CHAPTER 2

COMPOUND BARS

Summary

When a compound bar is constructed from members of different materials, lengths and areas and is subjected to an external tensile or compressive load W the load carried by any single member is given by

$$F_1 = \frac{\dfrac{E_1 A_1}{L_1}}{\Sigma \dfrac{EA}{L}} W$$

where suffix 1 refers to the single member and $\Sigma \dfrac{EA}{L}$ is the sum of all such quantities for all the members.

Where the bars have a common length the compound bar can be reduced to a single equivalent bar with an equivalent Young's modulus, termed a *combined E*.

$$\text{Combined } E = \frac{\Sigma EA}{\Sigma A}$$

The free expansion of a bar under a temperature change from T_1 to T_2 is

$$\alpha(T_2 - T_1)L$$

where α is the coefficient of linear expansion and L is the length of the bar.

If this expansion is prevented a stress will be induced in the bar given by

$$\alpha(T_2 - T_1)E$$

To determine the stresses in a compound bar composed of two members of different free lengths two principles are used:

(1) The tensile force applied to the short member by the long member is equal in magnitude to the compressive force applied to the long member by the short member.
(2) The extension of the short member plus the contraction of the long member equals the difference in free lengths.

This difference in free lengths may result from the tightening of a nut or from a temperature change in two members of different material (i.e. different coefficients of expansion) but of equal length initially.

If such a bar is then subjected to an additional external load the resultant stresses may be obtained by using the *principle of superposition*. With this method the stresses in the members

arising from the separate effects are obtained and the results added, taking account of sign, to give the resultant stresses.

N.B.: Discussion in this chapter is concerned with compound bars which are symmetrically proportioned such that no bending results.

2.1. Compound bars subjected to external load

In certain applications it is necessary to use a combination of elements or bars made from different materials, each material performing a different function. In overhead electric cables, for example, it is often convenient to carry the current in a set of copper wires surrounding steel wires, the latter being designed to support the weight of the cable over large spans. Such combinations of materials are generally termed *compound bars*. Discussion in this chapter is concerned with compound bars which are symmetrically proportioned such that no bending results.

When an external load is applied to such a compound bar it is shared between the individual component materials in proportions depending on their respective lengths, areas and Young's moduli.

Consider, therefore, a compound bar consisting of n members, each having a different length and cross-sectional area and each being of a different material; this is shown diagrammatically in Fig. 2.1. Let all members have a common extension x, i.e. the load is positioned to produce the same extension in each member.

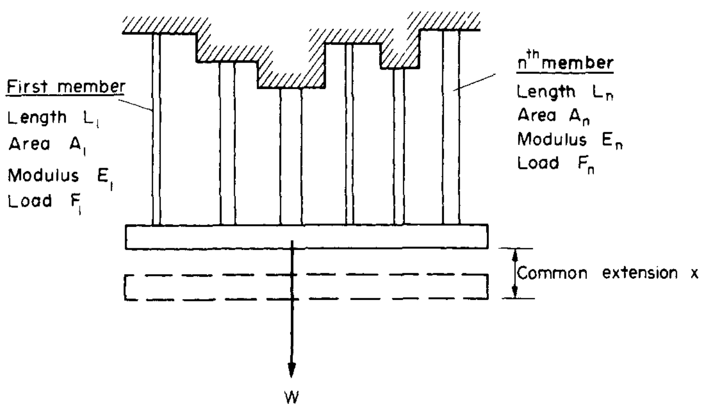

Fig. 2.1. Diagrammatic representation of a compound bar formed of different materials with different lengths, cross-sectional areas and Young's moduli.

For the nth member,

$$\frac{\text{stress}}{\text{strain}} = E_n = \frac{F_n L_n}{A_n x_n}$$

$$\therefore \quad F_n = \frac{E_n A_n x}{L_n} \qquad (2.1)$$

where F_n is the force in the nth member and A_n and L_n are its cross-sectional area and length.

The total load carried will be the sum of all such loads for all the members

i.e. $$W = \sum \frac{E_n A_n x}{L_n} = x \sum \frac{E_n A_n}{L_n} \qquad (2.2)$$

Now from eqn. (2.1) the force in member 1 is given by

$$F_1 = \frac{E_1 A_1 x}{L_1}$$

But, from eqn. (2.2),

$$x = \frac{W}{\sum \frac{E_n A_n}{L_n}}$$

$\therefore$ $$F_1 = \frac{\frac{E_1 A_1}{L_1}}{\sum \frac{EA}{L}} W \qquad (2.3)$$

i.e. *each member carries a portion of the total load W proportional to its EA/L value.*

If the wires are all of equal length the above equation reduces to

$$F_1 = \frac{E_1 A_1}{\Sigma EA} W \qquad (2.4)$$

The stress in member 1 is then given by

$$\sigma_1 = \frac{F_1}{A_1} \qquad (2.5)$$

2.2. Compound bars – "equivalent" or "combined" modulus

In order to determine the common extension of a compound bar it is convenient to consider it as a single bar of an imaginary material with an *equivalent* or *combined* modulus E_c. Here it is necessary to assume that both the extension and the original lengths of the individual members of the compound bar are the same; the strains in all members will then be equal.

Now total load on compound bar $= F_1 + F_2 + F_3 + \ldots + F_n$ where F_1, F_2, etc., are the loads in members 1, 2, etc.

But $$\text{force} = \text{stress} \times \text{area}$$

$\therefore$ $$\sigma(A_1 + A_2 + \ldots + A_n) = \sigma_1 A_1 + \sigma_2 A_2 + \ldots + \sigma_n A_n$$

where σ is the stress in the equivalent single bar.

Dividing through by the common strain ε,

$$\frac{\sigma}{\varepsilon}(A_1 + A_2 + \ldots + A_n) = \frac{\sigma_1}{\varepsilon} A_1 + \frac{\sigma_2}{\varepsilon} A_2 + \ldots + \frac{\sigma_n}{\varepsilon} A_n$$

i.e. $$E_c(A_1 + A_2 + \ldots + A_n) = E_1 A_1 + E_2 A_2 + \ldots + E_n A_n$$

where E_c is the *equivalent* or *combined* E of the single bar.

$$\therefore \qquad \text{combined } E = \frac{E_1A_1 + E_2A_2 + \ldots + E_nA_n}{A_1 + A_2 + \ldots + A_n}$$

i.e.

$$E_c = \frac{\Sigma EA}{\Sigma A} \tag{2.6}$$

With an external load W applied,

$$\text{stress in the equivalent bar} = \frac{W}{\Sigma A}$$

and

$$\text{strain in the equivalent bar} = \frac{W}{E_c \Sigma A} = \frac{x}{L}$$

$\therefore$ since

$$\frac{\text{stress}}{\text{strain}} = E$$

$$\textbf{common extension } x = \frac{WL}{E_c \Sigma A} \tag{2.7}$$

$$= \text{extension of single bar}$$

2.3. Compound bars subjected to temperature change

When a material is subjected to a change in temperature its length will change by an amount

$$\alpha Lt$$

where α is the coefficient of linear expansion for the material, L is the original length and t the temperature change. (An increase in temperature produces an increase in length and a decrease in temperature a decrease in length except in very special cases of materials with zero or negative coefficients of expansion which need not be considered here.)

If, however, the free expansion of the material is prevented by some external force, then a stress is set up in the material. This stress is equal in magnitude to that which would be produced in the bar by initially allowing the free change of length and then applying sufficient force to return the bar to its original length.

Now

$$\text{change in length} = \alpha Lt$$

$$\therefore \qquad \text{strain} = \frac{\alpha Lt}{L} = \alpha t$$

Therefore, the stress created in the material by the application of sufficient force to remove this strain

$$= \text{strain} \times E$$

$$= \boldsymbol{E\alpha t}$$

Consider now a compound bar constructed from two different materials rigidly joined together as shown in Fig. 2.2 and Fig. 2.3(a). For simplicity of description consider that the materials in this case are steel and brass.

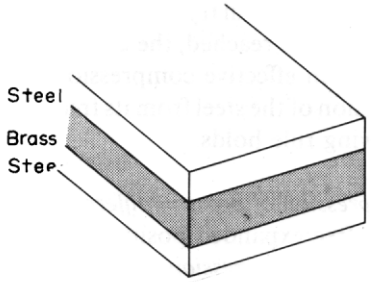

Fig. 2.2.

In general, the coefficients of expansion of the two materials forming the compound bar will be different so that as the temperature rises each material will attempt to expand by different amounts. Figure 2.3b shows the positions to which the individual materials will extend if they are completely free to expand (i.e. not joined rigidly together as a compound bar). The extension of any length L is given by

$$\alpha L t$$

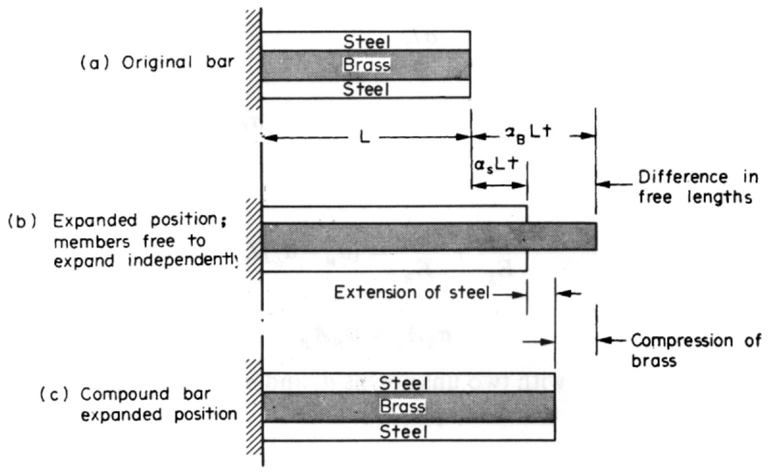

Fig. 2.3. Thermal expansion of compound bar.

Thus the difference of "free" expansion lengths or so-called *free lengths*

$$= \alpha_B Lt - \alpha_S Lt = (\alpha_B - \alpha_S) Lt$$

since in this case the coefficient of expansion of the brass α_B is greater than that for the steel α_S. The initial lengths L of the two materials are assumed equal.

If the two materials are now rigidly joined as a compound bar and subjected to the same temperature rise, each material will attempt to expand to its free length position but each will be affected by the movement of the other. The higher coefficient of expansion material (brass) will therefore seek to pull the steel up to its free length position and conversely the lower coefficient of expansion material (steel) will try to hold the brass back to the steel "free length" position. In practice a compromise is reached, the compound bar extending to the position shown in Fig. 2.3c, resulting in an effective compression of the brass from its free length position and an effective extension of the steel from its free length position. From the diagram it will be seen that the following rule holds.

Rule 1.

Extension of steel + compression of brass = difference in "free" lengths.

Referring to the bars in their free expanded positions the rule may be written as

Extension of "short" member + compression of "long" member = difference in free lengths.

Applying Newton's law of equal action and reaction the following second rule also applies.

Rule 2.

The tensile force applied to the short member by the long member is equal in magnitude to the compressive force applied to the long member by the short member.

Thus, in this case,

tensile force in steel = compressive force in brass

Now, from the definition of Young's modulus

$$E = \frac{\text{stress}}{\text{strain}} = \frac{\sigma}{\delta/L}$$

where δ is the change in length.

$$\therefore \qquad \delta = \frac{\sigma L}{E}$$

Also $\qquad$ force = stress × area = σA

where A is the cross-sectional area.

Therefore Rule 1 becomes

$$\frac{\sigma_S L}{E_S} + \frac{\sigma_B L}{E_B} = (\alpha_B - \alpha_S) L t \qquad (2.8)$$

and Rule 2 becomes

$$\sigma_S A_S = \sigma_B A_B \qquad (2.9)$$

We thus have two equations with two unknowns σ_S and σ_B and it is possible to evaluate the magnitudes of these stresses (see Example 2.2).

2.4. Compound bar (tube and rod)

Consider now the case of a hollow tube with washers or endplates at each end and a central threaded rod as shown in Fig. 2.4. At first sight there would seem to be no connection with the work of the previous section, yet, in fact, the method of solution to determine the stresses set up in the tube and rod when one nut is tightened is identical to that described in §2.3.

The compound bar which is formed after assembly of the tube and rod, i.e. with the nuts tightened, is shown in Fig. 2.4c, the rod being in a state of tension and the tube in compression. Once again Rule 2 applies, i.e.

compressive force in tube = tensile force in rod

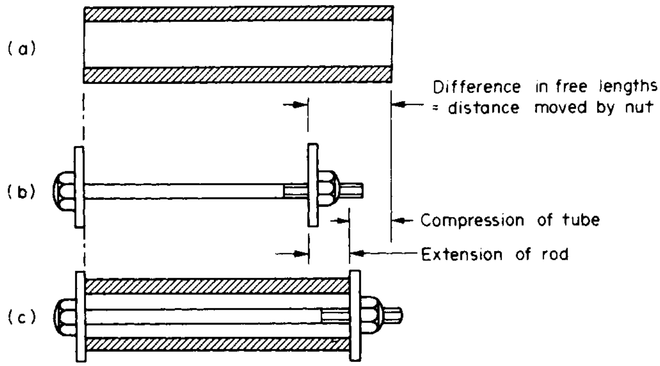

Fig. 2.4. Equivalent "mechanical" system to that of Fig. 2.3.

Figure 2.4a and b show, *diagrammatically*, the effective positions of the tube and rod before the nut is tightened and the two components are combined. As the nut is turned there is a simultaneous compression of the tube and tension of the rod leading to the final state shown in Fig. 2.4c. As before, however, the diagram shows that Rule 1 applies:

compression of tube + extension of rod = difference in free lengths = axial advance of nut

i.e. the axial movement of the nut (= number of turns n × threads per metre) is taken up by combined compression of the tube and extension of the rod.

Thus, with suffix t for tube and R for rod,

$$\frac{\sigma_t L}{E_t} + \frac{\sigma_R L}{E_R} = n \times \textbf{threads/metre} \tag{2.10}$$

also

$$\sigma_R A_R = \sigma_t A_t \tag{2.11}$$

If the tube and rod are now subjected to a change of temperature they may be treated as a normal compound bar of § 2.3 and Rules 1 and 2 again apply (Fig. 2.5),

i.e.

$$\frac{\sigma'_t L}{E_t} + \frac{\sigma'_R L}{E_R} = (\alpha_t - \alpha_R) L t \tag{2.12}$$

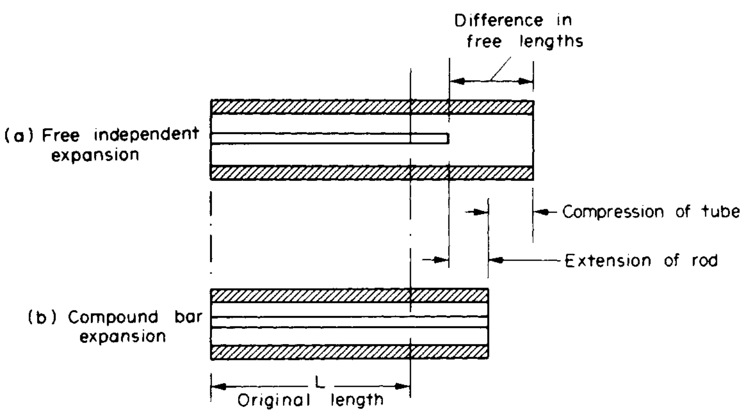

Fig. 2.5.

where σ'_t and σ'_R are the stresses in the tube and rod due to temperature change only and α_t is assumed greater than α_R. If the latter is not the case the two terms inside the final bracket should be interchanged.

Also

$$\sigma'_R A_R = \sigma'_t A_t$$

2.5. Compound bars subjected to external load and temperature effects

In this case the *principle of superposition* must be applied, i.e. provided that stresses remain within the elastic limit the effects of external load and temperature change may be assessed separately as described in the previous sections and the results added, taking account of sign, to determine the resultant total effect;

i.e. *total strain = sum of strain due to external loads and temperature strain*

2.6. Compound thick cylinders subjected to temperature changes

The procedure described in §2.3 has been applied to compound cylinders constructed from tubes of different materials on page 230.

Examples

Example 2.1

(a) A compound bar consists of four brass wires of 2.5 mm diameter and one steel wire of 1.5 mm diameter. Determine the stresses in each of the wires when the bar supports a load of 500 N. Assume all of the wires are of equal lengths.

(b) Calculate the "equivalent" or "combined" modulus for the compound bar and determine its total extension if it is initially 0.75 m long. Hence check the values of the stresses obtained in part (a).

For brass $E = 100\ \text{GN/m}^2$ and for steel $E = 200\ \text{GN/m}^2$.

Solution

(a) From eqn. (2.3) the force in the steel wire is given by

$$F_s = \frac{E_s A_s}{\Sigma EA} W$$

$$= \left[\frac{200 \times 10^9 \times \frac{\pi}{4} \times 1.5^2 \times 10^{-6}}{200 \times 10^9 \times \frac{\pi}{4} \times 1.5^2 \times 10^{-6} + 4(100 \times 10^9 \times \frac{\pi}{4} \times 2.5^2 \times 10^{-6})}\right] 500$$

$$= \left[\frac{2 \times 1.5^2}{(2 \times 1.5^2) + (4 \times 2.5^2)}\right] 500 = 76.27\ \text{N}$$

$\therefore$ total force in brass wires = 500 − 76.27 = 423.73 N

$$\therefore \quad \text{stress in steel} = \frac{\text{load}}{\text{area}} = \frac{76.27}{\frac{\pi}{4} \times 1.5^2 \times 10^{-6}} = \mathbf{43.2\ MN/m^2}$$

and
$$\text{stress in brass} = \frac{\text{load}}{\text{area}} = \frac{423.73}{4 \times \frac{\pi}{4} \times 2.5^2 \times 10^{-6}} = \mathbf{21.6\ MN/m^2}$$

(b) From eqn. (2.6)

$$\text{combined } E = \frac{\Sigma EA}{\Sigma A} = \frac{200 \times 10^9 \times \frac{\pi}{4} \times 1.5^2 \times 10^{-6} + 4(100 \times 10^9 \times \frac{\pi}{4} \times 2.5^2 \times 10^{-6})}{\frac{\pi}{4}(1.5^2 + 4 \times 2.5^2)^{10-6}}$$

$$= \frac{(200 \times 1.5^2 + 400 \times 2.5^2)}{(1.5^2 + 4 \times 2.5^2)} 10^9 = \mathbf{108.26\ GN/m^2}$$

Now
$$E = \frac{\text{stress}}{\text{strain}}$$

and the stress in the equivalent bar

$$= \frac{500}{\Sigma A} = \frac{500}{\frac{\pi}{4}(1.5^2 + 4 \times 2.5^2)10^{-6}} = 23.36\ \text{MN/m}^2$$

$$\therefore \quad \text{strain in the equivalent bar} = \frac{\text{stress}}{E} = \frac{23.36 \times 10^6}{108.26 \times 10^9} = 0.216 \times 10^{-3}$$

$$\therefore \quad \text{common extension} = \text{strain} \times \text{original length}$$
$$= 0.216 \times 10^{-3} \times 0.75 = 0.162 \times 10^{-3}$$
$$= \mathbf{0.162\ mm}$$

This is also the extension of any single bar, giving a strain in any bar

$$= \frac{0.162 \times 10^{-3}}{0.75} = 0.216 \times 10^{-3} \text{ as above}$$

$$\therefore \quad \text{stress in steel} = \text{strain} \times E_s = 0.216 \times 10^{-3} \times 200 \times 10^9$$
$$= \mathbf{43.2\ MN/m^2}$$

and
$$\text{stress in brass} = \text{strain} \times E_B = 0.216 \times 10^{-3} \times 100 \times 10^9$$
$$= \mathbf{21.6\ MN/m^2}$$

These are the same values as obtained in part (a).

Example 2.2

(a) A compound bar is constructed from three bars 50 mm wide by 12 mm thick fastened together to form a bar 50 mm wide by 36 mm thick. The middle bar is of aluminium alloy for which $E = 70$ GN/m^2 and the outside bars are of brass with $E = 100$ GN/m^2. If the bars are initially fastened at 18°C and the temperature of the whole assembly is then raised to 50°C, determine the stresses set up in the brass and the aluminium.

$$\alpha_B = 18 \times 10^{-6} \text{ per °C} \quad \text{and} \quad \alpha_A = 22 \times 10^{-6} \text{ per °C}$$

(b) What will be the changes in these stresses if an external compressive load of 15 kN is applied to the compound bar at the higher temperature?

Solution

With any problem of this type it is convenient to let the stress in one of the component members or materials, e.g. the brass, be x.

Then, since

$$\text{force in brass} = \text{force in aluminium}$$

and

$$\text{force} = \text{stress} \times \text{area}$$

$$x \times 2 \times 50 \times 12 \times 10^{-6} = \sigma_A \times 50 \times 12 \times 10^{-6}$$

i.e. stress in aluminium $\sigma_A = 2x$

Now, from eqn. (2.8),

$$\text{extension of brass} + \text{compression of aluminium} = \text{difference in free lengths}$$

$$= (\alpha_A - \alpha_B)(T_2 - T_1)L$$

$$\frac{xL}{100 \times 10^9} + \frac{2xL}{70 \times 10^9} = (22 - 18)10^{-6}(50 - 18)L$$

$$\frac{(7x + 20x)}{700 \times 10^9} = 4 \times 10^{-6} \times 32$$

$$27x = 4 \times 10^{-6} \times 32 \times 700 \times 10^9$$

$$\mathbf{x = 3.32\ MN/m^2}$$

The stress in the brass is thus **3.32 MN/m²** **(tensile)** and the stress in the aluminium is $2 \times 3.32 =$ **6.64 MN/m² (compressive).**

(b) With an external load of 15 kN applied each member will take a proportion of the total load given by eqn. (2.3).

$$\text{Force in aluminium} = \frac{E_A A_A}{\Sigma EA} W$$

$$= \left[\frac{70 \times 10^9 \times 50 \times 12 \times 10^{-6}}{(70 \times 50 \times 12 + 2 \times 100 \times 50 \times 12)10^9 \times 10^{-6}}\right] 15 \times 10^3$$

$$= \left[\frac{70}{(70 + 200)}\right] 15 \times 10^3$$

$$= 3.89\ \text{kN}$$

$\therefore$ force in brass $= 15 - 3.89 = 11.11$ kN

$\therefore$ stress in brass $= \dfrac{\text{load}}{\text{area}} = \dfrac{11.11 \times 10^3}{2 \times 50 \times 12 \times 10^{-6}}$

$$= \mathbf{9.26\ MN/m^2\ (compressive)}$$

$$\text{Stress in aluminium} = \frac{\text{load}}{\text{area}} = \frac{3.89 \times 10^3}{50 \times 12 \times 10^{-6}}$$

$$= \mathbf{6.5\ MN/m^2\ (compressive)}$$

These stresses represent the *changes* in the stresses owing to the applied load. The total or resultant stresses owing to combined applied loading plus temperature effects are, therefore,

$$\text{stress in aluminium} = -6.64 - 6.5 = -13.14\ \text{MN/m}^2$$

$$= \mathbf{13.14\ MN/m^2\ (compressive)}$$

$$\text{stress in brass} = +3.32 - 9.26 = -5.94\ \text{MN/m}^2$$

$$= \mathbf{5.94\ MN/m^2\ (compressive)}$$

Example 2.3

A 25 mm diameter steel rod passes concentrically through a bronze tube 400 mm long, 50 mm external diameter and 40 mm internal diameter. The ends of the steel rod are threaded and provided with nuts and washers which are adjusted initially so that there is no end play at 20°C.

(a) Assuming that there is no change in the thickness of the washers, find the stress produced in the steel and bronze when one of the nuts is tightened by giving it one-tenth of a turn, the pitch of the thread being 2.5 mm.
(b) If the temperature of the steel and bronze is then raised to 50°C find the changes that will occur in the stresses in both materials.

The coefficient of linear expansion per °C is 11×10^{-6} for steel and 18×10^{-6} for bronze. E for steel = 200 GN/m². E for bronze = 100 GN/m².

Solution

(a) Let x be the stress in the tube resulting from the tightening of the nut and σ_R the stress in the rod.

Then, from eqn. (2.11),

$$\text{force (stress} \times \text{area) in tube} = \text{force (stress} \times \text{area) in rod}$$

$$x \times \tfrac{\pi}{4}(50^2 - 40^2)10^{-6} = \sigma_R \times \tfrac{\pi}{4} \times 25^2 \times 10^{-6}$$

$$\sigma_R = \frac{(50^2 - 40^2)}{25^2} x = 1.44x$$

And since compression of tube + extension of rod = axial advance of nut, from eqn. (2.10),

$$\frac{x \times 400 \times 10^{-3}}{100 \times 10^9} + \frac{1.44x \times 400 \times 10^{-3}}{200 \times 10^9} = \frac{1}{10} \times 2.5 \times 10^{-3}$$

$$400 \frac{(2x + 1.44x)}{200 \times 10^9} 10^{-3} = 2.5 \times 10^{-4}$$

$$\therefore \qquad 6.88x = 2.5 \times 10^8$$

$$x = 36.3\ \text{MN/m}^2$$

The stress in the tube is thus **36.3 MN/m² (compressive)** and the stress in the rod is $1.44 \times 36.3 =$ **52.3 MN/m² (tensile)**.

(b) Let p be the stress in the tube resulting from temperature change. The relationship between the stresses in the tube and the rod will remain as in part (a) so that the stress in the rod is then $1.44p$. In this case, if free expansion were allowed in the independent members, the bronze tube would expand more than the steel rod and from eqn. (2.8)

compression of tube + extension of rod = difference in free length

$$\therefore \quad \frac{pL}{100 \times 10^9} + \frac{1.44pL}{200 \times 10^9} = (\alpha_B - \alpha_S)(T_2 - T_1)L$$

$$\frac{(2p + 1.44p)}{200 \times 10^9} = (18 - 11)10^{-6}(50 - 20)$$

$$3.44p = 7 \times 10^{-6} \times 30 \times 200 \times 10^9$$

$$p = 12.21 \text{ MN/m}^2$$

and

$$1.44p = 17.6 \text{ MN/m}^2$$

The changes in the stresses resulting from the temperature effects are thus 12.2 MN/m² (compressive) in the tube and 17.6 MN/m² (tensile) in the rod.

The final, resultant, stresses are thus:

stress in tube $= -36.3 - 12.2 =$ **48.5 MN/m² (compressive)**

stress in rod $= 52.3 + 17.6 =$ **69.9 MN/m² (tensile)**

Example 2.4

A composite bar is constructed from a steel rod of 25 mm diameter surrounded by a copper tube of 50 mm outside diameter and 25 mm inside diameter. The rod and tube are joined by two 20 mm diameter pins as shown in Fig. 2.6. Find the shear stress set up in the pins if, after pinning, the temperature is raised by 50°C.

For steel $E = 210$ GN/m² and $\alpha = 11 \times 10^{-6}$ per °C.

For copper $E = 105$ GN/m² and $\alpha = 17 \times 10^{-6}$ per °C.

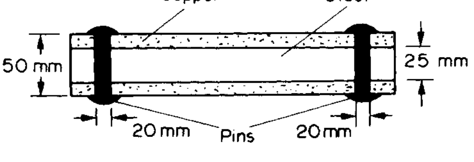

Fig. 2.6.

Solution

In this case the copper attempts to expand more than the steel, thus tending to shear the pins joining the two.

Let the stress set up in the steel be x, then, since

$$\text{force in steel} = \text{force in copper}$$

$$x \times \tfrac{\pi}{4} \times 25^2 \times 10^{-6} = \sigma_c \times \tfrac{\pi}{4}(50^2 - 25^2)\,10^{-6}$$

i.e.

$$\text{stress in copper } \sigma_c = \frac{x \times 25^2}{(50^2 - 25^2)} = 0.333x = \frac{x}{3}$$

Now the extension of the steel from its freely expanded length to its forced length in the compound bar is given by

$$\frac{\sigma L}{E} = \frac{xL}{210 \times 10^9}$$

where L is the original length.

Similarly, the compression of the copper from its freely expanded position to its position in the compound bar is given by

$$\frac{\sigma L}{E} = \frac{x}{3} \times \frac{L}{105 \times 10^9}$$

Now the extension of steel + compression of copper

$$= \text{difference in "free" lengths}$$

$$= (\alpha_2 - \alpha_1)(T_2 - T_1)L$$

$$\therefore \quad \frac{xL}{210 \times 10^9} + \frac{xL}{3 \times 105 \times 10^9} = (17 - 11)10^{-6} \times 50 \times L$$

$$\frac{3x + 2x}{6 \times 105 \times 10^9} = 6 \times 10^{-6} \times 50$$

$$5x = 6 \times 10^{-6} \times 50 \times 6 \times 105 \times 10^9$$

$$x = 37.8 \times 10^6 = 37.8\ \text{MN/m}^2$$

$$\therefore \quad \text{load carried by the steel} = \text{stress} \times \text{area}$$

$$= 37.8 \times 10^6 \times \tfrac{\pi}{4} \times 25^2 \times 10^{-6}$$

$$= 18.56\ \text{kN}$$

The pins will be in a state of double shear (see §1.15), the shear stress set up being given by

$$\tau = \frac{\text{load}}{2 \times \text{area}} = \frac{18.56 \times 10^3}{2 \times \frac{\pi}{4} \times 20^2 \times 10^{-6}}$$

$$= \mathbf{29.5\ MN/m^2}$$

Problems

2.1 (A). A power transmission cable consists of ten copper wires each of 1.6 mm diameter surrounding three steel wires each of 3 mm diameter. Determine the combined E for the compound cable and hence determine the extension of a 30 m length of the cable when it is being laid with a tension of 2 kN.
For steel, $E = 200$ GN/m^2; for copper, $E = 100$ GN/m^2. [151.3 GN/m^2; 9.6 mm.]

2.2 (A). If the maximum stress allowed in the copper of the cable of problem 2.1 is 60 MN/m^2, determine the maximum tension which the cable can support. [3.75 kN.]

2.3 (A). What will be the stress induced in a steel bar when it is heated from 15°C to 60°C, all expansion being prevented?

For mild steel, $E = 210$ GN/m^2 and $\alpha = 11 \times 10^{-6}$ per °C. [104 MN/m^2°.]

2.4 (A). A 75 mm diameter compound bar is constructed by shrinking a circular brass bush onto the outside of a 50 mm diameter solid steel rod. If the compound bar is then subjected to an axial compressive load of 160 kN determine the load carried by the steel rod and the brass bush and the compressive stress set up in each material.

For steel, $E = 210$ GN/m^2; for brass, $E = 100$ GN/m^2. [I. Struct. E.] [100.3, 59.7 kN; 51.1, 24.3 MN/m^2.]

2.5 (B). A steel rod of cross-sectional area 600 mm^2 and a coaxial copper tube of cross-sectional area 1000 mm^2 are firmly attached at their ends to form a compound bar. Determine the stress in the steel and in the copper when the temperature of the bar is raised by 80°C and an axial tensile force of 60 kN is applied.

For steel, $E = 200$ GN/m^2 with $\alpha = 11 \times 10^{-6}$ per °C.

For copper, $E = 100$ GN/m^2 with $\alpha \doteq 16.5 \times 10^{-6}$ per °C. [E.I.E.] [94.6, 3.3 MN/m^2.]

2.6 (B). A stanchion is formed by buttwelding together four plates of steel to form a square tube of outside cross-section 200 mm × 200 mm. The constant metal thickness is 10 mm. The inside is then filled with concrete.

(**a**) Determine the cross-sectional area of the steel and concrete

(**b**) If E for steel is 200 GN/m^2 and this value is twenty times that for the concrete find, when the stanchion carries a load of 368.8 kN,

(i) The stress in the concrete
(ii) The stress in the steel
(iii) The amount the stanchion shortens over a length of $2m$. [C.G.] [2, 40 MN/m^2; 40 mm]