CHAPTER 1

SIMPLE STRESS AND STRAIN

1.1. Load

In any engineering structure or mechanism the individual components will be subjected to external forces arising from the service conditions or environment in which the component works. If the component or member is in equilibrium, the resultant of the external forces will be zero but, nevertheless, they together place a load on the member which tends to deform that member and which must be reacted by internal forces which are set up within the material.

If a cylindrical bar is subjected to a direct pull or push along its axis as shown in Fig. 1.1, then it is said to be subjected to *tension* or *compression*. Typical examples of tension are the forces present in towing ropes or lifting hoists, whilst compression occurs in the legs of your chair as you sit on it or in the support pillars of buildings.
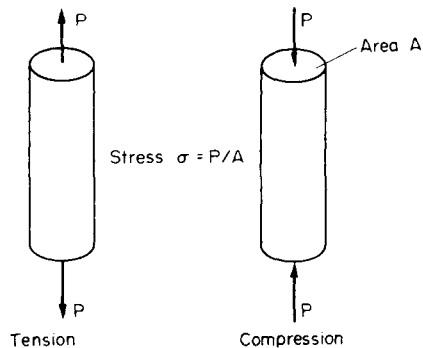

Fig. 1.1. Types of direct stress.

In the SI system of units load is measured in *newtons*, although a single newton, in engineering terms, is a very small load. In most engineering applications, therefore, loads appear in SI multiples, i.e. kilonewtons (kN) or meganewtons (MN).

There are a number of different ways in which load can be applied to a member. Typical loading types are:

(a) *Static* or *dead* loads, i.e. non-fluctuating loads, generally caused by gravity effects.
(b) *Live* loads, as produced by, for example, lorries crossing a bridge.
(c) *Impact* or *shock* loads caused by sudden blows.
(d) *Fatigue, fluctuating* or *alternating* loads, the magnitude and sign of the load changing with time.

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1.2. Direct or normal stress (σ)

It has been noted above that external force applied to a body in equilibrium is reacted by internal forces set up within the material. If, therefore, a bar is subjected to a uniform tension or compression, i.e. a direct force, which is uniformly or equally applied across the cross-section, then the internal forces set up are also distributed uniformly and the bar is said to be subjected to a uniform *direct or normal stress*, the stress being defined as

$$\text{stress }(\sigma) = \frac{\text{load}}{\text{area}} = \frac{P}{A}$$

Stress σ may thus be compressive or tensile depending on the nature of the load and will be measured in units of newtons per square metre (N/m^2) or multiples of this.

In some cases the loading situation is such that the stress will vary across any given section, and in such cases the stress at any point is given by the limiting value of $\delta P/\delta A$ as δA tends to zero.

1.3. Direct strain (ε)

If a bar is subjected to a direct load, and hence a stress, the bar will change in length. If the bar has an original length L and changes in length by an amount δL, the *strain* produced is defined as follows:

$$\text{strain }(\varepsilon) = \frac{\text{change in length}}{\text{original length}} = \frac{\delta L}{L}$$

Strain is thus a measure of the deformation of the material and is non-dimensional, i.e. it has no units; it is simply a ratio of two quantities with the same unit (Fig. 1.2).
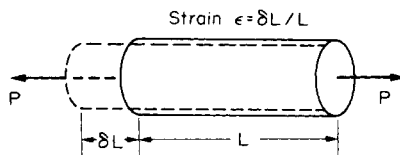

Fig. 1.2.

Since, in practice, the extensions of materials under load are very small, it is often convenient to measure the strains in the form of strain $\times 10^{-6}$, i.e. *microstrain*, when the symbol used becomes $\mu\varepsilon$.

Alternatively, strain can be expressed as a *percentage strain*

i.e.
$$\text{strain }(\varepsilon) = \frac{\delta L}{L} \times 100\%$$

1.4. Sign convention for direct stress and strain

Tensile stresses and strains are considered POSITIVE in sense producing an *increase* in length. Compressive stresses and strains are considered NEGATIVE in sense producing a *decrease* in length.

1.5. Elastic materials – Hooke's law

A material is said to be *elastic* if it returns to its original, unloaded dimensions when load is removed. A particular form of elasticity which applies to a large range of engineering materials, at least over part of their load range, produces deformations which are proportional to the loads producing them. Since loads are proportional to the stresses they produce and deformations are proportional to the strains, this also implies that, whilst materials are elastic, stress is proportional to strain. *Hooke's law*, in its simplest form*, therefore states that

$$\text{stress } (\sigma) \propto \text{strain } (\varepsilon)$$

i.e.
$$\frac{\text{stress}}{\text{strain}} = \text{constant*}$$

It will be seen in later sections that this law is obeyed within certain limits by most ferrous alloys and it can even be assumed to apply to other engineering materials such as concrete, timber and non-ferrous alloys with reasonable accuracy. Whilst a material is elastic the deformation produced by any load will be *completely* recovered when the load is removed; there is no permanent deformation.

Other classifications of materials with which the reader should be acquainted are as follows:

A material which has a uniform structure throughout without any flaws or discontinuities is termed a *homogeneous* material. *Non-homogeneous* or *inhomogeneous* materials such as concrete and poor-quality cast iron will thus have a structure which varies from point to point depending on its constituents and the presence of casting flaws or impurities.

If a material exhibits uniform properties throughout in all directions it is said to be *isotropic*; conversely one which does not exhibit this uniform behaviour is said to be *non-isotropic* or *anisotropic*.

An *orthotropic* material is one which has different properties in different planes. A typical example of such a material is wood, although some composites which contain systematically orientated "inhomogeneities" may also be considered to fall into this category.

1.6. Modulus of elasticity – Young's modulus

Within the elastic limits of materials, i.e. within the limits in which Hooke's law applies, it has been shown that

$$\frac{\text{stress}}{\text{strain}} = \text{constant}$$

This constant is given the symbol E and termed the *modulus of elasticity* or *Young's modulus*.

Thus
$$E = \frac{\text{stress}}{\text{strain}} = \frac{\sigma}{\varepsilon} \tag{1.1}$$

$$= \frac{P}{A} \div \frac{\delta L}{L} = \frac{PL}{A\,\delta L} \tag{1.2}$$

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* Readers should be warned that in more complex stress cases this simple form of Hooke's law will not apply and mis-application could prove dangerous; see §14.1, page 361.

Young's modulus E is generally assumed to be the same in tension or compression and for most engineering materials has a high numerical value. Typically, $E = 200 \times 10^9$ N/m^2 for steel, so that it will be observed from (1.1) that strains are normally very small since

$$\varepsilon = \frac{\sigma}{E} \tag{1.3}$$

In most common engineering applications strains do not often exceed 0.003 or 0.3 % so that the assumption used later in the text that deformations are small in relation to original dimensions is generally well founded.

The actual value of Young's modulus for any material is normally determined by carrying out a standard tensile test on a specimen of the material as described below.

1.7. Tensile test

In order to compare the strengths of various materials it is necessary to carry out some standard form of test to establish their relative properties. One such test is the standard tensile test in which a circular bar of uniform cross-section is subjected to a gradually increasing tensile load until failure occurs. Measurements of the change in length of a selected *gauge length* of the bar are recorded throughout the loading operation by means of extensometers and a graph of load against extension or stress against strain is produced as shown in Fig. 1.3; this shows a typical result for a test on a mild (low carbon) steel bar; other materials will exhibit different graphs but of a similar general form see Figs 1.5 to 1.7.
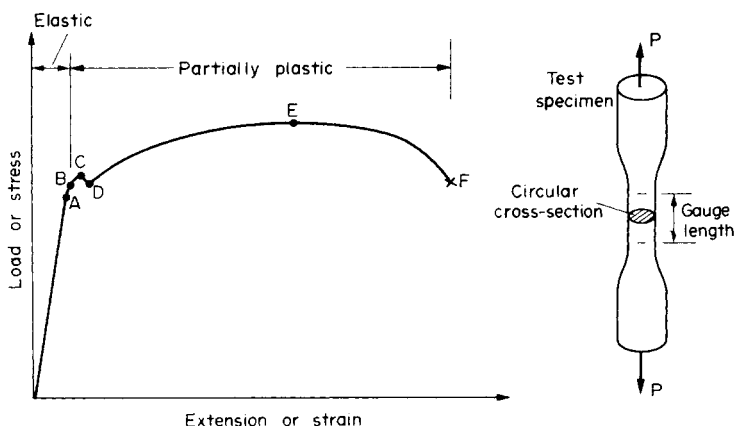

Fig. 1.3. Typical tensile test curve for mild steel.

For the first part of the test it will be observed that Hooke's law is obeyed, i.e. the material behaves elastically and stress is proportional to strain, giving the straight-line graph indicated. Some point A is eventually reached, however, when the linear nature of the graph ceases and this point is termed the *limit of proportionality*.

For a short period beyond this point the material may still be elastic in the sense that deformations are completely recovered when load is removed (i.e. strain returns to zero) but

Hooke's law does not apply. The limiting point B for this condition is termed the *elastic limit*. For most practical purposes it can often be assumed that points A and B are coincident.

Beyond the elastic limit *plastic deformation* occurs and strains are not totally recoverable. There will thus be some permanent deformation or *permanent set* when load is removed. After the points C, termed the *upper yield point*, and D, the *lower yield point*, relatively rapid increases in strain occur without correspondingly high increases in load or stress. The graph thus becomes much more shallow and covers a much greater portion of the strain axis than does the elastic range of the material. The capacity of a material to allow these large plastic deformations is a measure of the so-called *ductility* of the material, and this will be discussed in greater detail below.

For certain materials, for example, high carbon steels and non-ferrous metals, it is not possible to detect any difference between the upper and lower yield points and in some cases no yield point exists at all. In such cases a *proof stress* is used to indicate the onset of plastic strain or as a comparison of the relative properties with another similar material. This involves a measure of the permanent deformation produced by a loading cycle; the 0.1% proof stress, for example, is that stress which, when removed, produces a permanent strain or "set" of 0.1% of the original gauge length – see Fig. 1.4(a).
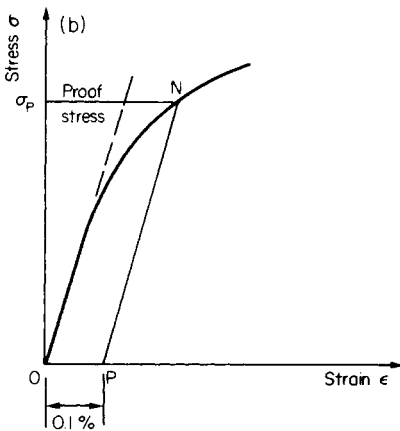

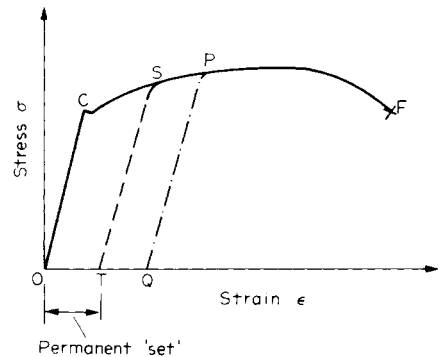Fig. 1.4. (a) Determination of 0.1 % proof stress.

Fig. 1.4. (b) Permanent deformation or "set" after straining beyond the yield point.

The 0.1% proof stress value may be determined from the tensile test curve for the material in question as follows:

Mark the point P on the strain axis which is equivalent to 0.1% strain. From P draw a line parallel with the initial straight line portion of the tensile test curve to cut the curve in N. The stress corresponding to N is then the *0.1 % proof stress*. A material is considered to satisfy its specification if the permanent set is no more than 0.1% after the proof stress has been applied for 15 seconds and removed.

Beyond the yield point some increase in load is required to take the strain to point E on the graph. Between D and E the material is said to be in the *elastic–plastic state*, some of the section remaining elastic and hence contributing to recovery of the original dimensions if load is removed, the remainder being plastic. Beyond E the cross-sectional area of the bar

begins to reduce rapidly over a relatively small length of the bar and the bar is said to *neck*. This necking takes place whilst the load reduces, and fracture of the bar finally occurs at point *F*.

The nominal stress at failure, termed the *maximum* or *ultimate tensile stress*, is given by the load at *E* divided by the original cross-sectional area of the bar. (This is also known as the *tensile strength* of the material of the bar.) Owing to the large reduction in area produced by the necking process the actual stress at fracture is often greater than the above value. Since, however, designers are interested in maximum loads which can be carried by the complete cross-section, the stress at fracture is seldom of any practical value.

If load is removed from the test specimen after the yield point *C* has been passed, e.g. to some position *S*, Fig. 1.4(b), the unloading line *ST* can, for most practical purposes, be taken to be linear. Thus, despite the fact that loading to *S* comprises both elastic (*OC*) and partially plastic (*CS*) portions, the unloading procedure is totally elastic. A second load cycle, commencing with the permanent elongation associated with the strain *OT*, would then follow the line *TS* and continue along the previous curve to failure at *F*. It will be observed, however, that the repeated load cycle has the effect of increasing the elastic range of the material, i.e. raising the effective yield point from *C* to *S*, while the tensile strength is unaltered. The procedure could be repeated along the line *PQ*, etc., and the material is said to have been *work hardened*.

In fact, careful observation shows that the material will no longer exhibit true elasticity since the unloading and reloading lines will form a small *hysteresis loop*, neither being precisely linear. Repeated loading and unloading will produce a yield point approaching the ultimate stress value but the elongation or strain to failure will be much reduced.

Typical stress–strain curves resulting from tensile tests on other engineering materials are shown in Figs 1.5 to 1.7.
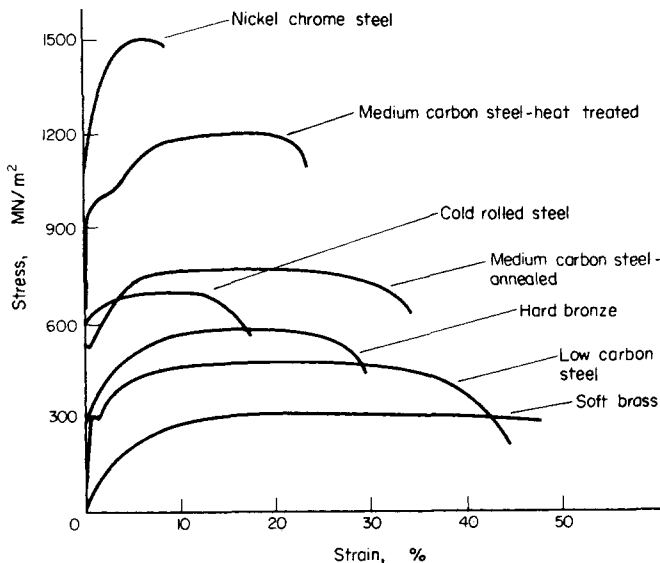

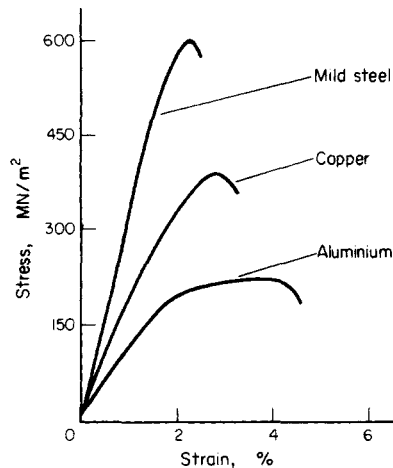Fig. 1.5. Tensile test curves for various metals.

Fig. 1.6. Typical stress–strain curves for hard drawn wire materials – note large reduction in strain values from those of Fig. 1.5.
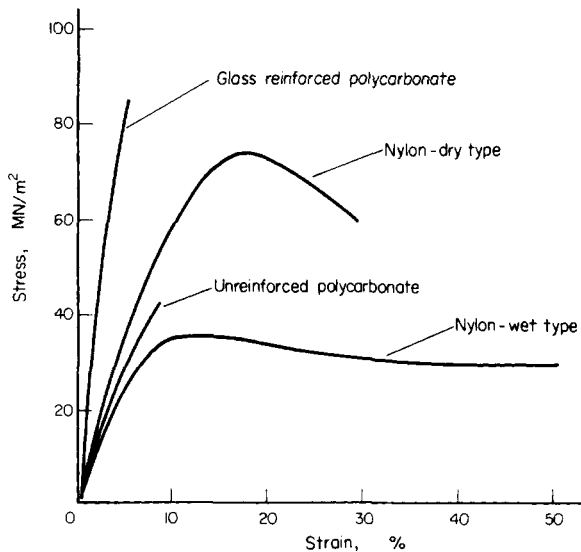

Fig. 1.7. Typical tension test results for various types of nylon and polycarbonate.

After completing the standard tensile test it is usually necessary to refer to some "British Standard Specification" or "Code of Practice" to ensure that the material tested satisfies the requirements, for example:

BS 4360   British Standard Specification for Weldable Structural Steels.
BS 970   British Standard Specification for Wrought Steels.
BS 153   British Standard Specification for Steel Girder Bridges.
BS 449   British Standard Specification for the use of Structural Steel in Building, etc.

1.8. Ductile materials

It has been observed above that the partially plastic range of the graph of Fig. 1.3 covers a much wider part of the strain axis than does the elastic range. Thus the extension of the material over this range is considerably in excess of that associated with elastic loading. The capacity of a material to allow these large extensions, i.e. the ability to be drawn out plastically, is termed its *ductility*. Materials with high ductility are termed *ductile* materials, members with low ductility are termed *brittle* materials. A quantitative value of the ductility is obtained by measurements of the *percentage elongation* or *percentage reduction in area*, both being defined below.

$$\text{Percentage elongation} = \frac{\text{increase in gauge length to fracture}}{\text{original gauge length}} \times 100$$

$$\text{Percentage reduction in area} = \frac{\text{reduction in cross-sectional area of necked portion}}{\text{original area}} \times 100$$

The latter value, being independent of any selected gauge length, is generally taken to be the more useful measure of ductility for reference purposes.

A property closely related to ductility is *malleability*, which defines a material's ability to be hammered out into thin sheets. A typical example of a malleable material is lead. This is used extensively in the plumbing trade where it is hammered or beaten into corners or joints to provide a weatherproof seal. Malleability thus represents the ability of a material to allow permanent extensions in all lateral directions under compressive loadings.

1.9. Brittle materials

A brittle material is one which exhibits relatively small extensions to fracture so that the partially plastic region of the tensile test graph is much reduced (Fig. 1.8). Whilst Fig. 1.3 referred to a low carbon steel, Fig. 1.8 could well refer to a much higher strength steel with a higher carbon content. There is little or no necking at fracture for brittle materials.
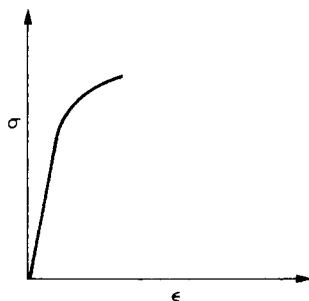

Fig. 1.8. Typical tensile test curve for a brittle material.

Typical variations of mechanical properties of steel with carbon content are shown in Fig. 1.9.
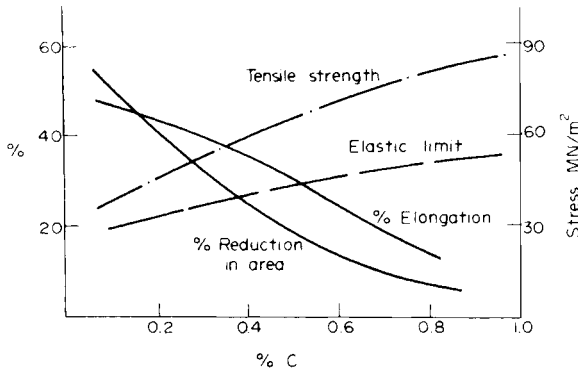
Fig. 1.9. Variation of mechanical properties of steel with carbon content.

1.10. Poisson's ratio

Consider the rectangular bar of Fig. 1.10 subjected to a tensile load. Under the action of this load the bar will increase in length by an amount δL giving a longitudinal strain in the bar of
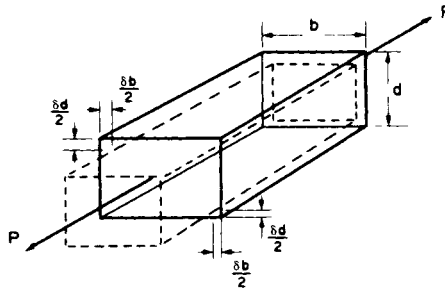
$$\varepsilon_L = \frac{\delta L}{L}$$

Fig. 1.10.

The bar will also exhibit, however, a *reduction* in dimensions laterally, i.e. its breadth and depth will both reduce. The associated lateral strains will both be equal, will be of opposite sense to the longitudinal strain, and will be given by

$$\varepsilon_{\text{lat}} = -\frac{\delta b}{b} = -\frac{\delta d}{d}$$

Provided the load on the material is retained within the elastic range the ratio of the lateral and longitudinal strains will always be constant. This ratio is termed *Poisson's ratio*.

i.e. $$\textbf{Poisson's ratio } (v) = \frac{\textbf{lateral strain}}{\textbf{longitudinal strain}} = \frac{(-\delta d/d)}{\delta L/L} \qquad (1.4)$$

The negative sign of the lateral strain is normally ignored to leave Poisson's ratio simply as

a ratio of strain magnitudes. It must be remembered, however, that the longitudinal strain induces a lateral strain of opposite sign, e.g. tensile longitudinal strain induces compressive lateral strain.

For most engineering materials the value of v lies between 0.25 and 0.33.
Since

$$\text{longitudinal strain} = \frac{\text{longitudinal stress}}{\text{Young's modulus}} = \frac{\sigma}{E} \tag{1.4a}$$

Hence

$$\textbf{lateral strain} = v\,\frac{\sigma}{E} \tag{1.4b}$$

1.11. Application of Poisson's ratio to a two-dimensional stress system

A two-dimensional stress system is one in which all the stresses lie within one plane such as the X–Y plane. From the work of §1.10 it will be seen that if a material is subjected to a tensile stress σ on one axis producing a strain σ/E and hence an extension on that axis, it will be subjected simultaneously to a lateral strain of v times σ/E on any axis at right angles. This lateral strain will be compressive and will result in a compression or reduction of length on this axis.

Consider, therefore, an element of material subjected to two stresses at right angles to each other and let both stresses, σ_x and σ_y, be considered tensile, see Fig. 1.11.
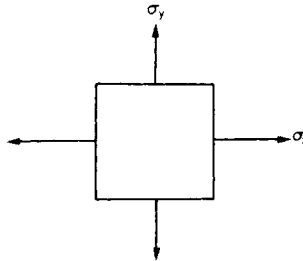

Fig. 1.11. Simple two-dimensional system of direct stresses.

The following strains will be produced:

(a) in the X direction resulting from $\sigma_x = \sigma_x/E$,
(b) in the Y direction resulting from $\sigma_y = \sigma_y/E$.
(c) in the X direction resulting from $\sigma_y = -v(\sigma_y/E)$,
(d) in the Y direction resulting from $\sigma_x = -v(\sigma_x/E)$.

strains (c) and (d) being the so-called *Poisson's ratio strain*, opposite in sign to the applied strains, i.e. compressive.

The total strain in the X direction will therefore be given by:

$$\varepsilon_x = \frac{\sigma_x}{E} - v\frac{\sigma_y}{E} = \frac{1}{E}(\sigma_x - v\sigma_y) \tag{1.5}$$

and the total strain in the Y direction will be:

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E} = \frac{1}{E}(\sigma_y - \nu\sigma_x) \tag{1.6}$$

If any stress is, in fact, compressive its value must be substituted in the above equations together with a negative sign following the normal sign convention.

1.12. Shear stress

Consider a block or portion of material as shown in Fig. 1.12a subjected to a set of equal and opposite forces Q. (Such a system could be realised in a bicycle brake block when contacted with the wheel.) There is then a tendency for one layer of the material to slide over another to produce the form of failure shown in Fig. 1.12b. If this failure is restricted, then a *shear stress* τ is set up, defined as follows:

$$\text{shear stress } (\tau) = \frac{\text{shear load}}{\text{area resisting shear}} = \frac{Q}{A}$$

This shear stress will always be *tangential* to the area on which it acts; direct stresses, however, are always *normal* to the area on which they act.
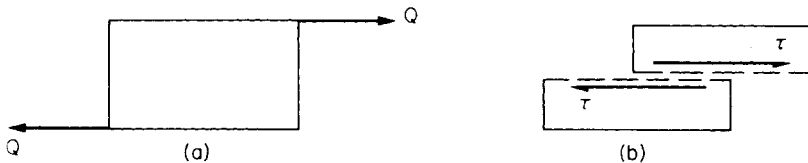

Fig. 1.12. Shear force and resulting shear stress system showing typical form of failure by relative sliding of planes.

1.13. Shear strain

If one again considers the block of Fig. 1.12a to be a bicycle brake block it is clear that the rectangular shape of the block will not be retained as the brake is applied and the shear forces introduced. The block will in fact change shape or "strain" into the form shown in Fig. 1.13. The angle of deformation γ is then termed the *shear strain*.

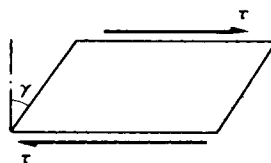Shear strain is measured in radians and hence is non-dimensional, i.e. it has no units.

Fig. 1.13. Deformation (shear strain) produced by shear stresses.

1.14. Modulus of rigidity

For materials within the elastic range the shear strain is proportional to the shear stress producing it,
i.e.

$$\frac{\text{shear stress}}{\text{shear strain}} = \frac{\tau}{\gamma} = \text{constant} = G \tag{1.7}$$

The constant G is termed the *modulus of rigidity* or *shear modulus* and is directly comparable to the modulus of elasticity used in the direct stress application. The term *modulus* thus implies a ratio of stress to strain in each case.

1.15. Double shear

Consider the simple riveted lap joint shown in Fig. 1.14a. When load is applied to the plates the rivet is subjected to shear forces tending to shear it on one plane as indicated. In the butt joint with two cover plates of Fig. 1.14b, however, each rivet is subjected to possible shearing on two faces, i.e. *double shear*. In such cases twice the area of metal is resisting the applied forces so that the shear stress set up is given by

$$\text{shear stress } \tau \text{ (in double shear)} = \frac{P}{2A} \tag{1.8}$$
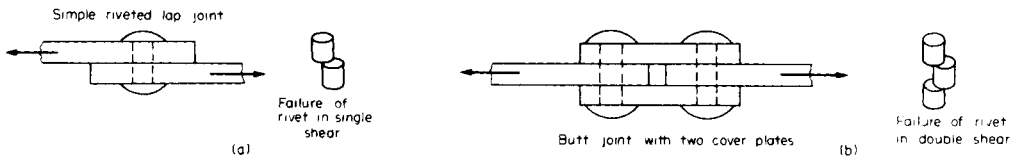

Fig. 1.14. (a) Single shear.   (b) Double shear.

1.16. Allowable working stress – factor of safety

The most suitable strength or stiffness criterion for any structural element or component is normally some maximum stress or deformation which must not be exceeded. In the case of stresses the value is generally known as the *maximum allowable working stress*.

Because of uncertainties of loading conditions, design procedures, production methods, etc., designers generally introduce a *factor of safety* into their designs, defined as follows:

$$\text{factor of safety} = \frac{\text{maximum stress}}{\text{allowable working stress}} \tag{1.9}$$

However, in view of the fact that plastic deformations are seldom accepted this definition is sometimes modified to

$$\text{factor of safety} = \frac{\text{yield stress (or proof stress)}}{\text{allowable working stress}}$$

In the absence of any information as to which definition has been used for any quoted value of safety factor the former definition must be assumed. In this case a factor of safety of 3 implies that the design is capable of carrying three times the maximum stress to which it is expected the structure will be subjected in any normal loading condition. There is seldom any realistic basis for the selection of a particular safety factor and values vary significantly from one branch of engineering to another. Values are normally selected on the basis of a consideration of the social, human safety and economic consequences of failure. Typical values range from 2.5 (for relatively low consequence, static load cases) to 10 (for shock load and high safety risk applications)—see §15.12.

1.17. Load factor

In some loading cases, e.g. buckling of struts, neither the yield stress nor the ultimate strength is a realistic criterion for failure of components. In such cases it is convenient to replace the safety factor, based on stresses, with a different factor based on loads. The *load factor* is therefore defined as:

$$\text{load factor} = \frac{\text{load at failure}}{\text{allowable working load}} \tag{1.10}$$

This is particularly useful in applications of the so-called plastic limit design procedures.

1.18. Temperature stresses

When the temperature of a component is increased or decreased the material respectively expands or contracts. If this expansion or contraction is not resisted in any way then the processes take place free of stress. If, however, the changes in dimensions are restricted then stresses termed *temperature stresses* will be set up within the material.

Consider a bar of material with a linear coefficient of expansion α. Let the original length of the bar be L and let the temperature *increase* be t. If the bar is free to expand the change in length would be given by

$$\Delta L = L\alpha t \tag{1.11}$$

and the new length

$$L' = L + L\alpha t = L(1 + \alpha t)$$

If this extension were totally prevented, then a compressive stress would be set up equal to that produced when a bar of length $L(1 + \alpha t)$ is compressed through a distance of $L\alpha t$. In this case the bar experiences a compressive strain

$$\varepsilon = \frac{\Delta L}{L} = \frac{L\alpha t}{L(1 + \alpha t)}$$

In most cases αt is very small compared with unity so that

$$\varepsilon = \frac{L\alpha t}{L} = \alpha t$$

But
$$\frac{\sigma}{\varepsilon} = E$$

∴   stress $\sigma = E\varepsilon = E\alpha t$ (1.12)

This is the stress set up owing to total restraint on expansions or contractions caused by a temperature rise, or fall, t. In the former case the stress is compressive, in the latter case the stress is tensile.

If the expansion or contraction of the bar is *partially* prevented then the stress set up will be less than that given by eqn. (1.10). Its value will be found in a similar way to that described above except that instead of being compressed through the total free expansion distance of $L\alpha t$ it will be compressed through some proportion of this distance depending on the amount of restraint.

Assuming some fraction n of $L\alpha t$ is *allowed*, then the extension which is prevented is $(1-n)L\alpha t$. This will produce a compressive strain, as described previously, of magnitude

$$\varepsilon = \frac{(1-n)L\alpha t}{L(1+\alpha t)}$$

or, approximately,

$$\varepsilon = (1-n)L\alpha t/L = (1-n)\alpha t.$$

The stress set up will then be E times ε.

i.e. $\sigma = (1-n)E\alpha t$ (1.13)

Thus, for example, if one-third of the free expansion is prevented the stress set up will be two-thirds of that given by eqn. (1.12).

1.19. Stress concentrations – stress concentration factor

If a bar of uniform cross-section is subjected to an axial tensile or compressive load the stress is assumed to be uniform across the section. However, in the presence of any sudden change of section, hole, sharp corner, notch, keyway, material flaw, etc., the local stress will rise significantly. The ratio of this stress to the nominal stress at the section in the absence of any of these so-called *stress concentrations* is termed the *stress concentration* factor.

1.20. Toughness

Toughness is defined as the ability of a material to withstand cracks, i.e. to prevent the transfer or propagation of cracks across its section hence causing failure. Two distinct types of toughness mechanism exist and in each case it is appropriate to consider the crack as a very high local stress concentration.

The first type of mechanism relates particularly to ductile materials which are generally regarded as tough. This arises because the very high stresses at the end of the crack produce local yielding of the material and local plastic flow at the crack tip. This has the action of blunting the sharp tip of the crack and hence reduces its stress-concentration effect considerably (Fig. 1.15).
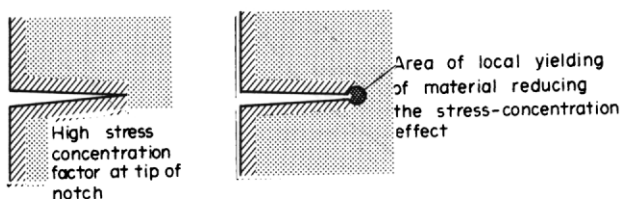
Fig. 1.15. Toughness mechanism – type

The second mechanism refers to fibrous, reinforced or resin-based materials which have weak interfaces. Typical examples are glass-fibre reinforced materials and wood. It can be shown that a region of local tensile stress always exists at the front of a propagating crack and provided that the adhesive strength of the fibre/resin interface is relatively low (one-fifth the cohesive strength of the complete material) this tensile stress opens up the interface and produces a crack sink, i.e. it blunts the crack by effectively increasing the radius at the crack tip, thereby reducing the stress-concentration effect (Fig. 1.16).

This principle is used on occasions to stop, or at least delay, crack propagation in engineering components when a temporary "repair" is carried out by drilling a hole at the end of a crack, again reducing its stress-concentration effect.
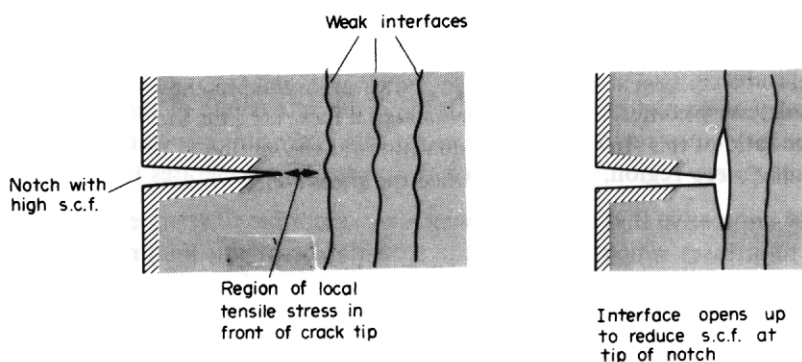

Fig. 1.16. Toughness mechanism – type 2.

1.21. Creep and fatigue

In the preceding paragraphs it has been suggested that failure of materials occurs when the ultimate strengths have been exceeded. Reference has also been made in §1.15 to cases where excessive deformation, as caused by plastic deformation beyond the yield point, can be considered as a criterion for effective failure of components. This chapter would not be complete, therefore, without reference to certain loading conditions under which materials can fail at stresses much less than the yield stress, namely *creep* and *fatigue*.

Creep is the gradual increase of plastic strain in a material with time at constant load. Particularly at elevated temperatures some materials are susceptible to this phenomenon and even under the constant load mentioned strains can increase continually until fracture. This form of fracture is particularly relevant to turbine blades, nuclear reactors, furnaces, rocket motors, etc.
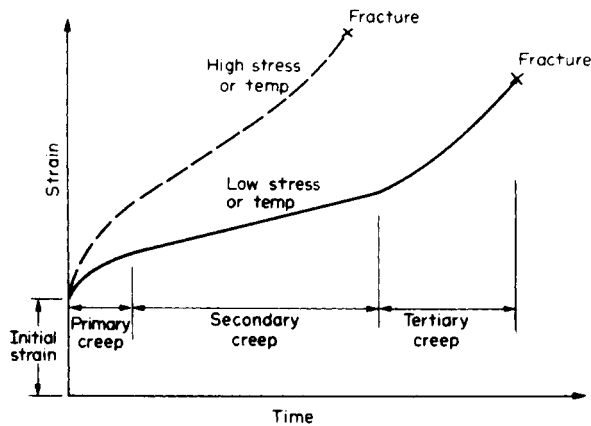
Fig. 1.17. Typical creep curve.

The general form of the strain versus time graph or *creep curve* is shown in Fig. 1.17 for two typical operating conditions. In each case the curve can be considered to exhibit four principal features.

(a) An *initial strain*, due to the initial application of load. In most cases this would be an elastic strain.
(b) A *primary creep* region, during which the creep rate (slope of the graph) diminishes.
(c) A *secondary creep* region, when the creep rate is sensibly constant.
(d) A *tertiary creep* region, during which the creep rate accelerates to final fracture.

It is clearly imperative that a material which is susceptible to creep effects should only be subjected to stresses which keep it in the secondary (straight line) region throughout its service life. This enables the amount of creep extension to be estimated and allowed for in design.

Fatigue is the failure of a material under fluctuating stresses each of which is believed to produce minute amounts of plastic strain. Fatigue is particularly important in components subjected to repeated and often rapid load fluctuations, e.g. aircraft components, turbine blades, vehicle suspensions, etc. Fatigue behaviour of materials is usually described by a *fatigue life* or *S–N curve* in which the number of stress cycles N to produce failure with a stress peak of S is plotted against S. A typical $S-N$ curve for mild steel is shown in Fig. 1.18. The particularly relevant feature of this curve is the limiting stress S_n since it is assumed that stresses below this value will not produce fatigue failure however many cycles are applied, i.e. there is *infinite life*. In the simplest design cases, therefore, there is an aim to keep all stresses below this limiting level. However, this often implies an over-design in terms of physical size and material usage, particularly in cases where the stress may only occasionally exceed the limiting value noted above. This is, of course, particularly important in applications such as aerospace structures where component weight is a premium. Additionally the situation is complicated by the many materials which do not show a defined limit, and modern design procedures therefore rationalise the situation by aiming at a prescribed, long, but *finite life*, and accept that service stresses will occasionally exceed the value S_n. It is clear that the number of occasions on which the stress exceeds S_n, and by how
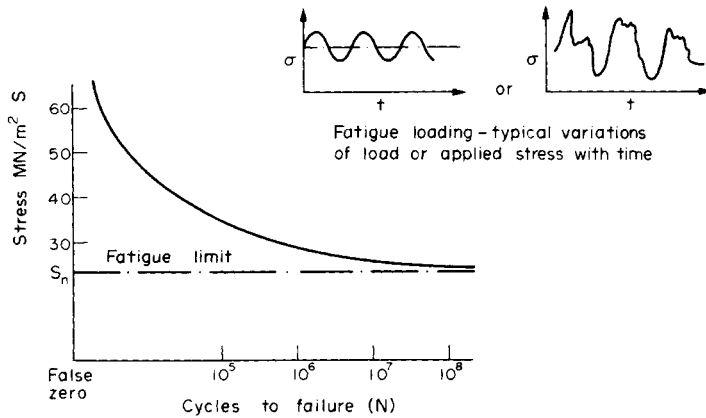
Fig. 1.18. Typical *S–N* fatigue curve for mild steel.

much, will have an important bearing on the prescribed life and considerable specimen, and often full-scale, testing is required before sufficient statistics are available to allow realistic life assessment.

The importance of the creep and fatigue phenomena cannot be over-emphasised and the comments above are only an introduction to the concepts and design philosophies involved. For detailed consideration of these topics and of the other materials testing topics introduced earlier the reader is referred to the texts listed at the end of this chapter.

Examples

Example 1.1

Determine the stress in each section of the bar shown in Fig. 1.19 when subjected to an axial tensile load of 20 kN. The central section is 30 mm square cross-section; the other portions are of circular section, their diameters being indicated. What will be the total extension of the bar? For the bar material $E = 210\,\text{GN/m}^2$.
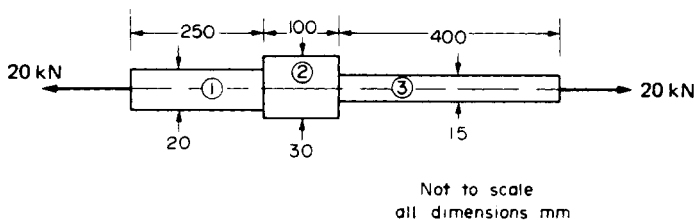

Fig. 1.19.

Solution

$$\text{Stress} = \frac{\text{force}}{\text{area}} = \frac{P}{A}$$

$$\text{Stress in section (1)} = \frac{20 \times 10^3}{\dfrac{\pi(20 \times 10^{-3})^2}{4}} = \frac{80 \times 10^3}{\pi \times 400 \times 10^{-6}} = 63.66\,\text{MN/m}^2$$

$$\text{Stress in section (2)} = \frac{20 \times 10^3}{30 \times 30 \times 10^{-6}} = 22.2\,\text{MN/m}^2$$

$$\text{Stress in section (3)} = \frac{20 \times 10^3}{\dfrac{\pi(15 \times 10^{-3})^2}{4}} = \frac{80 \times 10^3}{\pi \times 225 \times 10^{-6}} = 113.2\,\text{MN/m}^2$$

Now the extension of a bar can always be written in terms of the stress in the bar since

$$E = \frac{\text{stress}}{\text{strain}} = \frac{\sigma}{\delta/L}$$

i.e.

$$\delta = \frac{\sigma L}{E}$$

∴

$$\text{extension of section (1)} = 63.66 \times 10^6 \times \frac{250 \times 10^{-3}}{210 \times 10^9} = 75.8 \times 10^{-6}\,\text{m}$$

$$\text{extension of section (2)} = 22.2 \times 10^6 \times \frac{100 \times 10^{-3}}{210 \times 10^9} = 10.6 \times 10^{-6}\,\text{m}$$

$$\text{extension of section (3)} = 113.2 \times 10^6 \times \frac{400 \times 10^{-3}}{210 \times 10^9} = 215.6 \times 10^{-6}\,\text{m}$$

∴

$$\text{total extension} = (75.8 + 10.6 + 215.6)10^{-6}$$
$$= 302 \times 10^{-6}\,\text{m}$$
$$= \mathbf{0.302\,mm}$$

Example 1.2

(a) A 25 mm diameter bar is subjected to an axial tensile load of 100 kN. Under the action of this load a 200 mm gauge length is found to extend 0.19×10^{-3} mm. Determine the modulus of elasticity for the bar material.

(b) If, in order to reduce weight whilst keeping the external diameter constant, the bar is bored axially to produce a cylinder of uniform thickness, what is the maximum diameter of bore possible given that the maximum allowable stress is $240\,\text{MN/m}^2$? The load can be assumed to remain constant at 100 kN.

(c) What will be the change in the outside diameter of the bar under the limiting stress quoted in (b)? ($E = 210\,\text{GN/m}^2$ and $v = 0.3$).

Solution

(a) From eqn. (1.2),

$$\text{Young's modulus } E = \frac{PL}{A\,\delta L}$$

$$= \frac{100 \times 10^3 \times 200 \times 10^{-3}}{\dfrac{\pi(25 \times 10^{-3})^2}{4} \times 0.19 \times 10^{-3}}$$

$$= 214\,\text{GN/m}^2$$

(b) Let the required bore diameter be d mm; the cross-sectional area of the bar will then be reduced to

$$A = \left[\frac{\pi \times 25^2}{4} - \frac{\pi d^2}{4}\right]10^{-6} = \frac{\pi}{4}(25^2 - d^2)10^{-6}\,\text{m}^2$$

$$\therefore \qquad \text{stress in bar} = \frac{P}{A} = \frac{4 \times 100 \times 10^3}{\pi(25^2 - d^2)10^{-6}}$$

But this stress is restricted to a maximum allowable value of 240 MN/m².

$$\therefore \qquad 240 \times 10^6 = \frac{4 \times 100 \times 10^3}{\pi(25^2 - d^2)10^{-6}}$$

$$\therefore \qquad 25^2 - d^2 = \frac{4 \times 100 \times 10^3}{240 \times 10^6 \times \pi \times 10^{-6}} = 530.5$$

$$\therefore \qquad d^2 = 94.48 \quad \text{and} \quad d = 9.72\,\text{mm}$$

The maximum bore possible is thus **9.72 mm**.

(c) The change in the outside diameter of the bar will be obtained from the lateral strain,

$$\text{i.e.} \qquad \text{lateral strain} = \frac{\delta d}{d}$$

$$\text{But} \qquad \text{Poisson's ratio } v = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

$$\text{and} \qquad \text{longitudinal strain} = \frac{\sigma}{E} = \frac{240 \times 10^6}{210 \times 10^9}$$

$$\therefore \qquad \frac{\delta d}{d} = -v\frac{\sigma}{E} = -\frac{0.3 \times 240 \times 10^6}{210 \times 10^9}$$

$$\therefore \qquad \text{change in outside diameter} = -\frac{0.3 \times 240 \times 10^6}{210 \times 10^9} \times 25 \times 10^{-3}$$

$$= -8.57 \times 10^{-6}\,\text{m} \text{ (a reduction)}$$

Example 1.3

The coupling shown in Fig. 1.20 is constructed from steel of rectangular cross-section and is designed to transmit a tensile force of 50 kN. If the bolt is of 15 mm diameter calculate:

(a) the shear stress in the bolt;
(b) the direct stress in the plate;
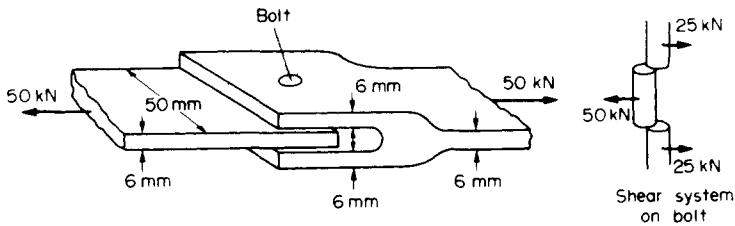(c) the direct stress in the forked end of the coupling.

Fig. 1.20.

Solution

(a) The bolt is subjected to double shear, tending to shear it as shown in Fig. 1.14b. There is thus twice the area of the bolt resisting the shear and from eqn. (1.8)

$$\text{shear stress in bolt} = \frac{P}{2A} = \frac{50 \times 10^3 \times 4}{2 \times \pi(15 \times 10^{-3})^2}$$

$$= \frac{100 \times 10^3}{\pi(15 \times 10^{-3})^2} = 141.5 \, \text{MN/m}^2$$

(b) The plate will be subjected to a direct tensile stress given by

$$\sigma = \frac{P}{A} = \frac{50 \times 10^3}{50 \times 6 \times 10^{-6}} = 166.7 \, \text{MN/m}^2$$

(c) The force in the coupling is shared by the forked end pieces, each being subjected to a direct stress

$$\sigma = \frac{P}{A} = \frac{25 \times 10^3}{50 \times 6 \times 10^{-6}} = 83.3 \, \text{MN/m}^2$$

Example 1.4

Derive an expression for the total extension of the tapered bar of circular cross-section shown in Fig. 1.21 when it is subjected to an axial tensile load W.
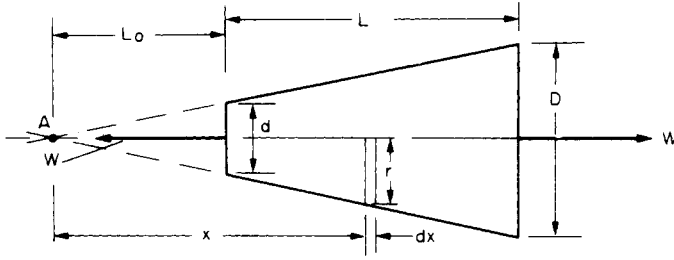
Fig. 1.21.

Solution

From the proportions of Fig. 1.21,

$$\frac{d/2}{L_0} = \frac{(D-d)/2}{L}$$

$\therefore$
$$L_0 = \frac{d}{(D-d)} L$$

Consider an element of thickness dx and radius r, distance x from the point of taper A.

$$\text{Stress on the element} = \frac{W}{\pi r^2}$$

But
$$\frac{r}{x} = \frac{d}{2L_0}$$

$\therefore$
$$r = d\left(\frac{D-d}{2dL}\right)x = \frac{x(D-d)}{2L}$$

$\therefore$
$$\text{stress on the element} = \frac{4WL^2}{\pi(D-d)^2 x^2}$$

$\therefore$
$$\text{strain on the element} = \frac{\sigma}{E}$$

$$\text{and extension of the element} = \frac{\sigma dx}{E}$$

$$= \frac{4WL^2}{\pi(D-d)^2 x^2 E} dx$$

$\therefore$
$$\text{total extension of bar} = \int_{L_0}^{L_0+L} \frac{4WL^2}{\pi(D-d)^2 E} \frac{dx}{x^2}$$

$$= \frac{4WL^2}{\pi(D-d)^2 E}\left[-\frac{1}{x}\right]_{L_0}^{L_0+L}$$

$$= \frac{4WL^2}{\pi(D-d)^2 E}\left[-\frac{1}{(L_0+L)} - \left(-\frac{1}{L_0}\right)\right]$$

But
$$L_0 = \frac{d}{(D-d)} L$$

$\therefore$
$$L_0 + L = \frac{d}{(D-d)} L + L = \frac{(d+D-d)}{D-d} L = \frac{DL}{(D-d)}$$

$\therefore$ total extension

$$= \frac{4WL^2}{\pi(D-d)^2 E} \left[-\frac{(D-d)}{DL} + \frac{(D-d)}{dL} \right] = \frac{4WL}{\pi(D-d)E} \left[\frac{(-d+D)}{Dd} \right]$$

$$= \frac{4WL}{\pi DdE}$$

Example 1.5

The following figures were obtained in a standard tensile test on a specimen of low carbon steel:

 diameter of specimen, 11.28 mm;
 gauge length, 56 mm;
 minimum diameter after fracture, 6.45 mm.

Using the above information and the table of results below, produce:

(1) a load/extension graph over the complete test range;
(2) a load/extension graph to an enlarged scale over the elastic range of the specimen.

Load (kN)	2.47	4.97	7.4	9.86	12.33	14.8	17.27	19.74	22.2	24.7
Extension (m × 10⁻⁶)	5.6	11.9	18.2	24.5	31.5	38.5	45.5	52.5	59.5	66.5

Load (kN)	27.13	29.6	32.1	33.3	31.2	32	31.5	32	32.2	34.5
Extension (m × 10⁻⁶)	73.5	81.2	89.6	112	224	448	672	840	1120	1680

Load (kN)	35.8	37	38.7	39.5	40	39.6	35.7	28		
Extension (m × 10⁻⁶)	1960	2520	3640	5600	7840	11200	13440	14560		

Using the two graphs and other information supplied, determine the values of:

(a) Young's modulus of elasticity;
(b) the ultimate tensile stress;
(c) the stress at the upper and lower yield points;
(d) the percentage reduction of area;
(e) the percentage elongation;
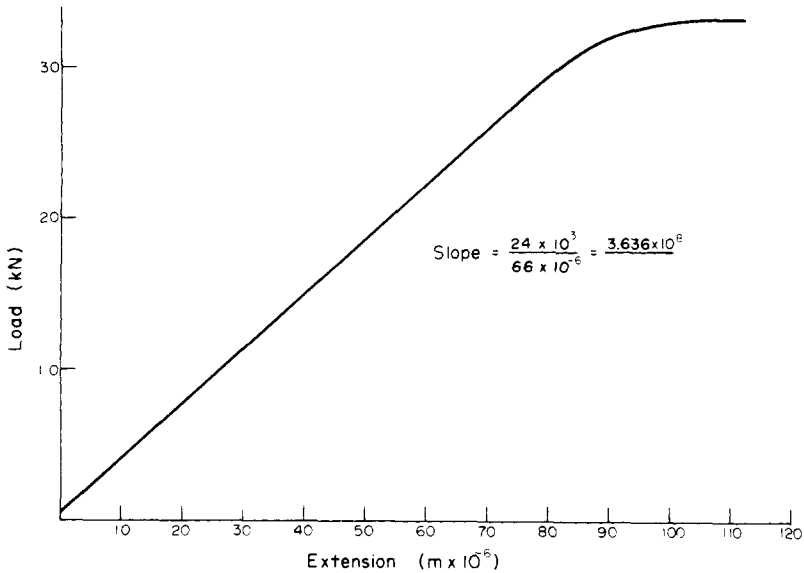(f) the nominal and actual stress at fracture.

Solution

Fig. 1.22. Load–extension graph for elastic range.

(a) $\qquad$ Young's modulus $E = \dfrac{\sigma}{\varepsilon} = \dfrac{\text{load}}{\text{area}} \times \dfrac{\text{gauge length}}{\text{extension}}$

$$= \dfrac{\text{load}}{\text{extension}} \times \dfrac{\text{gauge length}}{\text{area}}$$

i.e $\qquad E = \text{slope of graph} \times \dfrac{L}{A} = 3.636 \times 10^8 \times \dfrac{56 \times 10^{-3}}{100 \times 10^{-6}}$

$$= 203.6 \times 10^9 \text{ N/m}^2$$

$\therefore \qquad \boldsymbol{E = 203.6 \text{ GN/m}^2}$

(b) $\quad$ Ultimate tensile stress $= \dfrac{\text{maximum load}}{\text{cross-section area}} = \dfrac{40.2 \times 10^3}{100 \times 10^{-6}} = \boldsymbol{402 \text{ MN/m}^2}$

(see Fig. 1.23).

(c) $\qquad$ Upper yield stress $= \dfrac{33.3 \times 10^3}{100 \times 10^{-6}} = \boldsymbol{333 \text{ MN/m}^2}$

$\qquad$ Lower yield stress $= \dfrac{31.2 \times 10^3}{100 \times 10^{-6}} = \boldsymbol{312 \text{ MN/m}^2}$
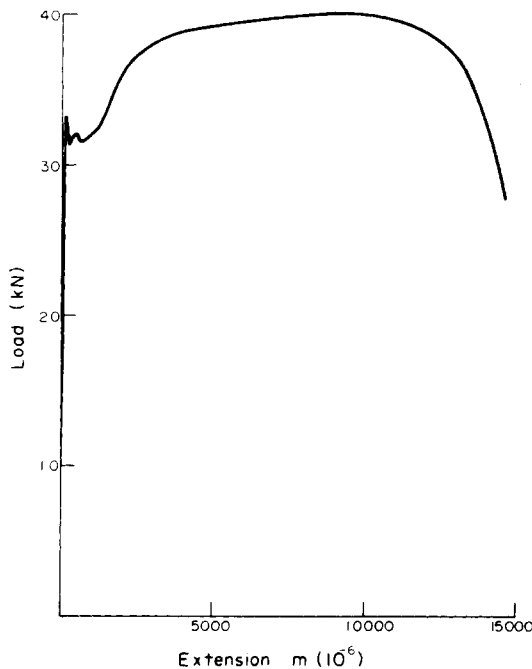
Fig. 1.23. Load–extension graph for complete load range.

(d)   Percentage reduction of area $= \dfrac{\left(\dfrac{\pi}{4}D^2 - \dfrac{\pi}{4}d^2\right)}{\dfrac{\pi}{4}D^2} \times 100$

$= \dfrac{(D^2 - d^2)}{D^2} \times 100$

$= \dfrac{(11.28^2 - 6.45^2)}{11.28^2} = \mathbf{67.3\%}$

(e)   Percentage elongation $= \dfrac{(70.56 - 56)}{56} \times 100$

$= \mathbf{26\%}$

(f)   Nominal stress at fracture $= \dfrac{28 \times 10^3}{100 \times 10^{-6}} = \mathbf{280\,MN/m^2}$

Actual stress at fracture $= \dfrac{28 \times 10^3}{\dfrac{\pi}{4}(6.45)^2 \times 10^{-6}} = \mathbf{856.9\,MN/m^2}$

Problems

1.1 (A). A 25 mm square-cross-section bar of length 300 mm carries an axial compressive load of 50 kN. Determine the stress set up in the bar and its change of length when the load is applied. For the bar material $E = 200 \, \text{GN/m}^2$. [80 MN/m²; 0.12 mm.]

1.2 (A). A steel tube, 25 mm outside diameter and 12 mm inside diameter, carries an axial tensile load of 40 kN. What will be the stress in the bar? What further increase in load is possible if the stress in the bar is limited to 225 MN/m²? [106 MN/m²; 45 kN.]

1.3 (A). Define the terms *shear stress* and *shear strain*, illustrating your answer by means of a simple sketch.
Two circular bars, one of brass and the other of steel, are to be loaded by a shear load of 30 kN. Determine the necessary diameter of the bars (a) in single shear, (b) in double shear, if the shear stress in the two materials must not exceed 50 MN/m² and 100 MN/m² respectively. [27.6, 19.5, 19.5, 13.8 mm.]

1.4 (A). Two fork-end pieces are to be joined together by a single steel pin of 25 mm diameter and they are required to transmit 50 kN. Determine the minimum cross-sectional area of material required in one branch of either fork if the stress in the fork material is not to exceed 180 MN/m². What will be the maximum shear stress in the pin? [$1.39 \times 10^{-4} \, \text{m}^2$; 50.9 MN/m².]

1.5 (A). A simple turnbuckle arrangement is constructed from a 40 mm outside diameter tube threaded internally at each end to take two rods of 25 mm outside diameter with threaded ends. What will be the nominal stresses set up in the tube and the rods, ignoring thread depth, when the turnbuckle carries an axial load of 30 kN? Assuming a sufficient strength of thread, what maximum load can be transmitted by the turnbuckle if the maximum stress is limited to 180 MN/m²? [39.2, 61.1 MN/m²; 88.4 kN.]

1.6 (A). An I-section girder is constructed from two 80 mm × 12 mm flanges joined by an 80 mm × 12 mm web. Four such girders are mounted vertically one at each corner of a horizontal platform which the girders support. The platform is 4 m above ground level and weighs 10 kN. Assuming that each girder supports an equal share of the load, determine the maximum compressive stress set up in the material of each girder when the platform supports an additional load of 15 kN. The weight of the girders may *not* be neglected. The density of the cast iron from which the girders are constructed is 7470 kg/m³. [2.46 MN/m².]

1.7 (A). A bar *ABCD* consists of three sections: *AB* is 25 mm square and 50 mm long, *BC* is of 20 mm diameter and 40 mm long and *CD* is of 12 mm diameter and 50 mm long. Determine the stress set up in each section of the bar when it is subjected to an axial tensile load of 20 kN. What will be the total extension of the bar under this load? For the bar material, $E = 210 \, \text{GN/m}^2$. [32, 63.7, 176.8 MN/m², 0.062 mm.]

1.8 (A). A steel bar *ABCD* consists of three sections: *AB* is of 20 mm diameter and 200 mm long, *BC* is 25 mm square and 400 mm long, and *CD* is of 12 mm diameter and 200 mm long. The bar is subjected to an axial compressive load which induces a stress of 30 MN/m² on the largest cross-section. Determine the total decrease in the length of the bar when the load is applied. For steel $E = 210 \, \text{GN/m}^2$. [0.272 mm.]

1.9 (A). During a tensile test on a specimen the following results were obtained:

Load (kN)	15	30	40	50	55	60	65
Extension (mm)	0.05	0.094	0.127	0.157	1.778	2.79	3.81

Load (kN)	70	75	80	82	80		70
Extension (mm)	5.08	7.62	12.7	16.0	19.05		22.9

Diameter of gauge length = 19 mm   Gauge length = 100 mm
Diameter at fracture   = 16.49 mm   Gauge length at fracture = 121 mm

Plot the complete load extension graph and the straight line portion to an enlarged scale. Hence determine:

(a) the modulus of elasticity;   (d) the nominal stress at fracture;
(b) the percentage elongation;   (e) the actual stress at fracture;
(c) the percentage reduction in area;   (f) the tensile strength.
[116 GN/m²; 21 %; 24.7 %; 247 MN/m²; 328 MN/m²; 289 MN/m².]

1.10 Figure 1.24 shows a special spanner used to tighten screwed components. A torque is applied at the tommy-bar and is transmitted to the pins which engage into holes located into the end of a screwed component.
(a) Using the data given in Fig. 1.24 calculate:
   (i) the diameter *D* of the shank if the shear stress is not to exceed 50 N/mm²,
   (ii) the stress due to bending in the tommy-bar,
   (iii) the shear stress in the pins.
(b) Why is the tommy-bar a preferred method of applying the torque?
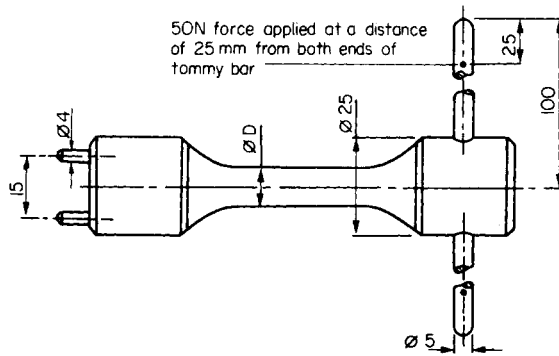[C.G.] [9.14 mm; 254.6 MN/m²; 39.8 MN/m².]

Fig. 1.24.

1.11 (a) A test piece is cut from a brass bar and subjected to a tensile test. With a load of 6.4 kN the test piece, of diameter 11.28 mm, extends by 0.04 mm over a gauge length of 50 mm. Determine:
(i) the stress, (ii) the strain, (iii) the modulus of elasticity.

(b) A spacer is turned from the same bar. The spacer has a diameter of 28 mm and a length of 250 mm, both measurements being made at 20°C. The temperature of the spacer is then increased to 100°C, the natural expansion being entirely prevented. Taking the coefficient of linear expansion to be $18 \times 10^{-6}/°C$ determine:
(i) the stress in the spacer, (ii) the compressive load on the spacer.

[C.G.] [64 MN/m^2, 0.0008, 80 GN/m^2, 115.2 MN/m^2, 71 kN.]

Bibliography

1. J. G. Tweedale, *Mechanical Properties of Metal*, George Allen & Unwin Ltd., 1964.
2. E. N. Simons, *The Testing of Metals*, David & Charles, Newton Abbot, 1972.
3. J. Y. Mann, *Fatigue of Materials – An Introductory Text*, Melbourne University Press, 1967.
4. P. G. Forrest, *Fatigue of Metals*, Pergamon, 1970.
5. R. B. Heywood, *Designing against Fatigue*, Chapman & Hall, 1962.
6. *Fatigue – An Interdisciplinary Approach*, 10th Sagamore Army Materials Research Conference Proceedings, Syracuse University Press, 1964.
7. A. J. Kennedy, *Processes of Creep and Fatigue in Metals*, Oliver & Boyd, Edinburgh and London, 1962.
8. R. K. Penny and D. L. Marriott, *Design for Creep*, McGraw-Hill (U.K.), 1971.
9. A. I. Smith and A. M. Nicolson, *Advances in Creep Design*, Applied Science Publishers, London, 1971.
10. J. F. Knott, *Fundamentals of Fracture Mechanics*, Butterworths, London, 1973.
11. H. Liebowitz, *Fracture – An Advanced Treatise*, vols. 1 to 7, Academic Press, New York and London, 1972.
12. W. D. Biggs, *The Brittle Fracture of Steel*, MacDonald & Evans Ltd., 1960.
13. D. Broek, *Elementary Engineering Fracture Mechanics*, Noordhoff International Publishing, Holland, 1974.
14. J. E. Gordon, *The New Science of Strong Materials*, Pelican 213, Penguin, 1970.
15. R. J. Roark and W. C. Young, *Formulas for Stress and Strain*, 5th Edition, McGraw-Hill, 1975.