CHAPTER 4

BENDING

Summary

The simple theory of elastic bending states that

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

where M is the applied bending moment (B.M.) at a transverse section, I is the second moment of area of the beam cross-section about the neutral axis (N.A.), σ is the stress at distance y from the N.A. of the beam cross-section, E is the Young's modulus of elasticity for the beam material, and R is the radius of curvature of the N.A. at the section.

Certain assumptions and conditions must obtain before this theory can strictly be applied: see page 64.

In some applications the following relationship is useful:

$$M = Z\sigma_{\rm max}$$

where $Z = I/y_{\text{max}}$ and is termed the section modulus; σ_{max} is then the stress at the maximum distance from the N.A.

The most useful standard values of the second moment of area I for certain sections are as follows (Fig. 4.1):

rectangle about axis through centroid =
$$\frac{bd^3}{12} = I_{NA}$$
.

rectangle about axis through side
$$=\frac{bd^3}{3}=I_{XX}$$

circle about axis through centroid
$$=\frac{\pi D^4}{64}=I_{\rm N.A.}$$



Fig. 4.1.

The *centroid* is the centre of area of the section through which the N.A., or axis of zero stress, is always found to pass.

In some cases it is convenient to determine the second moment of area about an axis other than the N.A. and then to use the *parallel axis theorem*.

$$I_{\rm NA} = I_G + Ah^2$$

For composite beams one material is replaced by an equivalent width of the other material given by

$$t'=\frac{E}{E'}\,t$$

where E/E' is termed the modular ratio. The relationship between the stress in the material and its equivalent area is then given by

$$\sigma = \frac{E}{E'}\sigma'$$

For skew loading of symmetrical sections the stress at any point (x, y) is given by

$$\sigma = \frac{M_{xx}}{I_{xx}} y \pm \frac{M_{yy}}{I_{yy}} x$$

the angle of the N.A. being given by

$$\tan\theta = \pm \frac{M_{yy}}{M_{xx}} \frac{I_{xx}}{I_{yy}}$$

For eccentric loading on one axis,

$$\sigma = \frac{P}{A} \pm \frac{Pey}{I}$$

the N.A. being positioned at a distance

$$y_N = \pm \frac{I}{Ae}$$

from the axis about which the eccentricity is measured.

For eccentric loading on two axes,

$$\sigma = \frac{P}{A} \pm \frac{Ph}{I_{yy}} x \pm \frac{Pk}{I_{xx}} y$$

For concrete or masonry rectangular or circular section columns, the load must be retained within the middle third or middle quarter areas respectively.

Introduction

If a piece of rubber, most conveniently of rectangular cross-section, is bent between one's fingers it is readily apparent that one surface of the rubber is stretched, i.e. put into tension, and the opposite surface is compressed. The effect is clarified if, before bending, a regular set of lines is drawn or scribed on each surface at a uniform spacing and perpendicular to the axis

of the rubber which is held between the fingers. After bending, the spacing between the set of lines on one surface is clearly seen to increase and on the other surface to reduce. The thinner the rubber, i.e. the closer the two marked faces, the smaller is the effect for the same applied moment. The change in spacing of the lines on each surface is a measure of the strain and hence the stress to which the surface is subjected and it is convenient to obtain a formula relating the stress in the surface to the value of the B.M. applied and the amount of curvature produced. In order for this to be achieved it is necessary to make certain simplifying assumptions, and for this reason the theory introduced below is often termed the simple theory of bending. The assumptions are as follows:

- (1) The beam is initially straight and unstressed.
- (2) The material of the beam is perfectly homogeneous and isotropic, i.e. of the same density and elastic properties throughout.
- (3) The elastic limit is nowhere exceeded.
- (4) Young's modulus for the material is the same in tension and compression.
- (5) Plane cross-sections remain plane before and after bending.
- (6) Every cross-section of the beam is symmetrical about the plane of bending, i.e. about an axis perpendicular to the N.A.
- (7) There is no resultant force perpendicular to any cross-section.

4.1. Simple bending theory

If we now consider a beam initially unstressed and subjected to a constant B.M. along its length, i.e. pure bending, as would be obtained by applying equal couples at each end, it will bend to a radius R as shown in Fig. 4.2b. As a result of this bending the top fibres of the beam will be subjected to tension and the bottom to compression. It is reasonable to suppose, therefore, that somewhere between the two there are points at which the stress is zero. The locus of all such points is termed the *neutral axis*. The radius of curvature R is then measured to this axis. For symmetrical sections the N.A. is the axis of symmetry, but whatever the section the N.A. will always pass through the centre of area or centroid.



Fig. 4.2. Beam subjected to pure bending (a) before, and (b) after, the moment M has been applied.

Consider now two cross-sections of a beam, HE and GF, originally parallel (Fig. 4.2a). When the beam is bent (Fig. 4.2b) it is assumed that these sections remain plane; i.e. H'E' and G'F', the final positions of the sections, are still straight lines. They will then subtend some angle θ .

Consider now some fibre AB in the material, distance y from the N.A. When the beam is bent this will stretch to A'B'.

Strain in fibre
$$AB = \frac{\text{extension}}{\text{original length}} = \frac{A'B' - AB}{AB}$$

But AB = CD, and, since the N.A. is unstressed, CD = C'D'.

strain =
$$\frac{A'B' - C'D'}{C'D'} = \frac{(R+y)\theta - R\theta}{R\theta} = \frac{y}{R}$$

stress = Young's modulus Estrain

$$strain = \frac{\sigma}{E}$$

Equating the two equations for strain,

$$\frac{\sigma}{E} = \frac{y}{R}$$

$$\frac{\sigma}{y} = \frac{E}{R}$$
(4.1)

Consider now a cross-section of the beam (Fig. 4.3). From eqn. (4.1) the stress on a fibre at distance y from the N.A. is

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$$\sigma = \frac{E}{R} y$$

Fig. 4.3. Beam cross-section.

If the strip is of area δA the force on the strip is

$$F = \sigma \delta A = \frac{E}{R} y \delta A$$

This has a moment about the N.A. of

$$Fy = \frac{E}{R} y^2 \delta A$$



...

But

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or

The total moment for the whole cross-section is therefore

$$M = \sum \frac{E}{R} y^2 \delta A$$
$$= \frac{E}{R} \sum y^2 \delta A$$

since E and R are assumed constant.

The term $\sum y^2 \delta A$ is called the second moment of area of the cross-section and given the symbol *I*.

$$M = \frac{E}{R}I$$
 and $\frac{M}{I} = \frac{E}{R}$ (4.2)

Combining eqns. (4.1) and (4.2) we have the bending theory equation

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$
(4.3)

From eqn. (4.2) it will be seen that if the beam is of uniform section, the material of the beam is homogeneous and the applied moment is constant, the values of I, E and M remain constant and hence the radius of curvature of the bent beam will also be constant. Thus for pure bending of uniform sections, beams will deflect into circular arcs and for this reason the term *circular bending* is often used. From eqn. (4.2) the radius of curvature to which any beam is bent by an applied moment M is given by:

$$R=\frac{EI}{M}$$

and is thus directly related to the value of the quantity EI. Since the radius of curvature is a direct indication of the degree of flexibility of the beam (the larger the value of R, the smaller the deflection and the greater the rigidity) the quantity EI is often termed the *flexural rigidity* or *flexural stiffness* of the beam. The relative stiffnesses of beam sections can then easily be compared by their EI values.

It should be observed here that the above proof has involved the assumption of pure bending without any shear being present. From the work of the previous chapter it is clear that in most practical beam loading cases shear and bending occur together at most points. Inspection of the S.F. and B.M. diagrams, however, shows that when the B.M. is a maximum the S.F. is, in fact, always zero. It will be shown later that bending produces by far the greatest magnitude of stress in all but a small minority of special loading cases so that beams designed on the basis of the maximum B.M. using the simple bending theory are generally more than adequate in strength at other points.

4.2. Neutral axis

As stated above, it is clear that if, in bending, one surface of the beam is subjected to tension and the opposite surface to compression there must be a region within the beam cross-section at which the stress changes sign, i.e. where the stress is zero, and this is termed the *neutral axis*.

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Further, eqn. (4.3) may be re-written in the form

$$\sigma = \frac{M}{I} y \tag{4.4}$$

which shows that at any section the stress is directly proportional to y, the distance from the N.A., i.e. σ varies linearly with y, the maximum stress values occurring in the outside surface of the beam where y is a maximum.

Consider again, therefore, the general beam cross-section of Fig. 4.3 in which the N.A. is located at some arbitrary position. The force on the small element of area is σdA acting perpendicular to the cross-section, i.e. parallel to the beam axis. The total force parallel to the beam axis is therefore $\int \sigma dA$.

Now one of the basic assumptions listed earlier states that when the beam is in equilibrium there can be no resultant force across the section, i.e. the tensile force on one side of the N.A. must exactly balance the compressive force on the other side.

$$\int \sigma dA = 0$$

Substituting from eqn. (4.1)

$$\int \frac{E}{R} y dA = 0 \quad \text{and hence} \quad \frac{E}{R} \int y dA = 0$$

This integral is the *first moment of area* of the beam cross-section about the N.A. since y is always measured from the N.A. Now the only first moment of area for the cross-section which is zero is that about an axis through the centroid of the section since this is the basic condition required of the centroid. It follows therefore that *the neutral axis must always pass through the centroid*.

It should be noted that this condition only applies with stresses maintained within the elastic range and different conditions must be applied when stresses enter the plastic range of the materials concerned.

Typical stress distributions in bending are shown in Fig. 4.4. It is evident that the material near the N.A. is always subjected to relatively low stresses compared with the areas most removed from the axis. In order to obtain the maximum resistance to bending it is advisable therefore to use sections which have large areas as far away from the N.A. as possible. For this reason beams with I- or T-sections find considerable favour in present engineering applications, such as girders, where bending plays a large part. Such beams have large moments of area about one axis and, provided that it is ensured that bending takes place about this axis, they will have a high resistance to bending stresses.



Fig. 4.4. Typical bending stress distributions.

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4.3. Section modulus

From eqn. (4.4) the maximum stress obtained in any cross-section is given by

$$\sigma_{\max} = \frac{M}{I} y_{\max} \tag{4.5}$$

For any given allowable stress the maximum moment which can be accepted by a particular shape of cross-section is therefore

$$M = \frac{I}{y_{\max}} \sigma_{\max}$$

For ready comparison of the strength of various beam cross-sections this is sometimes written in the form

$$M = Z\sigma_{\max} \tag{4.6}$$

where $Z(=I/y_{max})$ is termed the section modulus. The higher the value of Z for a particular cross-section the higher the B.M. which it can withstand for a given maximum stress.

In applications such as cast-iron or reinforced concrete where the properties of the material are vastly different in tension and compression two values of maximum allowable stress apply. This is particularly important in the case of unsymmetrical sections such as T-sections where the values of y_{max} will also be different on each side of the N.A. (Fig. 4.4) and here two values of section modulus are often quoted,

$$Z_1 = I/y_1$$
 and $Z_2 = I/y_2$ (4.7)

each being then used with the appropriate value of allowable stress.

Standard handbooks[†] are available which list section modulus values for a range of girders, etc; to enable appropriate beams to be selected for known section modulus requirements.

4.4. Second moment of area

Consider the rectangular beam cross-section shown in Fig. 4.5 and an element of area dA, thickness dy, breadth B and distance y from the N.A. which by symmetry passes through the



Fig. 4.5.

† Handbook on Structural Steelwork. BCSA/CONSTRADO. London, 1971, Supplement 1971, 2nd Supplement 1976 (in accordance with BS449, 'The use of structural steel in building'). Structural Steelwork Handbook for Standard Metric Sections. CONSTRADO. London, 1976 (in accordance with BS4848, 'Structural hollow sections').

centre of the section. The second moment of area I has been defined earlier as

 $I=\int y^2 dA$

Thus for the rectangular section the second moment of area about the N.A., i.e. an axis through the centre, is given by

$$H_{N.A.} = \int_{-D/2}^{D/2} y^2 B dy = B \int_{-D/2}^{D/2} y^2 dy$$

= $B \left[\frac{y^3}{3} \right]_{-D/2}^{D/2} = \frac{BD^3}{12}$ (4.8)

Similarly, the second moment of area of the rectangular section about an axis through the lower edge of the section would be found using the same procedure but with integral limits of 0 to D,

$$I = B \left[\frac{y^3}{3} \right]_0^D = \frac{BD^3}{3}$$
(4.9)

These standard forms prove very convenient in the determination of I_{NA} values for built-up sections which can be conveniently divided into rectangles. For symmetrical sections as, for instance, the I-section shown in Fig. 4.6,



Fig. 4.6.

 $I_{\rm NA} = I$ of dotted rectangle -I of shaded portions

$$=\frac{BD^{3}}{12} - 2\left(\frac{bd^{3}}{12}\right)$$
(4.10)

It will be found that any symmetrical section can be divided into convenient rectangles with the N.A. running through each of their centroids and the above procedure can then be employed to effect a rapid solution.

For unsymmetrical sections such as the T-section of Fig. 4.7 it is more convenient to divide the section into rectangles with their *edges* in the N.A., when the second type of standard form may be applied.

$$I_{\text{N.A.}} = I_{ABCD} - I_{\text{shaded areas}} + I_{EFGH}$$

(about DC) (about DC) (about HG)

(each of these quantities may be written in the form $BD^3/3$).





As an alternative procedure it is possible to determine the second moment of area of each rectangle about an axis through its own centroid $(I_G = BD^3/12)$ and to "shift" this value to the equivalent value about the N.A. by means of the *parallel axis theorem*.

$$I_{\mathrm{NA}} = I_G + Ah^2 \tag{4.11}$$

where A is the area of the rectangle and h the distance of its centroid G from the N.A. Whilst this is perhaps not so quick or convenient for sections built-up from rectangles, it is often the only procedure available for sections of other shapes, e.g. rectangles containing circular holes.

4.5. Bending of composite or flitched beams

(a) A composite beam is one which is constructed from a combination of materials. If such a beam is formed by rigidly bolting together two timber joists and a reinforcing steel plate, then it is termed a *flitched beam*.

Since the bending theory only holds good when a constant value of Young's modulus applies across a section it cannot be used directly to solve composite-beam problems where two different materials, and therefore different values of E, are present. The method of solution in such a case is to replace one of the materials by an *equivalent section* of the other.



Fig. 4.8. Bending of composite or flitched beams: original beam cross-section and equivalent of uniform material (wood) properties.

Consider, therefore, the beam shown in Fig. 4.8 in which a steel plate is held centrally in an appropriate recess between two blocks of wood. Here it is convenient to replace the steel by an equivalent area of wood, retaining the same bending strength, i.e. the moment at any section must be the same in the equivalent section as in the original so that the force at any given dy in the equivalent beam must be equal to that at the strip it replaces.

$$\sigma t = \sigma' t' dy$$

$$\sigma t = \sigma' t' \qquad (4.12)$$

$$\varepsilon E t = \varepsilon' E' t'$$

$$\frac{\sigma}{\varepsilon} = E$$

since

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i.e.

Again, for true similarity the strains must be equal,

 σ

Et = E't' or $\frac{t'}{t} = \frac{E}{E'}$ (4.13)

$$=\frac{E}{E'}t$$
(4.14)

Thus to replace the steel strip by an equivalent wooden strip the thickness must be multiplied by the modular ratio E/E'.

The equivalent section is then one of the same material throughout and the simple bending theory applies. The stress in the wooden part of the original beam is found directly and that in the steel found from the value at the same point in the equivalent material as follows:

from eqn. (4.12)
$$\frac{\sigma}{\sigma'} = \frac{t'}{t}$$

and from eqn. (4.13)
$$\frac{\sigma}{\sigma'} = \frac{E}{E'} \quad \text{or} \quad \sigma = \frac{E}{E'} \sigma' \qquad (4.15)$$

i.e. stress in steel = modular ratio × stress in equivalent wood

 $\varepsilon = \varepsilon'$

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The above procedure, of course, is not limited to the two materials treated above but applies equally well for any material combination. The wood and steel flitched beam was merely chosen as a convenient example.

4.6. Reinforced concrete beams-simple tension reinforcement

Concrete has a high compressive strength but is very weak in tension. Therefore in applications where tension is likely to result, e.g. bending, it is necessary to reinforce the concrete by the insertion of steel rods. The section of Fig. 4.9a is thus a compound beam and can be treated by reducing it to the equivalent concrete section, shown in Fig. 4.9b.

In calculations, the concrete is assumed to carry no tensile load; hence the gap below the N.A. in Fig. 4.9b. The N.A. is then fixed since it must pass through the centroid of the area assumed in this figure: i.e. moments of area about the N.A. must be zero.

Let t = tensile stress in the steel,

c =compressive stress in the concrete,

A =total area of steel reinforcement,

m =modular ratio, $E_{\text{steel}}/E_{\text{concrete}}$,

other symbols representing the dimensions shown in Fig. 4.9.

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Fig. 4.9. Bending of reinforced concrete beams with simple tension reinforcement.

$$bh\frac{h}{2} = mA(d-h) \tag{4.16}$$

which can be solved for h.

The moment of resistance is then the moment of the couple in Fig. 4.9c and d. Therefore

moment of resistance (based on compressive forces)

$$M = (bh) \times \frac{c}{2} \times \left(d - \frac{h}{3}\right) = \frac{bhc}{2} \left(d - \frac{h}{3}\right)$$
(4.17)
area
average lever
stress arm

Similarly,

Then

moment of resistance (based on tensile forces)

$$M = mA \times \frac{t}{m} \times \left(d - \frac{h}{3}\right) = At\left(d - \frac{h}{3}\right)$$
(4.18)
stress lever arm

Both t and c are usually given as the maximum allowable values, which may or may not be reached at the same time. Equations (4.17) and (4.18) must both be worked out, therefore, and the lowest value taken, since the larger moment would give a stress greater than the allowed maximum stress in the other material.

In design applications where the dimensions of reinforced concrete beams are required which will carry a known B.M. the above equations generally contain too many unknowns,

and certain simplifications are necessary. It is usual in these circumstances to assume a *balanced* section, i.e. one in which the maximum allowable stresses in the steel and concrete occur simultaneously. There is then no wastage of materials, and for this reason the section is also known as an *economic* or *critical* section.

For this type of section the N.A. is positioned by proportion of the stress distribution (Fig. 4.9c).

Thus by similar triangles

$$\frac{c}{h} = \frac{t/m}{(d-h)}$$
$$mc(d-h) = th$$
(4.19)

Thus d can be found in terms of h, and since the moment of resistance is known this relationship can be substituted in eqn. (4.17) to solve for the unknown depth d.

Also, with a balanced section,

moment of resistance (compressive) = moment of resistance (tensile)

$$bh\frac{c}{2}\left(d-\frac{h}{3}\right) = mA\frac{t}{m}\left(d-\frac{h}{3}\right)$$
$$\frac{bhc}{2} = At \qquad (4.20)$$

By means of eqn. (4.20) the required total area of reinforcing steel A can thus be determined.

4.7. Skew loading (bending of symmetrical sections about axes other than the axes of symmetry)

Consider the simple rectangular-section beam shown in Fig. 4.10 which is subjected to a load inclined to the axes of symmetry. In such cases bending will take place about an inclined axis, i.e. the N.A. will be inclined at some angle θ to the XX axis and deflections will take place perpendicular to the N.A.



Fig. 4.10. Skew loading of symmetrical section.

In such cases it is convenient to resolve the load P, and hence the applied moment, into its components parallel with the axes of symmetry and to apply the simple bending theory to the resulting bending about both axes. It is thus assumed that simple bending takes place

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simultaneously about both axes of symmetry, the total stress at any point (x, y) being given by combining the results of the separate bending actions algebraically using the normal conventions for the signs of the stress, i.e. tension-positive, compression-negative.

Thus
$$\sigma = \frac{M_{xx}}{I_x} y \pm \frac{M_{yy}}{I_{yy}} x$$
(4.21)

The equation of the N.A. is obtained by setting eqn. (4.21) to zero,

i.e.
$$\tan \theta = \frac{y}{x} = \pm \frac{M_{yy}}{M_{xx}} \frac{I_{xx}}{I_{yy}}$$
(4.22)

4.8. Combined bending and direct stress-eccentric loading

(a) Eccentric loading on one axis

There are numerous examples in engineering practice where tensile or compressive loads on sections are not applied through the centroid of the section and which thus will introduce not only tension or compression as the case may be but also considerable bending effects. In concrete applications, for example, where the material is considerably weaker in tension than in compression, any bending and hence tensile stresses which are introduced can often cause severe problems. Consider, therefore, the beam shown in Fig. 4.11 where the load has been applied at an eccentricity e from one axis of symmetry. The stress at any point is determined by calculating the bending stress at the point on the basis of the simple bending theory and combining this with the direct stress (load/area), taking due account of sign,

i.e.
$$\sigma = \frac{P}{A} \pm \frac{My}{l}$$
(4.23)

where

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$$M = applied moment = Pe$$

$$\sigma = \frac{P}{A} \pm \frac{Pey}{I} \tag{4.24}$$

The positive sign between the two terms of the expression is used when both parts have the same effect and the negative sign when one produces tension and the other compression.



Fig. 4.11. Combined bending and direct stress - eccentric loading on one axis.

It should now be clear that any eccentric load can be treated as precisely equivalent to a direct load acting through the centroid plus an applied moment about an axis through the centroid equal to load \times eccentricity. The distribution of stress across the section is then given by Fig. 4.12.



Fig. 4.12. Stress distributions under eccentric loading.

The equation of the N.A. can be obtained by setting σ equal to zero in eqn. (4.24),

i.e.
$$y = \pm \frac{I}{Ae} = y_N$$
 (4.25)

Thus with the load eccentric to one axis the N.A. will be parallel to that axis and a distance y_N from it. The larger the eccentricity of the load the nearer the N.A. will be to the axis of symmetry through the centroid for given values of A and I.

(b) Eccentric loading on two axes

It some cases the applied load will not be applied on either of the axes of symmetry so that there will now be a direct stress effect plus simultaneous bending about both axes. Thus, for the section shown in Fig. 4.13, with the load applied at P with eccentricities of h and k, the total stress at any point (x, y) is given by



Fig. 4.13. Eccentric loading on two axes showing possible position of neutral axis SS.

Again the equation of the N.A is obtained by equating eqn. (4.26) to zero, when

$$\frac{P}{A} \pm \frac{Phx}{I_{yy}} \pm \frac{Pky}{I_{xx}} = 0$$

or

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$$\frac{Ahx}{I_{yy}} \pm \frac{Aky}{I_{xx}} = \pm 1 \tag{4.27}$$

This equation is a linear equation in x and y so that the N.A. is a straight line such as SS which may or may not cut the section.

4.9. "Middle-quarter" and "middle-third" rules

It has been stated earlier that considerable problems may arise in the use of cast-iron or concrete sections in applications in which eccentric loads are likely to occur since both materials are notably weaker in tension than in compression. It is convenient, therefore, that for rectangular and circular cross-sections, provided that the load is applied within certain defined areas, no tension will be produced whatever the magnitude of the applied compressive load. (Here we are solely interested in applications such as column and girder design which are principally subjected to compression.)

Consider, therefore, the rectangular cross-section of Fig. 4.13. The stress at any point (x, y) is given by eqn. (4.26) as

$$\sigma = \frac{P}{A} \pm \frac{Phx}{I_{yy}} \pm \frac{Pky}{I_{xx}}$$

Thus, with a compressive load applied, the most severe tension stresses are introduced when the last two terms have their maximum value and are tensile in effect,

i.e.
$$\sigma = \frac{P}{A} - \frac{Ph}{I_{yy}} \times \frac{b}{2} - \frac{Pk}{I_{xx}} \times \frac{d}{2}$$
$$= \frac{P}{bd} - \frac{Phb}{2} \times \frac{12}{db^3} - \frac{Pkd}{2} \times \frac{12}{bd^3}$$

For no tension to result in the section, σ must be equated to zero,

$$0 = \frac{1}{bd} - \frac{6h}{db^2} - \frac{6k}{bd^2}$$

or
$$\frac{bd}{6} = dh + bk$$

This is a linear expression in h and k producing the line SS in Fig. 4.13. If the load is now applied in each of the other three quadrants the total limiting area within which P must be applied to produce zero tension in the section is obtained. This is the diamond area shown shaded in Fig. 4.14 with diagonals of b/3 and d/3 and hence termed the *middle third*.

For circular sections of diameter d, whatever the position of application of P, an axis of symmetry will pass through this position so that the problem reduces to one of eccentricity about a single axis of symmetry.

Now from eqn. (4.23)

$$\sigma = \frac{P}{A} \pm \frac{Pe}{I} y$$



Fig. 4.14. Eccentric loading of rectangular sections-"middle third".

Therefore for zero tensile stress in the presence of an eccentric compressive load

$$\frac{P}{A} = \frac{Pey}{\pi d^2}$$
$$\frac{4}{\pi d^2} = e \times \frac{d}{2} \times \frac{64}{\pi d^4}$$
$$e = \frac{d}{8}$$

Thus the limiting region for application of the load is the shaded circular area of diameter d/4 (shown in Fig. 4.15) which is termed the *middle quarter*.



Fig. 4.15. Eccentric loading of circular sections-"middle quarter".

4.10. Shear stresses owing to bending

It can be shown that any cross-section of a beam subjected to bending by transverse loads experiences not only direct stresses as given by the bending theory but also shear stresses. The magnitudes of these shear stresses at a particular section is always such that they sum up to the total shear force Q at that section. A full treatment of the procedures used to determine the distribution of the shear stresses is given in Chapter 7.

4.11. Strain energy in bending

For beams subjected to bending the total strain energy of the system is given by

$$U = \int_{0}^{L} \frac{M^2 ds}{2EI}$$

For uniform beams, or parts of beams, subjected to a constant B.M. M, this reduces to

$$U = \frac{M^2 L}{2EI}$$

In most beam-loading cases the strain energy due to bending far exceeds that due to other forms of loading, such as shear or direct stress, and energy methods of solution using *Castigliano* or *unit load* procedures based on the above equations are extremely powerful methods of solution. These are covered fully in Chapter 11.

4.12. Limitations of the simple bending theory

It has been observed earlier that the theory introduced in preceding sections is often termed the "simple theory of bending" and that it relies on a number of assumptions which either have been listed on page 64 or arise in the subsequent proofs. It should thus be evident that in practical engineering situations the theory will have certain limitations depending on the degree to which these assumptions can be considered to hold true. The following paragraphs give an indication of when some of the more important assumptions can be taken to be valid and when alternative theories or procedures should be applied.

Assumption:	Stress is proportional to the distance from the axis of zero stress (neutral axis), i.e. $\sigma = E_V/R = E\varepsilon$.		
	Correct	for homogeneous beams within the elastic range.	
	Incorrect	(a) for loading conditions outside the elastic range when $\sigma \neq E$,	
		(b) for composite beams with different materials or pro- perties when 'equivalent sections' must be used; see §4.5	
Assumption:	Strain is proportional to the distance from the axis of zero strain, i.e. $\varepsilon = y/R$.		
	Correct	for initially straight beams or, for engineering purposes,	
		beams with $R/d > 10$ (where $d =$ total depth of section).	
	Incorrect	for initially curved beams for which special theories have	
		been developed or to which correction factors $\sigma = K (My/I)$ may be applied.	
Assumption:	Neutral axis passes through the centroid of the section.		
	Correct	for pure bending with no axial load.	
	Incorrect	for combined bending and axial load systems such as eccentric loading. In such cases the loading effects must be separated, stresses arising from each calculated and the results superimposed – see 84.8	

Assumption:	Plane cross-sections remain plane.		
	Correct	(a) for cross-sections at a reasonable distance from points of	
		iocal loading of stress concentration (usually taken to be	
		(b) when change of cross section with length is gradual	
		(b) when change of eros-section with length is gradual,	
	These cond	(c) In the absence of end-condition spurious chects.	
	Incorrect	(a) for points of local loading:	
	meoneer	(b) at positions of stress concentration such as holes,	
		keyways, fillets and other changes in geometry;	
		(c) in regions of rapid change of cross-section.	
		In such cases appropriate stress concentration factors † must	
		be applied or experimental stress/strain analysis techniques adopted.	
Assumption:	The axis of the applied bending moment is coincident with the neutral axis.		
	Correct	when the axis of bending is a principal axis $(I_{xy} = 0)$ e.g. on axis of symmetry	
	Incorrect	for so-called unsymmetrical bending cases when the axis of	
		the applied bending moment is not a principal axis. In such cases the moment should be resolved into components about the principal axes.	
Assumption:	Lateral contraction or expansion is not prevented.		
	Correct	when the beam can be considered narrow (i.e. width the same order as the depth)	
	Incorrect	for wide beams or plates in which the width may be many times the depth. Special procedures apply for such cases.	

It should now be evident that care is required in the application of "simple" theory and reference should be made where necessary to more advanced theories.

Examples

Example 4.1

An I-section girder, 200 mm wide by 300 mm deep, with flange and web of thickness 20 mm is used as a simply supported beam over a span of 7 m. The girder carries a distributed load of 5 kN/m and a concentrated load of 20 kN at mid-span. Determine: (a) the second moment of area of the cross-section of the girder, (b) the maximum stress set-up.

Solution

(a) The second moment of area of the cross-section may be found in two ways. Method 1-Use of standard forms

For sections with symmetry about the N.A., use can be made of the standard I value for a rectangle about an axis through its centroid, i.e. $bd^3/12$. The section can thus be divided into convenient rectangles for each of which the N.A. passes through the centroid, e.g. in this case, enclosing the girder by a rectangle (Fig. 4.16).



Fig. 4.16.

For sections without symmetry about the N.A., e.g. a T-section, a similar procedure can be adopted, this time dividing the section into rectangles with their edges in the N.A. and applying the standard $I = bd^3/3$ for this condition (see Example 4.2).

Method 2-Parallel axis theorem

Consider the section divided into three parts-the web and the two flanges.

$$I_{\text{N.A.}} \text{ for the web} = \frac{bd^3}{12} = \left[\frac{20 \times 260^3}{12}\right] 10^{-12}$$

I of flange about $AB = \frac{bd^3}{12} = \left[\frac{200 \times 20^3}{12}\right] 10^{-12}$

Therefore using the parallel axis theorem

 $I_{\rm N.A.}$ for flange = $I_{AB} + Ah^2$

where h is the distance between the N.A. and AB,

$$I_{\text{NA.}}$$
 for flange = $\left[\frac{200 \times 20^3}{12}\right] 10^{-12} + \left[(200 \times 20)140^2\right] 10^{-12}$

Therefore total I_{NA} of girder

$$= 10^{-12} \left\{ \left[\frac{20 \times 260^3}{12} \right] + 2 \left[\frac{200 \times 20^3}{12} \right] + 200 \times 20 \times 140^2 \right\}$$

= 10⁻⁶ (29.3 + 0.267 + 156.8)
= 1.86 × 10⁻⁴ m⁴

Both methods thus yield the same value and are equally applicable in most cases. Method 1, however, normally yields the quicker solution.

(b) The maximum stress may be found from the simple bending theory of eqn. (4.4),

i.e.
$$\sigma_{\max} = \frac{M_{\max} y_{\max}}{I}$$

Now the maximum B.M. for a beam carrying a u.d.l. is at the centre and given by $wL^2/8$. Similarly, the value for the central concentrated load is WL/4 also at the centre. Thus, in this case,

$$M_{\text{max}} = \frac{WL}{4} + \frac{WL^2}{8} = \left[\frac{20 \times 10^3 \times 7}{4}\right] + \left[\frac{5 \times 10^3 \times 7^2}{8}\right] \text{N m}$$
$$= (35.0 + 30.63)10^3 = 65.63 \text{ kN m}$$
$$\sigma_{\text{max}} = \frac{65.63 \times 10^3 \times 150 \times 10^{-3}}{1.9 \times 10^{-4}} = 51.8 \text{ MN/m}^2$$

The maximum stress in the girder is 52 MN/m^2 , this value being compressive on the upper surface and tensile on the lower surface.

Example 4.2

÷.

A uniform T-section beam is 100 mm wide and 150 mm deep with a flange thickness of 25 mm and a web thickness of 12 mm. If the limiting bending stresses for the material of the beam are 80 MN/m^2 in compression and 160 MN/m^2 in tension, find the maximum u.d.l. that the beam can carry over a simply supported span of 5 m.

Solution

The second moment of area value I used in the simple bending theory is that about the N.A. Thus, in order to determine the I value of the T-section shown in Fig. 4.17, it is necessary first to position the N.A.

Since this always passes through the centroid of the section we can take moments of area about the base to determine the position of the centroid and hence the N.A.

Thus

$$(100 \times 25 \times 137.5)10^{-9} + (125 \times 12 \times 62.5)10^{-9} = 10^{-6} [(100 \times 25) + (125 \times 12)\bar{y}]$$
$$(343750 + 93750)10^{-9} = 10^{-6} (2500 + 1500)\bar{y}$$
$$\bar{y} = \frac{437.5 \times 10^{-6}}{4000 \times 10^{-6}} = 109.4 \times 10^{-3} = 109.4 \text{ mm},$$



Fig. 4.17.

Thus the N.A. is positioned, as shown, a distance of 109.4 mm above the base. The second moment of area I can now be found as suggested in Example 4.1 by dividing the section into convenient rectangles with their edges in the neutral axis.

$$I = \frac{1}{3} [(100 \times 40.6^3) - (88 \times 15.6^3) + (12 \times 109.4^3)] 10^{-12}$$

= $\frac{1}{3} (6.69 - 0.33 + 15.71) 10^{-6} = 7.36 \times 10^{-6} \text{ m}^4$

Now the maximum compressive stress will occur on the upper surface where y = 40.6 mm, and, using the limiting compressive stress value quoted,

$$M = \frac{\sigma I}{v} = \frac{80 \times 10^6 \times 7.36 \times 10^{-6}}{40.6 \times 10^{-3}} = 14.5 \,\mathrm{kN}\,\mathrm{m}$$

This suggests a maximum allowable B.M. of 14.5 kN m. It is now necessary, however, to check the tensile stress criterion which must apply on the lower surface,

i.e. $M = \frac{\sigma I}{y} = \frac{160 \times 10^6 \times 7.36 \times 10^{-6}}{109.4 \times 10^{-3}} = 10.76 \text{ kN m}$

The greatest moment that can therefore be applied to retain stresses within both conditions quoted is therefore M = 10.76 kN m.

But for a simply supported beam with u.d.l.,

$$M_{\rm max} = \frac{wL^2}{8}$$
$$w = \frac{8M}{L^2} = \frac{8 \times 10.76 \times 10^3}{5^2}$$
$$= 3.4 \text{ kN/m}$$

The u.d.l. must be limited to 3.4 kN m.

Example 4.3

A flitched beam consists of two 50 mm \times 200 mm wooden beams and a 12 mm \times 80 mm steel plate. The plate is placed centrally between the wooden beams and recessed into each so that, when rigidly joined, the three units form a 100 mm \times 200 mm section as shown in Fig. 4.18. Determine the moment of resistance of the flitched beam when the maximum

bending stress in the timber is 12 MN/m^2 . What will then be the maximum bending stress in the steel?

For steel $E = 200 \text{ GN/m}^2$; for wood $E = 10 \text{ GN/m}^2$.



Fig. 4.18.

Solution

The flitched beam may be considered replaced by the equivalent wooden section shown in Fig. 4.18. The thickness t' of the wood equivalent to the steel which it replaces is given by eqn. (4.14),

$$t' = \frac{E}{E'} t = \frac{200 \times 10^9}{10 \times 10^9} \times 12 = 240 \text{ mm}$$

Then, for the equivalent section

$$I_{\text{N.A.}} = 2 \left[\frac{50 \times 200^3}{12} \right] - 2 \left[\frac{6 \times 80^3}{12} \right] + \left[\frac{240 \times 80^3}{12} \right] 10^{-12}$$
$$= (66.67 - 0.51 + 10.2) 10^{-6} = 76.36 \times 10^{-6} \text{ m}^4$$

Now the maximum stress in the timber is 12 MN/m^2 , and this will occur at y = 100 mm; thus, from the bending theory,

$$M = \frac{\sigma I}{v} = \frac{12 \times 10^6 \times 76.36 \times 10^{-6}}{100 \times 10^{-3}} = 9.2 \text{ kN m}$$

The moment of resistance of the beam, i.e. the bending moment which the beam can withstand within the given limit, is 9.2 kN m.

The maximum stress in the steel with this moment applied is then determined by finding first the maximum stress in the equivalent wood at the same position, i.e. at y = 40 mm.

Therefore maximum stress in equivalent wood

$$\sigma'_{\text{max}} = \frac{My}{I} = \frac{9.2 \times 10^3 \times 40 \times 10^{-3}}{76.36 \times 10^{-6}} = 4.82 \times 10^6 \text{ N/m}^2$$

Therefore from eqn. (4.15), the maximum stress in the steel is given by

$$\sigma_{\max} = \frac{E}{E'} \sigma'_{\max} = \frac{200 \times 10^9}{10 \times 10^9} \times 4.82 \times 10^6$$
$$= 96 \times 10^6 = 96 \text{ MN/m}^2$$

Example 4.4

(a) A reinforced concrete beam is 240 mm wide and 450 mm deep to the centre of the reinforcing steel rods. The rods are of total cross-sectional area 1.2×10^{-3} m² and the maximum allowable stresses in the steel and concrete are 150 MN/m² and 8 MN/m² respectively. The modular ratio (steel: concrete) is 16. Determine the moment of resistance of the beam.

(b) If, after installation, it is required to up-rate the service loads by 30% and to replace the above beam with a second beam of increased strength but retaining the same width of 240 mm, determine the new depth and area of steel for tension reinforcement required.

Solution



Fig. 4.19.

(a) From eqn. (4.16) moments of area about the N.A. of Fig. 4.19.

$$10^{-9} \left(240 \times h \times \frac{h}{2} \right) = 16 \times 1.2 \times 10^{-3} (450 - h) 10^{-3}$$
$$120h^2 = (8640 - 19.2h) 10^3$$
$$h^2 + 160h - 72000 = 0$$
$$h = 200 \text{ mm}$$

From which

Substituting in eqn. (4.17),

moment of resistance (compressive) = $(240 \times 200 \times 10^{-6}) \frac{8 \times 10^{6}}{2} (450 - 66.7) 10^{-3}$

and from eqn. (4.18)

moment of resistance (tensile) = $(16 \times 1.2 \times 10^{-3}) \frac{150 \times 10^{6}}{16} (450 - 66.7) 10^{-3}$

= 69.0 kN m

Thus the safe moment which the beam can carry within both limiting stress values is 69 kN m.

(b) For this part of the question the dimensions of the new beam are required and it is necessary to assume a *critical* or *economic* section. The position of the N.A. is then determined from eqn. (4.19) by consideration of the proportions of the stress distribution (i.e. assuming that the maximum stresses in the streel and concrete occur together).

Thus from eqn. (4.19)

$$\frac{h}{d} = \frac{1}{1 + \frac{t}{mc}} = \frac{1}{1 + \frac{150 \times 10^6}{16 \times 8 \times 10^6}} = 0.46$$

From (4.17)
$$M = \frac{bhc}{2}\left(d - \frac{h}{3}\right) = \frac{h}{2d}\left(1 - \frac{h}{3d}\right)c bd^2$$

Substituting for $\frac{h}{d} = 0.46$ and solving for d gives

$$d = 0.49 \text{ m}$$

$$h = 0.46 \times 0.49 = 0.225 \text{ m}$$

$$\therefore \text{ From (4.20)} \qquad A = \frac{0.24 \times 0.225 \times 8 \times 10^6}{2 \times 150 \times 10^6}$$
i.e.
$$A = 1.44 \times 10^{-3} \text{ m}^2$$

Example 4.5

(a) A rectangular masonry column has a cross-section 500 mm \times 400 mm and is subjected to a vertical compressive load of 100 kN applied at point P shown in Fig. 4.20. Determine the value of the maximum stress produced in the section. (b) Is the section *at any point* subjected to tensile stresses?



Fig. 4.20.

Solution

In this case the load is eccentric to both the XX and YYaxes and bending will therefore take place simultaneously about both axes.

Moment about $XX = 100 \times 10^3 \times 80 \times 10^{-3} = 8000 \text{ N m}$ Moment about $YY = 100 \times 10^3 \times 100 \times 10^{-3} = 10000 \text{ N m}$

Therefore from eqn. (4.26) the maximum stress in the section will be compressive at point A since at this point the compressive effects of bending about both XX and YY add to the direct compressive stress component due to P,

i.e

$$\sigma_{\max} = -\left[\frac{P}{A} + \frac{M_{xx}Y_A}{I_{xx}} + \frac{M_{yy}X_A}{I_{yy}}\right]$$
$$= -\left[\frac{100 \times 10^3}{500 \times 400 \times 10^{-6}} + \frac{8000 \times 200 \times 10^{-3} \times 12}{(500 \times 400^3)10^{-12}} + \frac{10000 \times 250 \times 10^{-3} \times 12}{(400 \times 500^3)10^{-12}}\right]$$
$$= -(0.5 + 0.6 + 0.6)10^6 = -1.7 \text{ MN/m}^2$$

For the section to contain no tensile stresses, P must be applied within the middle third. Now since b/3 = 133 mm and d/3 = 167 mm it follows that the maximum possible values of the coordinates x or y for P are $y = \frac{1}{2} \times 133 = 66.5$ mm and $x = \frac{1}{2} \times 167 = 83.5$ mm.

The given position for P lies outside these values so that tensile stresses will certainly exist in the section.

(The full middle-third area is in fact shown in Fig. 4.20 and P is clearly outside this area.)

Example 4.6

The crank of a motor vehicle engine has the section shown in Fig. 4.21 along the line AA. Derive an expression for the stress at any point on this section with the con-rod thrust P



Fig. 4.21.

applied at some angle θ as shown. Hence, if the maximum tensile stress in the section is not to exceed 100 MN/m², determine the maximum value of P which can be permitted with $\theta = 60^{\circ}$. What will be the distribution of stress along the section AA with this value of P applied?

Solution

Assuming that the load P is applied in the plane of the crank the stress at any point along the section AA will be the result of (a) a direct compressive load of magnitude $P \cos \theta$, and (b) a B.M. in the plane of the crank of magnitude $P \sin \theta \times h$; i.e. stress at any point along AA, distance s from the centre-line, is given by eqn. (4.26) as

$$\sigma = \frac{P\cos\theta}{A} \pm \frac{(P\sin\theta \cdot h)}{I_{AA}} \cdot s$$
$$= \frac{P\cos\theta}{A} \left[1 \pm \frac{hs\tan\theta}{k_{AA}^2} \right]$$

where k_{AA} is the radius of gyration of the section AA about its N.A.

Now $A = [(2 \times 20 \times 8) + (24 \times 10)]10^{-6} = 560 \times 10^{-6} \text{ m}^2$

and

$$I_{\text{NA.}} = \frac{1}{12} [20 \times 40^3 - 10 \times 24^3] \ 10^{-12} = 9.51 \times 10^{-8} \text{ m}^4$$

$$\therefore \qquad \sigma = -P \left[\frac{0.5}{560 \times 10^{-6}} \pm \frac{0.866 \times 80 \times 10^{-3} \times 10^{-3}}{9.51 \times 10^{-8}} \right]$$
$$= -P \left[0.893 \pm 0.729s \right] 10^3 \text{ N/m}^2$$

where s is measured in millimetres,

i.e. maximum tensile stress = $P[-0.893 + 0.729 \times 20]10^3 \text{ N/m}^2$

 $= 13.69P \text{ kN/m}^2$

In order that this stress shall not exceed 100 MN/m^2

$$100 \times 10^6 = 13.69 \times P \times 10^3$$

 $P = 7.3 \text{ kN}$

With this value of load applied the direct stress on the section will be

 $-0.893P \times 10^3 = -6.52 \text{ MN/m}^2$

and the bending stress at each edge

$$\pm 0.729 \times 20 \times 10^{3} P = 106.4 \text{ MN/m}^{2}$$

The stress distribution along AA is then obtained as shown in Fig. 4.22.



Problems

4.1 (A). Determine the second moments of area about the axes XX for the sections shown in Fig. 4.23. [15.69, 7.88, 41.15, 24; all $\times 10^{-6}$ m⁴.]



Fig. 4.23.

4.2 (A). A rectangular section beam has a depth equal to twice its width. It is the same material and mass per unit length as an I-section beam 300 mm deep with flanges 25 mm thick and 150 mm wide and a web 12 mm thick. Compare the flexural strengths of the two beams. [8.59:1.]

4.3 (A). A conveyor beam has the cross-section shown in Fig. 4.24 and it is subjected to a bending moment in the plane YY. Determine the maximum permissible bending moment which can be applied to the beam (a) for bottom flange in tension, and (b) for bottom flange in compression, if the safe stresses for the material in tension and compression are 30 MN/m^2 and 150 MN/m^2 respectively. [32.3, 84.8 kNm.]



4.4 (A/B). A horizontal steel girder has a span of 3 m and is built-in at the left-hand end and freely supported at the other end. It carries a uniformly distributed load of 30 kN/m over the whole span, together with a single concentrated load of 20 kN at a point 2 m from the left-hand end. The supporting conditions are such that the reaction at the left-hand end is 65 kN.

- (a) Determine the bending moment at the left-hand end and draw the B.M. diagram.
- (b) Give the value of the maximum bending moment.
- (c) If the girder is 200 mm deep and has a second moment of area of 40 × 10⁻⁶ m⁴ determine the maximum stress resulting from bending.
 [I.Mech.E.] [40 kN m; 100 MN/m².]

4.5 (A/B). Figure 4.25 represents the cross-section of an extruded alloy member which acts as a simply supported beam with the 75 mm wide flange at the bottom. Determine the moment of resistance of the section if the maximum permissible stresses in tension and compression are respectively 60 MN/m^2 and 45 MN/m^2 .

[1.E.I.] [2.62 kN m.]



Fig. 4.25.

4.6 (A/B). A trolley consists of a pressed steel section as shown in Fig. 4.26. At each end there are rollers at 350 mm centres.

If the trolley supports a mass of 50 kg evenly distributed over the 350 mm length of the trolley calculate, using the data given in Fig. 4.26, the maximum compressive and tensile stress due to bending in the pressed steel section. State clearly your assumptions. [C.G.] [14.8, 42.6 MN/m²]



Fig. 4.26.

4.7 (A/B). The channel section of Fig. 4.21 is used as a simply-supported beam over a span of 2.8 m. The channel is used as a guide for a roller of an overhead crane gantry and can be expected to support a maximum load (taken to be a concentrated point load) of 40 kN. At what position of the roller will the bending moment of the channel be a maximum and what will then be the maximum tensile bending stress? If the maximum allowable stress for the material of the beam is 320 MN/m^2 what safety factor exists for the given loading condition.

4.8 (A/B). A $120 \times 180 \times 15$ mm uniform I-section steel girder is used as a cantilever beam carrying a uniformly distributed load ω kN/m over a span of 2.4 m. Determine the maximum value of ω which can be applied before yielding of the outer fibres of the beam cross-section commences. In order to strengthen the girder, steel plates are attached to the outer surfaces of the flanges to double their effective thickness. What width of plate should be added (to the nearest mm) in order to reduce the maximum stress by 30%? The yield stress for the girder material is 320 MN/m^2 .

4.9 (A/B). A 200 mm wide \times 300 mm deep timber beam is reinforced by steel plates 200 mm wide \times 12 mm deep on the top and bottom surfaces as shown in Fig. 4.27. If the maximum allowable stresses for the steel and timber are 120 MN/m² and 8 MN/m² respectively, determine the maximum bending moment which the beam can safely carry. For steel $E = 200 \text{ GN/m}^2$; for timber $E = 10 \text{ GN/m}^2$. [I.Mech.E.] [103.3 kN m.]



Fig. 4.27.

4.10 (A/B). A composite beam is of the construction shown in Fig. 4.28. Calculate the allowable u.d.l. that the beam can carry over a simply supported span of 7 m if the stresses are limited to 120 MN/m^2 in the steel and 7 MN/m^2 in the timber.

Modular ratio = 20.

[1.13 kN/m.]



Fig. 4.28.

4.11 (A/B). Two bars, one of steel, the other of aluminium alloy, are each of 75 mm width and are rigidly joined together to form a rectangular bar 75 mm wide and of depth $(t_s + t_A)$, where t_s = thickness of steel bar and t_A = thickness of alloy bar.

Determine the ratio of t_s to t_A in order that the neutral axis of the compound bar is coincident with the junction of the two bars. ($E_s = 210 \text{ GN/m}^2$; $E_A = 70 \text{ GN/m}^2$.)

If such a beam is 50 mm deep determine the maximum bending moment the beam can withstand if the maximum stresses in the steel and alloy are limited to 135 MN/m^2 and 37 MN/m^2 respectively. [0.577; 1.47 kN m.]

4.12 (A/B). A brass strip, $50 \text{ mm} \times 12 \text{ mm}$ in section, is riveted to a steel strip, $65 \text{ mm} \times 10 \text{ mm}$ in section, to form a compound beam of total depth 22 mm, the brass strip being on top and the beam section being symmetrical about the vertical axis. The beam is simply supported on a span of 1.3 m and carries a load of 2 kN at mid-span.

(a) Determine the maximum stresses in each of the materials owing to bending.

(b) Make a diagram showing the distribution of bending stress over the depth of the beam.

Take *E* for steel = 200 GN/m² and *E* for brass = 100 GN/m². [U.L.] [σ_b = 130 MN/m²; σ_s = 162.9 MN/m².]

4.13 (B). A concrete beam, reinforced in tension only, has a rectangular cross-section, width 200 mm and effective depth to the tensile steel 500 mm, and is required to resist a bending moment of 70 kN m. Assuming a modular ratio of 15, calculate (a) the minimum area of reinforcement required if the stresses in steel and concrete are not to exceed 190 MN/m² and 8 MN/m² respectively, and (b) the stress in the non-critical material when the bending moment is applied. [E.I.E.] [0.916 × 10^{-3} m²; 177 MN/m².]

4.14 (B). A reinforced concrete beam of rectangular cross-section, b = 200 mm, d (depth to reinforcement) = 300 mm, is reinforced in tension only, the steel ratio, i.e. the ratio of reinforcing steel area to concrete area (neglecting cover), being 1%. The maximum allowable stresses in concrete and steel are 8 MN/m² and 135 MN/m² respectively. The modular ratio may be taken as equal to 15. Determine the moment of resistance capable of being developed in the beam. [1.Struct.E.] [20.9 kN m.]

4.15 (B). A rectangular reinforced concrete beam is 200 mm wide and 350 mm deep to reinforcement, the latter consisting of three 20 mm diameter steel rods. If the following stresses are not to be exceeded, calculate: (a) the maximum bending moment which can be sustained, and (b) the steel stress and the maximum concrete stress when the section is subjected to this maximum moment.

Maximum stress in concrete in bending not to exceed 8 MN/m².

Maximum steel stress not to exceed 150 MN/m².

Modular ratio m = 15.

[l.Struct.E.] [38.5 kN m; 138, 8 MN/m².]

4.16 (B). A reinforced concrete beam has to carry a bending moment of 100 kN m. The maximum permissible stresses are 8 MN/m^2 and 135 MN/m^2 in the concrete and steel respectively. The beam is to be of rectangular cross-section 300 mm wide. Design a suitable section with "balanced" reinforcement if $E_{steel}/E_{concrete} = 12$.

[I.Mech.E.] $[d = 482.4 \text{ mm}; A = 1.782 \times 10^{-3} \text{ m}^2.]$