library of 108 macro-cells has been put together. Even the most complex 2-D woven



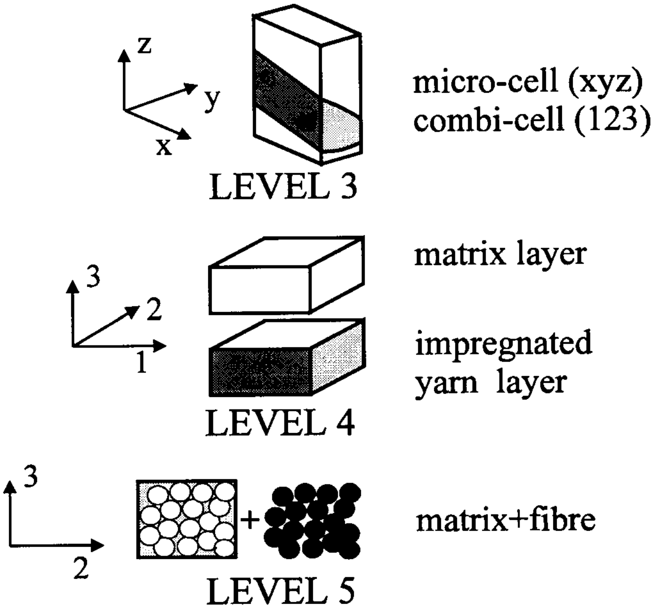
3.7 Schematic of a block cell or macro-cell (level 2) and a micro-cell (level 3).

structures can be composed with this library of rectangular macro-cells or building blocks.

It should be pointed out that only the weave construction pattern provided by the weaving company is sufficient to determine *automatically* the number of each type of macro-cell present in the unit cell. No extra fabric information, nor any geometric assumptions, nor operator interventions are needed. Therefore, the macro-partition is simple in concept and easy to apply. It provides a theoretical basis for the design of woven fabric composites.

The decomposition of the block in *micro-cells* is called the micro-partition. It is assumed that within each small micro-cell the yarn follows a straight yarn path. The 2-D micro-partition results in a fully 3-D division of the unit cell, so that the variation of properties within the unit cell can be properly analysed. This is essential for understanding the mechanical performance of woven fabric composites. For the local stress analysis, it will further be necessary to define the micro-cell also in the 123 local axis system, where 1 corresponds to the longitudinal yarn direction. This is called a *combi-cell*. Therefore the label micro-cell (*xyz*) or combi-cell (123) is selected depending on the reference axis system.

Then, the multilevel decomposition approach is based on simplifying the geometry within each combi-cell by assuming one matrix layer and one



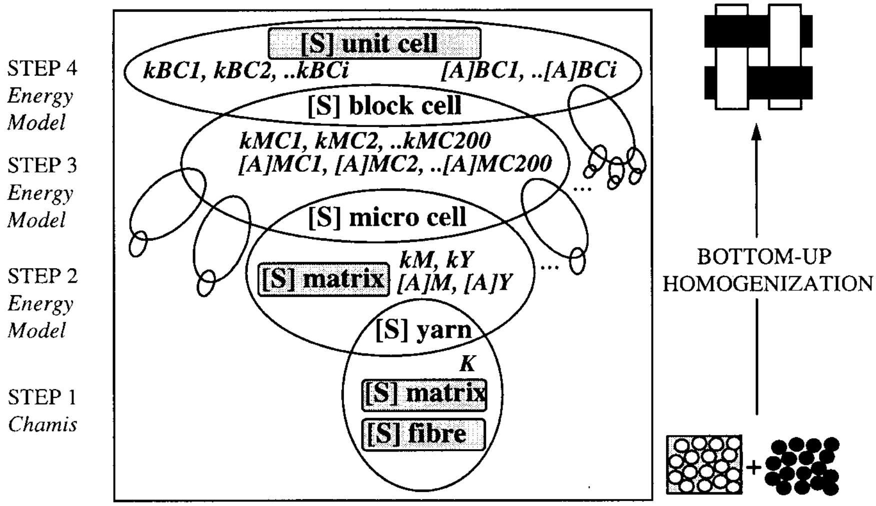
3.8 Schematic of a micro-cell or combi-cell (level 3), matrix and yarn layers (level 4) and a unidirectional lamina with fibre and matrix phases (level 5).

impregnated yarn layer (Fig. 3.8). This is level 4. Finally, each impregnated yarn is treated as a unidirectional lamina with *matrix* and *fibre phases*. This is level 5.

3.3.4 The complementary energy elastic model

Previously, by constructing a multilevel decomposition scheme, the composite unit cell was split automatically into matrix and yarn cells. Presently, by a multistep homogenization procedure, a link is established between the external loading and the internal stresses. The principal idea lies in the interpretation that the stress 'concentration factors' can be computed at each step by applying the *complementary variational principle*. This principle states that from all the admissible stress fields, the true field is that which minimizes the total complementary energy (hence the name: complementary energy model, CEM). We achieve a straightforward analytical stress model for woven fabric composites.

The *four-step homogenization model CEM* is now developed. The Venn diagrams in Fig. 3.9 show the link between the fractional cell volumes k, the stress concentration factors A and the compliance matrices S. With this



3.9 Multistep homogenization of woven fabric composites: CEM [*S*].

bottom-up homogenization scheme, from geometric level 5 to level 1, it is easy to compute the effective stiffness matrix of the woven fabric composite.

In the *first homogenization step*, the effective lamina or impregnated yarn layer properties are predicted in terms of their constituent material properties, and the fibre packing density K in the yarn. The empirical Chamis expressions [37], describing the elastic properties of a unidirectional lamina composed of transversely anisotropic fibres in an isotropic matrix, are used in this first step.

Consider now the micro-cell homogenization problem. This is *step two*. The complementary variational principle is used to compute the stress concentration factors for both layers. One average stress tensor is specified for each layer. These computed stress concentration factors are extremely useful because they link the stress tensor applied on the micro-cell with the average stress tensors on both layers and because they allow a straightforward computation of the micro-cell compliance matrix $[S_{MC}]$. More information on this topic can be found in [33].

As a next step, *homogenization step three*, the effective properties for the block cell are also determined by applying the complementary variational principle, taking into account the position and the properties of the 200 micro-cell constituents. The calculation procedure is shown in the Venn diagram of Fig. 3.9. Obviously, the 3-D compliance matrix of the block-cell is related to the fractional volume k, the compliance matrix $[S]$ and the concentration factors $[A]$ for the micro-cell's components as follows:

$$[S_{\text{BC}}] = \sum_{r=1}^{10}\sum_{s=1}^{10}\sum_{t=1}^{2} k_{\text{MCrst}}[A_{\text{MCrst}}]^{\text{T}}[S_{\text{MCrst}}][A_{\text{MCrst}}] \quad [3.1]$$

where the subscripts r, s and t refer to the position of the micro-cell in the block-cell as shown in Fig. 3.7. Already in the geometric model, the fractional volume k_{MC} for each of the micro-cells has been expressed and calculated as a function of the yarn spacings, the yarn width and the yarn thickness. The compliance matrix $[S_{\text{MC}}]$ for each of the micro-cells was calculated in the previous homogenization step. Applying the complementary variational principle [39] will yield the concentration factors $[A_{\text{MC}}]$ which link the average block stress and the micro-cell stresses. Now, the variational problem is solved in the global xyz coordinate system. Because one *constant stress tensor* is assigned to each of the 200 micro-cells, there are a total of 1200 unknown stress constants to be determined. The assumed and admissible stress fields are imposed by conditions 3.2 and 3.3:

$$\begin{aligned}
&\sum_{r=1}^{10}\sum_{s=1}^{10}\sum_{t=1}^{2} k_{rst}\sigma_{x\text{MCrst}} = \sigma_{x\text{BC}} \qquad \sum_{r=1}^{10}\sum_{s=1}^{10}\sum_{t=1}^{2} k_{rst}\tau_{yz\text{MCrst}} = \tau_{yz\text{BC}}\\
&\sum_{r=1}^{10}\sum_{s=1}^{10}\sum_{t=1}^{2} k_{rst}\sigma_{y\text{MCrst}} = \sigma_{y\text{BC}} \qquad \sum_{r=1}^{10}\sum_{s=1}^{10}\sum_{t=1}^{2} k_{rst}\tau_{zx\text{MCrst}} = \tau_{zx\text{BC}}\\
&\sum_{r=1}^{10}\sum_{s=1}^{10}\sum_{t=1}^{2} k_{rst}\sigma_{z\text{MCrst}} = \sigma_{z\text{BC}} \qquad \sum_{r=1}^{10}\sum_{s=1}^{10}\sum_{t=1}^{2} k_{rst}\tau_{xy\text{MCrst}} = \tau_{xy\text{BC}}
\end{aligned} \quad [3.2]$$

$$\begin{aligned}
&\text{(a)}\ \sigma_{x\text{MC}1st} = \sigma_{x\text{MC}2st} = \ldots = \sigma_{x\text{MC}10st} \quad s = 1 \ldots 10;\ t = 1 \ldots 2\\
&\text{(b)}\ \sigma_{y\text{MC}r1t} = \sigma_{y\text{MC}r2t} = \ldots = \sigma_{y\text{MC}r10t} \quad r = 1 \ldots 10;\ t = 1 \ldots 2\\
&\text{(c)}\ \sigma_{z\text{MC}rs1} = \sigma_{z\text{MC}rs2} \quad r = 1 \ldots 10;\ s = 1 \ldots 10\\
&\text{(d)}\ \tau_{yz\text{MC}r11} = \tau_{yz\text{MC}r21} = \ldots = \tau_{yz\text{MC}r101} = \tau_{yz\text{MC}r12}\\
&\qquad = \tau_{yz\text{MC}r22} = \ldots = \tau_{yz\text{MC}r102} \quad r = 1 \ldots 10\\
&\text{(e)}\ \tau_{zx\text{MC}1s1} = \tau_{zx\text{MC}2s1} = \ldots = \tau_{zx\text{MC}10s1} = \tau_{zx\text{MC}1s2}\\
&\qquad = \tau_{zx\text{MC}2s2} = \ldots = \tau_{zx\text{MC}10s2} \quad s = 1 \ldots 10\\
&\text{(f)}\ \tau_{xy\text{MC}11t} = \tau_{xy\text{MC}21t} = \tau_{xy\text{MC}12t} = \ldots = \tau_{xy\text{MC}1010t} \quad \tau = 1 \ldots 2
\end{aligned} \quad [3.3]$$

Mathematically, these stress constraints are considered by the method of the Lagrangian multipliers [38]. Therefore, the optimization problem is replaced by a set of equations that can be solved directly for the unknown stress constants. Proceeding from the geometric block cell level to the assembled composite unit cell level, the complementary variational principle is used for the last time. This is *homogenization step four*. The unknown block cell stresses and block cell concentration factors $[A_{\text{BC}}]$ are computed directly by the method of Lagrange [38]. Here, only the final expression for the compliance matrix of the unit cell is presented in Equation 3.4. The subscripts m and n refer to the position of the block in the unit cell (Fig. 3.6), the constants F and W refer to the number of weft and warp yarns in the unit cell:

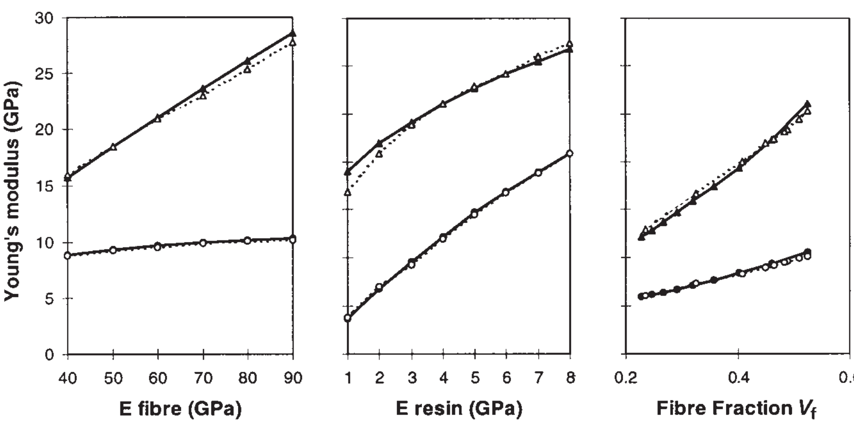
$$[S_{\mathrm{UC}}] = \sum_{m=1}^{F}\sum_{n=1}^{W} k_{\mathrm{BC}mn}[A_{\mathrm{BC}mn}]^{\mathrm{T}}[S_{\mathrm{BC}mn}][A_{\mathrm{BC}mn}] \qquad [3.4]$$

Basically, the four-step procedure defined in the foregoing will result directly in the computation of the overall symmetric 3-D compliance matrix of the woven fabric composite unit cell. Moreover, a direct link is established between the average unit cell stress and the cell stresses at each geometric level. This most important result will serve as a solid basis for further strength modelling in the next section.

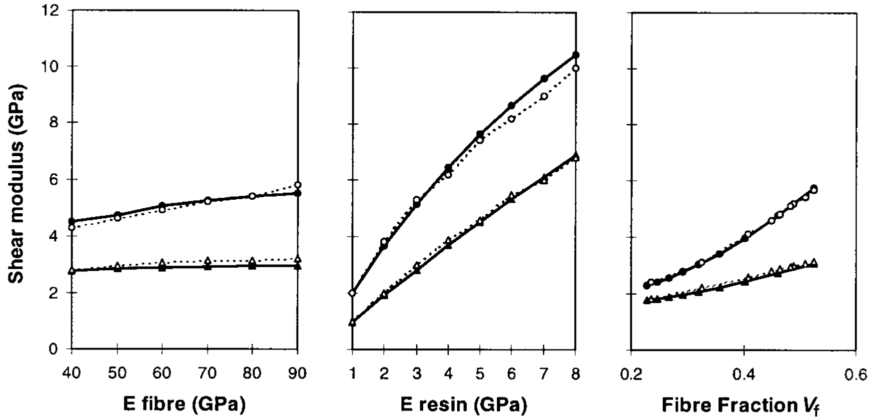
In conclusion, Fig. 3.10 and 3.11 present a benchmark parametric study for a glass/epoxy plain weave fabric composite. The analytical model yields elastic moduli predictions comparable to those obtained by 3-D finite element modelling. This fact is put forward as an indication of the appropriateness of the present multilevel, multistep technique.

3.3.5 Conclusion

The ability to specify the woven fabric geometry gives the designer control over the composite material. Many of the properties that influence how a composite can be used are determined by the 'averaged' behaviours of the fibres and the matrix. The 'averaged' stiffness properties are shaped by the internal yarn distribution, i.e. yarn *orientation* and *position*. With CEM, we now have a fast and efficient tool to predict the effect of each geometric



3.10 Predicted Young's moduli for the benchmark composite: comparing results from an FEM study [24] and our CEM calculations (material: glass-epoxy plain weave). ▲ $E_x = E_y$ (CEM); ● E_z (CEM); △ $E_x = E_y$ (FEM); ○ E_z (FEM).



3.11 Predicted shear moduli for the benchmark composite: comparing results from an FEM study [24] and our CEM calculations (material: glass-epoxy plain weave). ▲ $G_{yz} = G_{zx}$ (CEM); ● G_{xy} (CEM); △ $G_{yz} = G_{zx}$ (FEM); ○ G_{xy} (FEM).

variable on the 3-D elastic performance of the 2-D woven fabric composite. The proposed model can easily be extended to calculate the so-called thermal concentration tensors for the computation of the effective thermal expansion constants [33]. Moreover, the model can be generalized to 3-D preforms by simply extending the set of macro-cells.

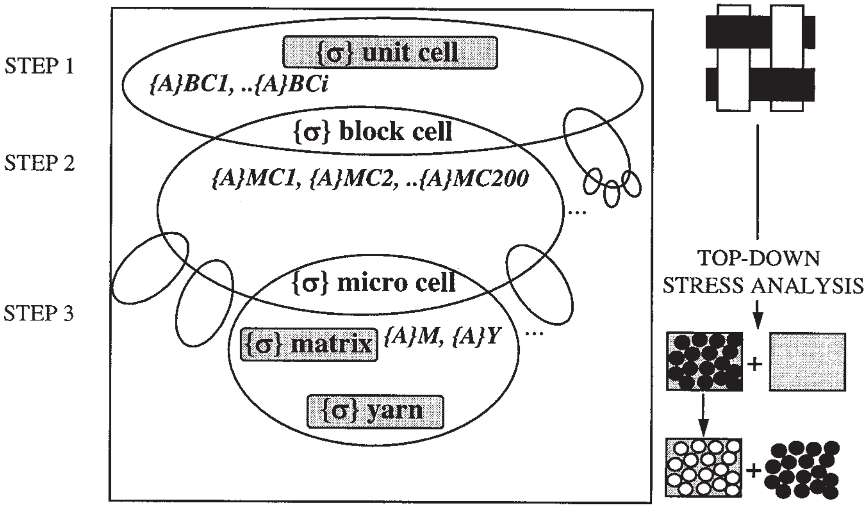
3.4 Strength model

3.4.1 Introduction

Woven fabric composite components are subjected to a variety of loading conditions during their service life. Therefore, an understanding of the mechanical response of these materials to various loading conditions is necessary for the safe design of a component. The prediction of strength is certainly one of the outstanding problems in the analysis of fibre composites. This section presents a method to predict the micro-stress fields, the first cell failure and the ultimate strength of woven fabric composites.

3.4.2 The complementary energy stress model

In order to predict strength *accurately*, a sufficiently detailed stress distribution must be available for composites subjected to arbitrary combinations of applied stresses. The need for *computationally efficient* predictive tools is clear when one considers the large range of fibres, matrices and



3.12 Multistep stress analysis of woven fabric composites: CEM.

woven fabric types available. The Venn diagrams in Fig. 3.12 show the link between the external mechanical loading, the stress concentration factors and the internal stress tensors. With this top-down stress calculation scheme it is easy to compute the stress fields at each geometric level. The entities corresponding to one of the stress calculation steps are grouped together in one Venn diagram.

As an example, the calculation of the yarn and matrix layer stresses for an arbitrary mechanical loading of the composite unit cell is explained below. The overall average stress tensor on the unit cell is denoted as:

$$\{\sigma_{UC}\}^{T} = \{\sigma_{xUC}, \sigma_{yUC}, \sigma_{zUC}, \tau_{yzUC}, \tau_{zxUC}, \tau_{xyUC}\} \quad [3.5]$$

By considering the computed concentration factors $[A]$ at each step and the yarn orientation through the calculation of the stress transformation matrix $[T_\sigma]$, the matrix and yarn layer mechanical stresses are given by

$$\{\sigma_M\} = [A_M][T_\sigma][A_{MC}][A_{BC}]\{\sigma_{UC}\} \quad [3.6]$$

$$\{\sigma_Y\} = [A_Y][T_\sigma][A_{MC}][A_{BC}]\{\sigma_{UC}\}$$

Two important observations are made. First, the *yarn orientation* and *yarn position* effects are included in the stress model due to the calculation of the concentration factors using the multilevel, multistep CEM. For example, the matrix material is not characterized by one stress state but by multiple stress states depending on the position of the matrix cell. Second, it should be stressed that any type of *simple* or *combined* 3-D mechanical loading can be applied, irrespective of symmetry, without resorting to different boundary-condition application strategies, as in the case of the finite element procedure. This is particularly important in the analysis of realistic woven fabric composite components where different loading conditions exist throughout the structure.

3.4.3 Development of the failure model

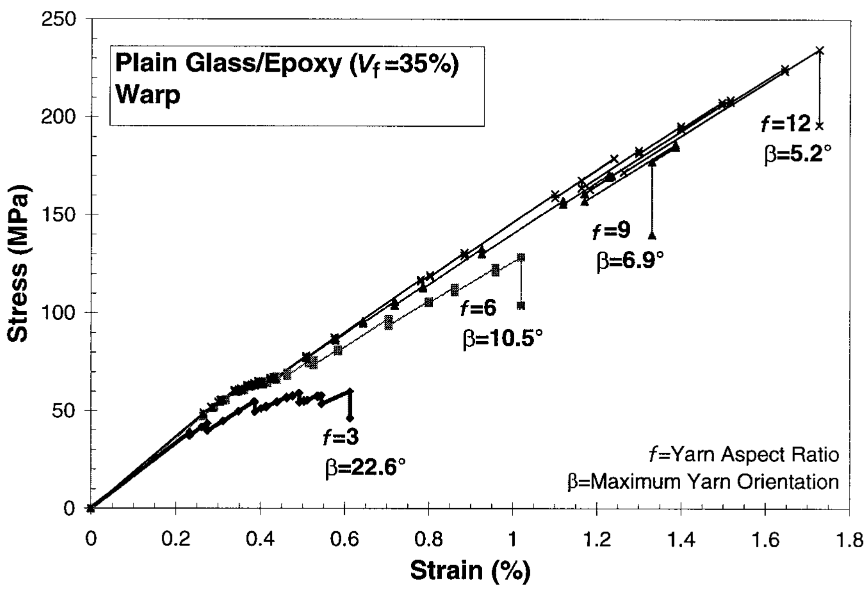
The computed micro-stress fields are very useful to predict the appearance of damage. That is, the model is capable of predicting the point of initial failure using only strength values of the constituent matrix and yarn cells. This point of initial failure is called the *first cell failure*.

For the isotropic matrix material, the paraboloid failure locus applied on the principal stresses is used for its simplicity. It is a flexible failure criterion that yields a unique solution for each loading path. Moreover, it conforms with the basic physical laws and experimental evidence [40]. For the transversely isotropic yarn material, a maximum stress criterion is used. The current yarn stresses $\{\sigma_Y\}$ are computed in the local 123 coordinate system and compared with respective ultimate strengths. It is assumed here that five strength parameters of the impregnated yarn can be estimated from available unidirectional composite strengths. These are: the longitudinal tensile strength X_T, the longitudinal compressive strength X_C, the transverse tensile strength Y_T, the transverse compressive strength Y_C and the shear strength S.

In the *progressive failure analysis*, the effects of matrix and yarn failure are taken into account in an average sense. It is based on the assumption that the damaged material could be replaced with an equivalent material of degraded properties. The properties of the damaged material are adjusted as the loading and progression of damage continue. However, it is not an easy task to determine the degraded properties of the degraded material with certainty [27]. In the present study, the stiffness reduction method as proposed by Blackketter [27] will be used. First, the method accounts for the damage mode when modelling degradation of yarn materials (Table 3.1). If failure is detected, appropriate moduli are reduced. Second, the matrix failure is introduced by reducing the Young's modulus to 1% and the shear modulus to 20% of their original values. After failure, the matrix is no longer isotropic.

After the implementation of the damaged elastic properties in the CEM, another global load increment is applied on the composite. The detailed stress state in the woven fabric composite is updated and compared with the strength properties. The load is increased until (1) a new material cell has failed, or (2) another failure mode is detected for a damaged cell, or (3) catastrophic failure of the total unit cell has occurred. Catastrophic failure is determined by a large displacement or stress fall compared with the previous values. Finally, to clarify the present approach, some important remarks are provided:

- The woven fabric composite is assumed to be *initially free of damage* (cracks, voids, etc.).
- The *non-linearity of the matrix* is not taken into account for two reasons. First, the non-linear behaviour of the different matrix zones in the composite is different from that of the bulk material. This is mainly caused by the presence of local multiaxial stress states and thermal stresses. This information is usually not available. Second, the non-linear stress–strain behaviour of woven fabric composites was shown to be mainly influenced by damage propagation [27] and not by the non-linearity of the matrix.
- The model does not calculate the *fabric geometric deformation* at each load step. This is acceptable for on-axis loading because the strain-to-failure is low. However, it is expected that the model will predict less accurate results for off-axis tensile tests, where the strain-to-failure is much higher.
- The proposed *deterministic modelling* approach will yield a typical 'peaked' stress–strain curve as shown in Fig. 3.13 because several cells fail at the same moment. There is no drop in stress in the experimen-



3.13 Predicted warp stress–strain curves as a function of the yarn aspect ratio *f*.

Table 3.5. Thermo-elastic properties of the matrix and the fibre material

Material	*E* (GPa)	ν	α (/K)
Epoxy matrix	3.13	0.34	6.60×10^{-5}
Glass fibre	73	0.2	4.80×10^{-6}

Table 3.6. Strength parameters for the matrix and the impregnated yarn cells (MPa)

S_C	S_T	X_C	X_T	Y_C	Y_T	S
83	56	610	1462	118	50	72

tally obtained stress–strain curve because the failure of matrix and yarn cells is spread out over a large strain range.

3.4.4 Parametric study: strength analysis

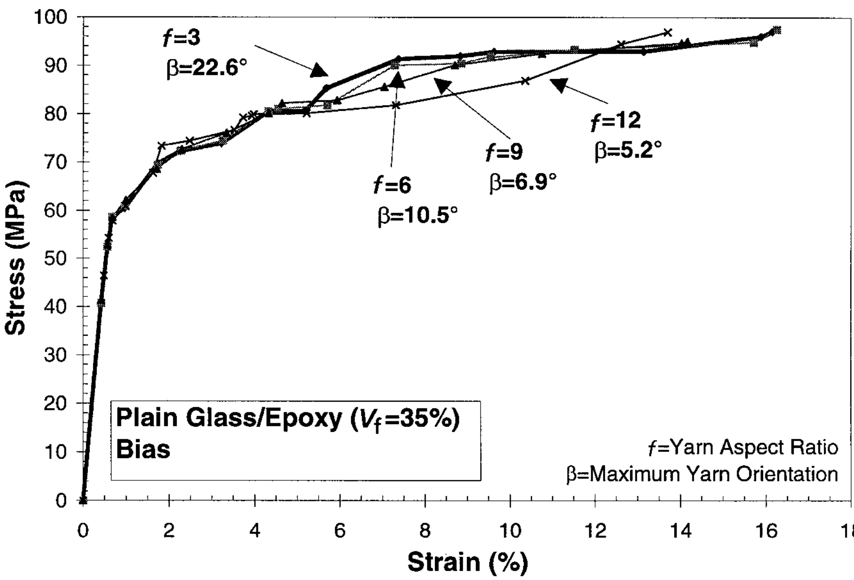
A Fortran code, called WCUnix, has been developed for computing the micro-stress fields, the first cell failure and the ultimate strength of woven fabric composites. The most novel feature offered is the simulation of progressive failure by a stiffness degradation scheme. That is, the analysis of loading becomes a series of elastic analyses. The source code has been compiled on a SUN SparcStation 10. A single load step for a plain fabric composite required about 30 seconds' calculation time. Blackketter reports a calculation time of 30 minutes for each iteration or load step using 3-D finite element modelling on a VAX 8800 computer [27]. Therefore, the WCUnix code is certainly not computationally intensive. Recently, by introducing more time efficient mathematical subroutines, the calculation time has been further reduced drastically.

In this parametric study, a glass/epoxy plain weave fabric composite is considered. The elastic constants of the matrix and fibre constituents are presented in Table 3.5. The strength parameters for the matrix and the transversely isotropic impregnated yarn cells are listed in Table 3.6. Geometric characteristics of the warp and weft yarns are given in Table 3.7.

The format of this study is to change the yarn aspect ratio *f* for both yarn systems. The ratio is set equal to 3, 6, 9 and 12, ranging from rather round to very flat yarn cross-sections. Then, the yarn spacings in warp and weft direction are computed as the width of the yarn plus 20%. The thickness of the composite is computed as the thickness of the plain weave fabric plus

Table 3.7. Yarn characteristics of the plain weave composite

Number of fibres, N_f	Fibre diameter, *d* (mm)	Fibre packing density, *K*
1000	0.01	0.70



3.14 Predicted bias stress–strain curves as a function of the yarn aspect ratio *f*.

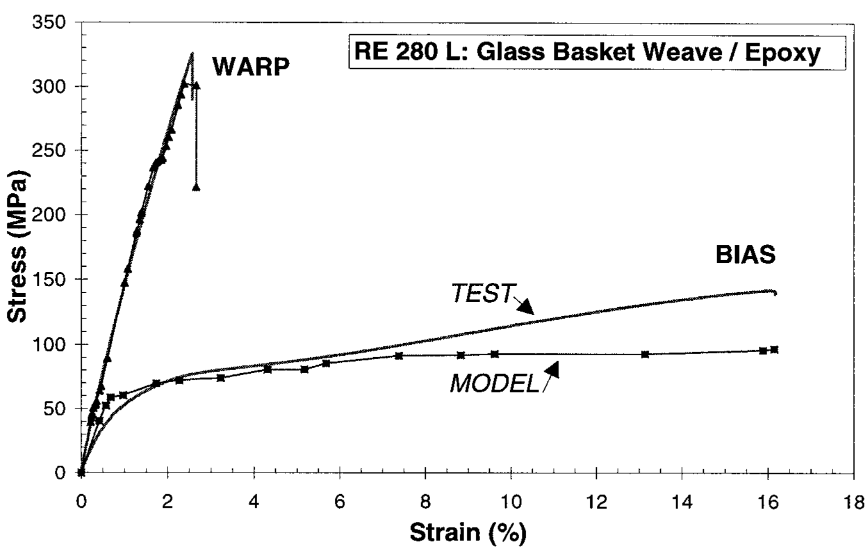
an extra 10%. The predicted fibre volume fraction of all the resulting woven fabric composites equals 35%. Hence, a comparison of the results is possible.

The tensile stress–strain curves are predicted with the WCUnix code. Figures 3.13 and 3.14 plot the computed stress–strain curves in warp and bias direction, respectively. The stiffness reduction scheme does not affect the stress level at which damage initiates; only the final shape of the stress–strain plot after the first cell failure occurs is influenced.

In the *warp direction load case*, we observe a very strong influence of the yarn aspect ratio *f* on the failure behaviour. If the yarn aspect ratio *f* equals 3, the ultimate strength is only 60 MPa and the first cell failure is due to matrix cell failure. However, if the ratio equals 12 (a very flat yarn cross-section), the strength reaches 240 MPa and the first cell failure is due to transverse weft yarn failures at points of maximum yarn curvature. These differences should be attributed to the different geometric architectures. The maximum yarn orientation β (which is directly linked to the yarn cross-section) plays a key role. Basically, the predicted strength decreases considerably with increased yarn undulation. For all four woven fabric composites considered, the ultimate failure is due to warp fibre breakage. The non-linearity of the curves is a result of progressive damage development.

In the *bias direction load case*, we observe only a minor influence of the yarn aspect ratio *f* on the failure behaviour. The first cell failure is always due to transverse yarn failure. The strength of the woven fabric composite is related to the failure of a undirectional lamina or yarn cell. The failure of a single yarn cell in the bias direction is only weakly dependent on the out-of-plane lamina orientation. Therefore, the predicted strength of the composite is constant.

Figure 3.15 compares the experimental and theoretical stress–strain curves for the RE280 glass/epoxy composite. The RE280 basket weave fabric has been supplied by the Syncoglas company, Belgium. The elastic and strength properties of the constituent materials are readily available and listed in Tables 3.5 and 3.6. Three important observations are presented here. First, a *very good agreement* is observed between experiment and theory on glass/epoxy composites. Second, the theoretical and experimen-



3.15 Theoretical and experimental stress–strain curves in warp and bias directions.

tal stress–strain curves in the *warp direction* are apparently straight lines. However, the curves are nonlinear due to the characteristic knee behaviour for woven fabric composites. The knee is the result of transverse weft yarn failures. The position of the knee is predicted very accurately with the CEM. Finally, the yarn reorientation in the loading direction plays an important role in the *bias specimen*. At large strains, the experimental stresses are higher than the predicted ones. This is because the model does not account for the fabric yarn reorientation, which becomes significant at strains higher than 10%. The yarns rotate towards a smaller angle with respect to the loading direction, resulting in a stiffness increase. Therefore, the ultimate bias strength is not predicted very accurately.

3.5 Conclusions

This chapter has addressed the important issue of developing micro-mechanical models for woven fabric composites. Besides an extensive literature review on the modelling of 2-D woven fabric composites, a fully 3-D geometric, elastic and strength analysis has been presented as an example of the vigour in textile composites research. A brief summary of the principal conclusions follows.

- Many researchers have used the *isostrain technique* to model the 3-D elastic properties of 2-D woven fabric composites. The model is indeed a 'quick' method to calculate an upper and lower bound for the effective stiffness matrix because it only requires yarn orientation data. Most researchers agree that the upper bound yields much better results than the lower bound. The technique can easily be applied to other textile composites. We have experienced that for most woven fabric composites, the isostrain models predict correct in-plane elastic properties but incorrect out-of-plane properties. The relative position of the predicted shear moduli is always false [21].
- Development of a *geometric decomposition scheme* for woven fabric composites with an arbitrary 2-D architecture. It is new because of its clear geometric concept (a library of building blocks for woven fabric composites), its easy bookkeeping of geometric data and its ability to describe non-traditional fabrics. Moreover, by extending the library of building blocks, solid 3-D woven fabric composites and braided fabric composites are actually analysed, using the same approach.
- For modelling woven fabric composites one can find inspiration in the extensive modelling efforts on unidirectional and short fibre random composites. However, *analogies can be misleading*. The 'yarn distribution' of a textile composite is determined by the textile processing route (yarn level analysis). For woven fabric composites the characteristic

yarn interlacings, curvatures and locations should be taken into account, reflecting the actual geometry. One of the critical determinants of effective woven fabric composites properties is definitely the *yarn orientation*. However, the *yarn position* is also critical for the accurate prediction of both stiffness and strength properties. Because it is not always easy to account for the position effect, it is neglected, minimized or concealed by most researchers.

- The development of mechanical models or the application of optimization models to the design of large woven fabric composite structures is difficult because the number of geometric design variables and constraints is large. A remedy is to break the problem into several smaller *subproblems*. Although this is not in itself a new discovery, it is important to be fully aware of that fact and draw the appropriate conclusions. We are able to report that a 'multilevel decomposition, multistep homogenization' approach has been developed to solve the stiffness, stress and strength analysis problem for 2-D woven fabric composites. Although we have adopted fully the complementary energy approach, it should be stressed that different modelling strategies can be used to solve the different subproblems. The method is only limited by the capability to come up with an appropriate decomposition in subproblems.
- The new and successful *complementary energy model* is based on solving the stress analysis problem first. Second, the computed stress concentration factors yield a solution for the 3-D stiffness properties. To the best of our knowledge, this kind of model has not yet been presented in the literature for woven fabric composites. Finally, the accurate stress fields serve as a useful tool for the strength analysis. One major advantage is that any type of simple or combined multiaxial loading can be applied. Another advantage is that the model can be extended to take into account the presence of residual thermal stresses.

Although a considerable body of knowledge has been generated in the past years, more research is required to develop design guidelines for optimizing material performance by manipulating the woven fabric architecture. Future research could address the following topics:

- Design of structural components such as aircraft parts, automobile chassis elements or bicycle frames tends to be very complex and time-consuming. The development of efficient analytical *pre-processors for woven fabric composites* can decrease cost and make finite element modelling an economic and easy-to-use solution. Using pre-processors, the production of a correct finite element model will only require the generation of the component geometry, but not any more the detailed and elaborate description of unit cells, textile style, yarn size and yarn undulation.

- The design of woven fabric composite materials will grow dependent on computer models. The optimization of a microstructure will require the marriage of micromechanical and *optimization models*. Therefore, optimization models should be developed which can optimize at once certain geometric, mechanical, thermal, process and economic properties.

3.6 Acknowledgements

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